H. T. I.

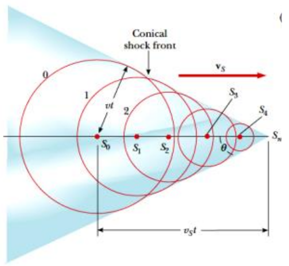
Publications of

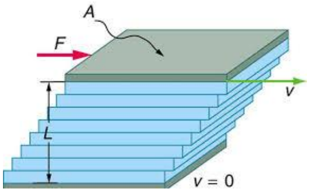
THE HIGHER TECHNOLOGICAL INSTITUTE – TENTH OF RAMADAN CITY

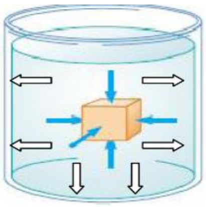
Course Code: PHY 001 **Course Name: Physics (1)**

Lecture notes in

ENGINEERING PHYSICS







Assist. Prof. Dr: Enas Hassan El-khawas **Assist. Prof. Dr: Ahmed A. Galahom**

Dr: Mohamed Ragab Abdou **Dr: Mustafa Muhammad Fadel**

2021

Preface

Physics is the branch of science that describes matter, energy, space and time at the most fundamental level. Whether you are planning to study engineering, biology, medicine, music, chemistry or art, some principles of physics are relevant to your field. Physicists look for patterns in the physical phenomena that occur in the universe. They try to explain what is happening and they perform experiments to see if the proposed explanation is valid. *The goal is to find the most basic laws that govern the universe and to formulate those laws in the most precise way possible.*

In this book, we will present a brief description of some of the physical materials scheduled for students of preparatory engineering to facilitate the process of retrieval and follow-up, as will be mentioned in ten chapters that included the properties of matter, waves and heat, which coincides with what the students studying in the global engineering colleges.

We also thank Dr. Saeed Abdullah for his efforts and active participation in the scientific content of this book in this manner.

At the conclusion of this humble work, we hope that all students will benefit and that this book will serve as a guide for them in this semester.

With our best wishes

Authors,

CHAPTER (1)
UNITS AND DIMENSIONS

1.1 Introduction

Science and engineering are based on measurements and comparisons. Thus, we need rules about how things are measured and compared, and we need experiments to establish the units for those measurements and comparisons. One purpose of physics (and engineering) is to design and conduct those experiments. For example, physicists strive to develop clocks of extreme accuracy so that any time or time interval can be precisely determined and compared. You may wonder whether such accuracy is actually needed or worth the effort. Here is one example of the worth: Without clocks of extreme accuracy, the Global Positioning System (GPS) that is now vital to worldwide navigation would be useless.

1.2 Units

There are three systems of units, International System (SI or MKS), Gaussian System (CGS) and British System. **Table** 1-1 illustrates the four units of basic quantities. A metric system of units has been used for many years in scientific work and in European countries. In 1960, the general conference of Weights and Measures, an international authority on units, proposed a revised metric system called the systéme International d' Unités in French (abbreviated SI). **Table** 1-2 illustrates the units of basic physical quantities.

Table 1-1: Different Systems of Units

Quantity	**Systems of Units**		
	SI or MKS System	**Gaussian or GCS System**	**British System**
Length	Meter (m)	Cm	Foot (ft)
Mass	Kg	Gm	slug
Time	Sec	Sec	Sec
Temperature	Kelvin (°K)	°K	Fahrenheit (°F)

Length

We can identify length as the distance between two points in space. In 1120, the king of England decreed that the standard of length in his country would be named the yard and would be precisely equal to the distance from the tip of his nose to the end of his outstretched arm. Similarly, the original standard for the foot adopted by the French was the length of the royal foot of King Louis XIV. Neither of these standards is constant in time; when a new king took the throne, length measurements changed! The French standard prevailed until 1799, when the legal standard of length in France became the meter (m), defined as one ten-millionth of the distance from the equator to the North Pole along one particular longitudinal line that passes through Paris. Notice that this value is an Earth-based standard that does not satisfy the requirement that it can be used throughout the Universe. As recently as 1960, the length of the meter was defined as the distance between two lines on a specific platinum–iridium bar stored under controlled conditions in France. Current requirements of science and technology, however, necessitate more accuracy than that with which the separation between the lines on the bar can be determined. In the $1960s$ and $1970s$, the meter was defined as 1 650 763.73 wavelengths1 of orange-red light emitted from a krypton-86 $lamp$. In October 1983, however, the meter was redefined as the distance traveled by light in vacuum during a time of 1/299 792 458 second. In effect, this latest definition establishes that the speed of light in vacuum is precisely 299 792 458 meters per second. This definition of the meter is valid throughout the Universe based on our assumption that light is the same everywhere.

Table 1-2: Basic Physical Quantities and Their Units

Quantity	Unit Name	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	K
Amount of substance	Mole	mol
Luminous intensity	Candela	Cd

Mass

The SI fundamental unit of mass, the kilogram (kg), is defined as the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France. This mass standard was established in 1887 and has not been changed since that time because platinum–iridium is an unusually stable alloy. A duplicate of the Sèvres cylinder is kept at the National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland.

Time

Before 1967, the standard of time was defined in terms of the mean solar day. (A solar day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day.) The fundamental unit of a second (s) was defined as $(1/60)\ (1/60)\ (1/24)$ of a mean solar day. This definition is based on the rotation of one planet, the Earth. Therefore, this motion does not provide a time standard that is universal.

In 1967, the second was redefined to take advantage of the high precision attainable in a device known as an atomic clock, which measures vibrations of cesium atoms. One second is now defined as $9\ 192\ 631\ 770$ times the period of vibration of radiation from the cesium-133 $atom$.

The SI system is based on powers of ten. Some of the most frequently used prefixes for various powers of ten and their abbreviation are listed in **table** 1-3.

Table 1-3: SI Prefixes and Symbols

Factor	Decimal Representation	Prefix	Symbol
10^{18}	1,000,000,000,000,000,000	exa	E
10^{15}	1,000,000,000,000,000	peta	P
10^{12}	1,000,000,000,000	tera	T
10^{9}	1,000,000,000	giga	G
10^{6}	1,000,000	mega	M
10^{3}	1,000	kilo	k
10^{2}	100	hecto	h
10^{1}	10	deka	da
10^{0}	1		
10^{-1}	0.1	deci	d
10^{-2}	0.01	centi	c
10^{-3}	0.001	milli	m
10^{-6}	0.000 001	micro	µ
10^{-9}	0.000 000 001	nano	n
10^{-12}	0.000 000 000 001	pico	p
10^{-15}	0.000 000 000 000 001	femto	f
10^{-18}	0.000 000 000 000 000 001	atto	a

1.3 Dimensional Analysis

In physics, the word dimension denotes the physical nature of a quantity. The distance between two points, for example, can be measured in feet, meters, or furlongs, which are all different ways of expressing the dimension of length. The quantities that are independent of other quantities are called fundamental quantities. The units that are used to measure these fundamental quantities are called fundamental units. And the quantities that are derived using the fundamental quantities are called derived quantities. The units that are used to measure these derived quantities are called derived units. The symbols we use in this book to specify the dimensions as illustrated in **table** 1-4.

Table 1-4: Basic Physical Quantities and Their Dimensions

Basic quantity	**Dimension**
Length	L
Mass	M
Time	T

Uses of Dimensional Analysis

- To establish the relationship between some related physical quantities.
- To find the dimensions of dimensional constants.
- To check the correctness of a physical relation/formula.

Limitations of dimensional analysis

1. Dimensionless quantities cannot be determined by this method. Constant of proportionality cannot be determined by this method. They can be found either by experiment (or) by theory.
2. This method is not applicable to trigonometric, logarithmic and exponential functions.
3. In the case of physical quantities which are dependent upon more than three physical quantities, this method will be difficult.

From these basic physical quantities, we can derive the dimensions of some Common Physical Quantities as illustrated in **table** 1-5.

Table 1-5: Most Common Physical Quantities and Their Dimensions

Physical quantity	Law	Dimension	Units
Area مساحه	Length * Width	L^2	m^2
Volume حجم	Length * Width* Height	L^3	m^3
Density كثافة	Mass/Volume	ML^{-3}	kg/ m^3
Linear density كثافة طولية	Mass/ Length	ML^{-1}	kg/ m
Velocity سرعة	Distance / Time	LT^{-1}	m/s
Acceleration عجلة	Velocity/ Time	LT^{-2}	m/s^2
Force = Weight = Tension الشـــد = الـوزن = القـوة	Mass * Acceleration	MLT^{-2}	kg. m/ s^2
Torque عزم الازدواج	Force * Distance	ML^2T^{-2}	kg. m^2/ s^2
Pressure الضغط	Force / Area	$ML^{-1}T^{-2}$	kg/ m s^2
Work or Energy الشغل أو الطاقة	Force * Distance	ML^2T^{-2}	kg. m^2/ s^2
Power القدرة	Energy / Time	ML^2T^{-3}	kg. m^2/ s^3
Intensity الشدة	Power / Area	MT^{-3}	kg/ s^3
Velocity gradient تدرج السرعة	Velocity / Distance	T^{-1}	s^{-1}
Volumetric rate معدل التدفق الحجمي	Volume / Time	L^3T^{-1}	m^3/s
Momentum كمية الحركة	Mass * Velocity	MLT^{-1}	kg. m/ s
Coefficient of viscosity معامل اللزوجة	$\eta = \frac{Force * Distance}{Area * Velocity}$	$ML^{-1}T^{-1}$	Kg/m s

Example (1.1):

A body is falling from rest under gravity. Find the relation between the **distance traveled** by the body and between the **gravitational acceleration** and **time elapsed**.

Solution:

$$d \ \alpha \ a^x \ t^y$$

$$d = k\, a^x\, t^y$$

$$[d] = [k]\,[a]^x [t]^y$$

$$L = (LT^{-2})^x\,(T)^y$$

$$L^1T^0 = L^x\,T^{y-2x}$$

$$x = 1 \quad and\; y = 2$$

$$\boldsymbol{d = k\, a\, t^2}$$

Example (1.2):

If the viscous force between any two layers in a viscous fluid is given by $\boldsymbol{F = \eta\, A\frac{dv}{dh}}$, where $\boldsymbol{\eta}$ is the coefficient of viscosity and ***dv/dh*** is the velocity gradient. Find the dimension of $\boldsymbol{\eta}$.

Solution:

$$F = \eta\, A\frac{dv}{dh}$$

$$\eta = \frac{F}{A}\frac{dh}{dv}$$

$$[\eta] = \frac{[F]}{[A]}\frac{[dh]}{[dv]}$$

$$[\eta] = \frac{(MLT^{-2})(L)}{(L^2)(LT^{-1})} = ML^{-1}T^{-1}$$

Example (1.3):

Show that $\boldsymbol{x - x_o = v_o t + \frac{1}{2}at^2}$ is dimensionally correct, where ***x*** and $\boldsymbol{x_o}$ are the final and initial distances, $\boldsymbol{v_o}$ is the velocity, ***a*** is the acceleration and ***t*** is the time.

Solution:

$$[L.H.S] = [x - x_o] = L$$

$$[R.H.S] = [v_o t + \frac{1}{2}at^2] = [v_o][t] + \left[\frac{1}{2}\right][a][t^2] = (LT^{-1}\,T) + (LT^{-2}\,)(T^2\,) = L$$

$$[L.H.S] = [R.H.S]$$

$x - x_o = v_o t + \frac{1}{2}at^2$ is dimensionally correct.

PROBLEMS

1. Newton's law of universal gravitation is represented by: $\left[\boldsymbol{F = G\frac{Mm}{r^2}}\right]$. where, $(\boldsymbol{F})$ is the magnitude of the gravitational force exerted by one small object on another, $(\boldsymbol{M})$ and $(\boldsymbol{m})$ are the masses of the objects, and $(\boldsymbol{r})$ is a distance. Find the dimension of the gravitational constant $(\boldsymbol{G})$.

2. Use dimensional analysis to determine how the linear speed of particle $(\boldsymbol{v})$ traveling in a circle depends on some or all of the following properties; the radius of the circle $(\boldsymbol{r})$, the angular frequency $(\boldsymbol{\omega})$ with which the particle orbits about the circle and the mass of particle $(\boldsymbol{m})$. There is no dimensionless constant involved in the relation.

3. The relationship between kinetic energy $(\boldsymbol{K})$ and momentum $(\boldsymbol{P})$ is $[\boldsymbol{K = P^2/2m}]$, where $(\boldsymbol{m})$ stands for mass. What is the SI unit of momentum?

4. Shows which of the following equations are dimensionally correct;

(a) $\boldsymbol{F_b = \rho V g}$,where $(\boldsymbol{F_b})$ is the buoyant force that is acting by a fluid of density $(\boldsymbol{\rho})$ on an object of submerged volume $(\boldsymbol{V})$ and $(\boldsymbol{g})$ is gravitational acceleration.

(b) $\boldsymbol{P = P_a + \rho g h}$,where $(\boldsymbol{P})$ is the pressure at a point placed at a depth $(\boldsymbol{h})$ from the surface of a fluid of density $(\boldsymbol{\rho})$, $(\boldsymbol{P_a})$ is the atmospheric pressure and $(\boldsymbol{g})$ is gravitational acceleration.

5. Verify the truth of the relation $\left[\boldsymbol{\frac{dV}{dt} = \frac{Pa^4}{\eta L}}\right]$, where $\boldsymbol{\frac{dV}{dt}}$ is the volumetric rate of flow of a liquid of viscosity coefficient $(\boldsymbol{\eta})$ flowing through a tube of radius $(\boldsymbol{a})$ and length $(\boldsymbol{L})$ due to pressure difference $(\boldsymbol{P})$.

6. Find the relation between the **velocities** of transverse waves produced from the vibration of thin homogeneous string and between the **tension** in the string and **mass per unit length** of it.

7. Obtain an expression for the viscous **force** acting on a sphere of **radius** $(\boldsymbol{r})$ moving in a viscous fluid with **velocity** $(\boldsymbol{v})$, if the **coefficient of viscosity** is $(\boldsymbol{\eta})$.

CHAPTER (2)
ELASTICITY

2.1 Introduction

In the study of mechanics thus far, we have assumed that objects remain un-deformed when external forces act on them. In reality, all objects are deformable. That is, it is possible to change the shape or the size of an object (or both) by applying external forces. As these changes take place, however, internal forces in the object resist the deformation. In this case the object return to its original shape or size after the external force is removed and the object is called elastic. Therefore, elasticity is defined as *the power of substance to regain its original shape after deformation.*

We shall discuss the deformation of solids in terms of the concepts of *stress* and *strain*. Stress is *a quantity that is proportional to the force causing a deformation*; more specifically, stress is the *external force acting on an object per unit cross-sectional area.* The result of a stress is strain, which is a *measure of the degree of deformation*. It is found that, for sufficiently small stresses, stress is proportional to strain; the constant of proportionality depends on the material being deformed and on the nature of the deformation. We call this proportionality constant the elastic modulus. The elastic modulus is therefore defined as the ratio of the stress to the resulting strain:

$$Elastic\ modulus = \frac{stress}{strain} \qquad (2-1)$$

The elastic modulus in general relates what is done to a solid object (a force is applied) to how that object responds (it deforms to some extent). It is similar to the spring constant *k* in Hooke's law that relates a force applied to a spring and the resultant deformation of the spring, measured by its extension or compression.

Three elastic moduli are used to describe the elastic behavior (deformations) of objects as they respond to forces that act on them.

2.2 Tensile stress or compressive stress

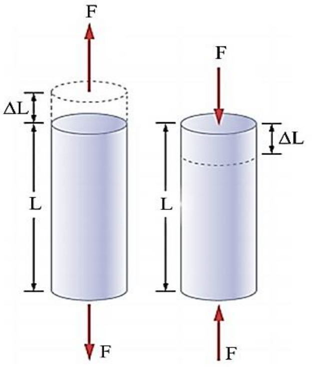
For simple tension or compression, as shown in figure 2.1 (a, b) respectively, the stress on a long rod is defined as:

$$Tensile\ stress\ = \frac{F}{A} \qquad "N\ /m^2"$$

where (F) is the magnitude of the force applied perpendicularly to an area (A) on the rod. The strain, or unit deformation, is then the dimensionless quantity and defined as:

$$Tensile\ strain = \frac{\Delta L}{L} \qquad \text{"}unitless\text{"}$$

where, ΔL is the change in a length of the specimen.



(a) (b)

Figure 2.1: Tensile and compressive stress and strain

If the stress does not exceed the yield strength, the modulus for tensile and compressive stresses is called the Young's modulus and is represented in engineering practice by the symbol Y, which defined as *the force (tension or compression) normal to unit area over the change of length per the original length.* Then, equation (2-1) becomes

$$Y = \frac{F/_A}{\Delta L/_L} \qquad \text{"}N/m^2\text{"} \qquad (2-2)$$

The strain $\left(\Delta L/_L\right)$ in a specimen can often be measured conveniently with a strain gage, which can be attached directly to operating machinery with an adhesive. Its electrical properties are dependent on the strain it undergoes.

Although the Young's modulus for an object may be almost the same for tension and compression, the object's ultimate strength may well be different for the two types of stress. Concrete, for example, is very strong in compression but is so weak in tension that it is almost never used in that manner. **Table** 2-1 shows the Young's modulus and other elastic properties for some materials of engineering interest.

Example (2.1):

Steel wire has a length of $\mathbf{5\ m}$ and cross – sectional area $\mathbf{0.12\ cm^2}$. A compressive force acted on the wire reduces its length by **3**% from its original length find: the stress, the force and the contraction, ($\boldsymbol{Y = 2 \times 10^{11}\ N/m^2}$).

Solution:

$$stress = Y\ strain = (2 \times 10^{11}\ N/m^2)(0.03) = 6 \times 10^9\ N/m^2$$

$$F = stress \times A = (6 \times 10^9\ \ N/m^2)(0.12 \times 10^{-4}\ m^2) = 7.2 \times 10^4\ N$$

$$\Delta L = L \times strain = (500 \times 10^{-2}\ m)(0.03) = 0.15\ m$$

Example (2.2):

In an experiment to measure Young's modules, a load of $\mathbf{500\ kg}$ hanging from a steel wire of length $\mathbf{3\ m}$ and cross section $\mathbf{0.2\ cm^2}$, was found to stretch the wire $\mathbf{0.4\ cm}$ above its no-load length. What were the stress, the strain, and the value of Young's modulus for the steel of which the wire was composed?

Solution:

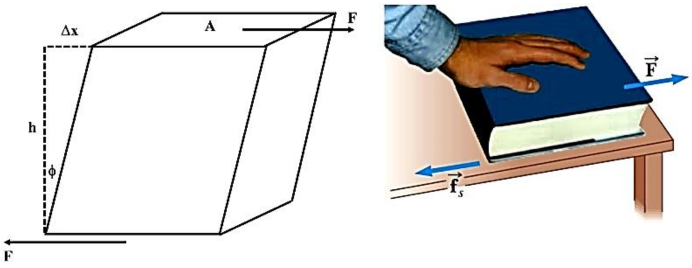
$$Stress = \frac{F}{A} = \frac{(500\ kg)(9.81\ m/s^2)}{(0.2 \times 10^{-4}\ m^2)} = 2.45 \times 10^8\ \ N/m^2$$

$$Strain = \frac{\Delta L}{L} = \frac{(0.4 \times 10^{-2}\ m)}{(3\ m)} = 0.00133$$

$$Y = \frac{Stress}{Strain} = \frac{(2.45 \times 10^8\ \ N/m^2)}{(0.00133)} = 1.84 \times 10^{11}\ \ N/m^2$$

2.3 Surface or Shear Deformation (Elasticity shape)

Another type of deformation occurs when an object is subjected to a force tangential to one of its faces while the opposite face is held fixed by another force (figure 2.2). The stress in this case is called a shear stress. If the object is originally a rectangular block, a shear stress results in a shape whose cross-section is a parallelogram. Book pushed sideways, as shown in figure 2.2b, is an example of an object subjected to a shear stress. To a first approximation (for small distortions), no change in volume occurs with this deformation.



(a) (b)

Figure 2.2: Shear Elasticity

Shear stress is defined as the *ratio of the tangential force* (F) *to the area* A *of the face being sheared*

$$Shear\ Stress\ = \frac{F}{A} \qquad "N\ /m^2"$$

Shear strain, or unit deformation, is then is defined as:

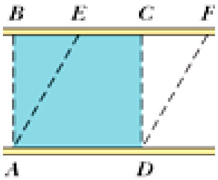
$$Shear\ Streain\ = \frac{\Delta x}{h} = tan\,\varphi \simeq \varphi \quad "for\ small\ \theta"$$

where Δx is the horizontal distance that the sheared face moves and h is the height of the object.

The elastic modulus in this case is called Shear modulus or rigidity modulus (S) and can be written as:

$$S = \frac{F\ /A}{\tan\varphi} = \frac{Shear\ Stress}{Shear\ Strain} \qquad "N\ /m^2" \qquad (2-3)$$

Example (2.3):



Suppose the object in the figure is a brass plate $\mathbf{1\ m}$ square and $\mathbf{0.5\ cm}$ thick. If the displacement CF is $\mathbf{0.02\ cm}$, how large a force F must be exerted on each of its edges? (The shear modulus of brass is $\mathbf{0.36\times\ 10^{11} Pa}$.)

Solution:

The shear stress of each edge is:

$$Shear\ Stress\ = \frac{F}{A} = \frac{F}{(1\ m)(0.005\ m)} = (200\ m^{-2})F$$

$$Shear\ Strain\ = \frac{\Delta x}{h} = \frac{(2\times 10^{-4}\ m)}{(1\ m)} = 2\times 10^{-4}$$

$$S = \frac{Shear\ Stress}{Shear\ Strain}$$

$$(0.36\times 10^{11}\ N/m^2) = \frac{(200\ m^{-2})F}{(2\times 10^{-4}\ m)}$$

$$F\ = 3.6\times 10^4\ N$$

2.4 Volume Elasticity (volume change)

In this type of elasticity, we characterize the response of a substance to uniform squeezing or to a reduction in pressure when the object is placed in a partial vacuum. Suppose that the external forces acting on an object are at right angles to all its faces, as shown in figure 2.3, and that they are distributed uniformly over all the faces. Such a uniform distribution of forces occurs when an object is immersed in a fluid. An object subject to this type of deformation undergoes a change in volume but no change in shape.

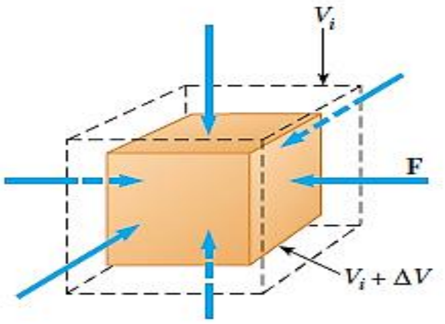


Figure 2.3: Volume elasticity

Volume stress is defined as *the ratio of the magnitude of the change of the normal force* ΔF *to the area* A *of the object.*

$$Volume\ Stress\ = \frac{\Delta F}{A} = \Delta P \qquad "N\ /m^2"$$

The object will experience a volume change. The volume strain is equal to *the change in volume* ΔV *divided by the initial volume* V_i

$$Volume\ Strain\ = \frac{\Delta V}{V_i} \qquad "dimensionless"$$

Thus, from equation (2-1), we can characterize a volume ("bulk") compression in terms of the Bulk modulus (B), which is defined as *the ratio of the volume stress to volume strain.*

$$Bulk\ modulus\ (B) = \frac{Volume\ Stress}{Volume\ Strain} = -\frac{\Delta F/A}{\Delta V/V_i} = -\frac{\Delta P}{\Delta V/V_i} \qquad "N\ /m^2" \qquad (2-4)$$

Example (2.4):

A solid has a volume of $\mathbf{2.5}\ \boldsymbol{liters}$ when the external pressure is $\mathbf{1}\ \boldsymbol{atm}$. (a) What is the change in the volume when the body is subjected to pressure of $\mathbf{16}\ \boldsymbol{atm}$. ($\mathbf{1}\ \boldsymbol{atm} = \mathbf{1.013 \times 10^5}\ \boldsymbol{N/m^2}, \boldsymbol{B} = \mathbf{2 \times 10^{11}}\ \boldsymbol{N/m^2}$)?

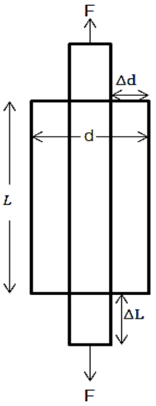
Solution:

$$\Delta V = -\frac{\Delta P\ V}{B} = \frac{(15 \times 1.013 \times 10^5\ N/m^2)(2.5 \times 10^{-3}\ m^3)}{(\ 2 \times 10^{11}\ N/m^2)} = -1.9 \times 10^{-8}\ m^3$$

Table 2-1: Typical Values for Elastic Modulus

Substance	Young's modulus (N/m²)	Shear modulus (N/m²)	Bulk modulus (N/m²)
Tungsten	$35\ \times 10^{10}$	$14\ \times 10^{10}$	$20\ \times 10^{10}$
Steel	$20\ \times 10^{10}$	$8.4\ \times 10^{10}$	$6\ \times 10^{10}$
Copper	$11\ \times 10^{10}$	$4.2\ \times 10^{10}$	$14\ \times 10^{10}$
Brass	$9.1\ \times 10^{10}$	$3.5\ \times 10^{10}$	$6.1\ \times 10^{10}$
Aluminum	$7\ \times 10^{10}$	$2.5\ \times 10^{10}$	$7\ \times 10^{10}$
Glass	$6.5 - 7.8\ \times 10^{10}$	$2.6 - 3.2\ \times 10^{10}$	$5 - 5.5\ \times 10^{10}$
Quartz	$5.6\ \times 10^{10}$	$2.6\ \times 10^{10}$	$2.7\ \times 10^{10}$
Water	-	-	$0.21\ \times 10^{10}$
Mercury	-	-	$2.8\ \times 10^{10}$

2.5 Poisson's ratio



When a sample of material is stretched in one direction it tends to get thinner in the lateral direction - and if a sample is compressed in one direction it tends to get thicker in the lateral direction.

The ratio of the relative contraction strain (transverse, lateral or radial strain) normal to the applied load - to the relative extension strain (or axial strain) in the direction of the applied load

$$Poisson's\ ratio, \sigma = \frac{Lateral\ strain}{longitudinal\ (Tensile) strain} = \frac{\Delta r/r}{\Delta L/L} \quad "dimensionless" \qquad (2-5)$$

$$\text{where,} \qquad \frac{\Delta r}{r} = \frac{\Delta d}{d} = \frac{Change\ of\ radius\ (diameter)}{Original\ radius\ (diameter)}$$

Example (2.5):

A nylon rope used by maintainers elongates $\mathbf{1.5\ m}$ under the weight of an $\mathbf{80\ kg}$ Climber. a) If the rope is $\mathbf{50\ m}$ in length and $\mathbf{9\ mm}$ in diameter find the elastic modulus ($\boldsymbol{Y}$)? b) If Poisson's ratio for nylon is $\mathbf{0.2}$, find the change in diameter under this stress?

Solution:

$$Y = \frac{FL}{A\Delta L} = \frac{FL}{\pi r^2 \Delta L} = \frac{(80 \times 9.81\ N)(50\ m)}{\pi (4.5 \times 10^{-3}\ m)^2 (1.5\ m)} = 4.11 \times 10^8\ N/m^2$$

$$\boldsymbol{Poisson'sratio}, \sigma = \frac{\Delta d/d}{\Delta L/L} \Longrightarrow \Delta d = \frac{\sigma d\ \Delta L}{L} = \frac{(0.2)(9 \times 10^{-3}\ m)(1.5\ m)}{(50\ m)} = 5.4 \times 10^5\ m$$

2.6 Energy Stored in elastic body (work done)

Suppose that a wire has original length (L_i) and is stretched by a length (ΔL) when a force (F) is applied at one end. If the elastic limit is not exceeded (the extension is directly proportional to the applied force

$$\mathrm{W} = \int_{x=0}^{x=\Delta L} Kx\, dx = K \int_{x=0}^{x=\Delta L} x\, dx = K \left[\frac{x^2}{2}\right]_0^{\Delta L} = \frac{1}{2} K \Delta L^2 = \frac{1}{2}(K\Delta L)(\Delta L) = \frac{1}{2} F \Delta L$$

This work is stored in the wire as elastic potential energy (U)

$$\text{elastic potential energy}, U = \frac{1}{2} F \Delta L$$

Multiply right hand side by $\frac{AL}{AL}$,we get

$$U = \frac{1}{2}\left[\frac{F}{A}\frac{\Delta L}{L}\ AL\right]$$

$$Stress = \frac{F}{A}, Strain = \frac{\Delta L}{L}$$

$$U = \frac{1}{2}[Stress \times Strain \times AL]$$

$$U = \frac{1}{2}[Stress \times Strain \times Volume] \qquad "Joule" \qquad (2-6)$$

And also elastic potential energy density (u) is defined as the elastic potential energy per unit volume, and can be written as

$$u = \frac{U}{Volume} = \frac{1}{2}[Stress \times Strain] \qquad "Joule/m^3" \qquad (2-7)$$

2.7 Hook's law

The stress is directly proportional to the strain within the elastic limit.

$$F\ \alpha\ \Delta L$$

or

$$F\ =\ k\,\Delta L \qquad (2-8)$$

where, (K) is the force or elastic constant.

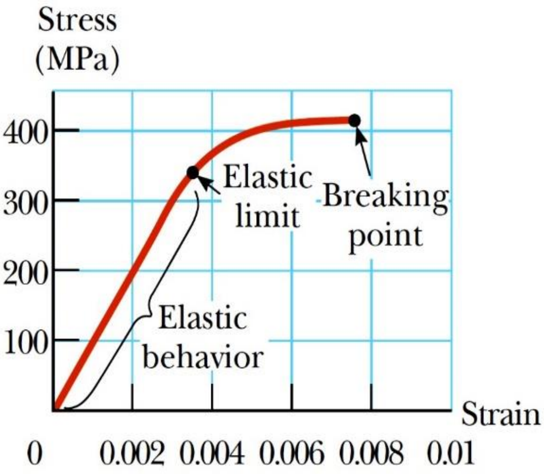


Figure 2.4: The stress – strain curve

The stress – strain is shown in figure 2.4. Initially, a stress-versus-strain curve is a straight line and the substance returns to its initial length, shape and volume when the force is removed. It is possible to exceed the elastic limit of a substance by applying a sufficiently large stress as seen in figure 2.3. ***The elastic limit of a substance is defined as the maximum stress that can be applied to the substance before it becomes permanently deformed and does not return to its initial length***. As the stress increases, however, the curve is no longer a straight line. When the stress exceeds the elastic limit, the object is permanently distorted and does not return to its original shape after the stress is removed. As the stress is increased even further, the material ultimately breaks. *Breaking stress: (Ultimate strength): is the maximum stress at which the material breaks. Or: The stress required to cause actual fracture of a material.* Elastic fatigue: As a result of exerting stress on the material several times and for long time, *the material loss part of its elasticity and does not regain its original shape.*

PROBLEMS

1. If Young's modulus for steel $\boldsymbol{Y = 1.9 \times 10^{12}\ D/cm^2}$, how much force will be required to stretch a sample of wire $\boldsymbol{1\ mm^2}$ in cross – section by $\boldsymbol{10}$% of its original length?

2. A copper wire has a length of $\boldsymbol{150\ cm}$ and radius of $\boldsymbol{r = 1.25\ mm}$ fixed at its upper end and a weight of $\boldsymbol{8\ kg}$ is attached to its lower end. Find the stress and strain. ($\boldsymbol{Y = 1.2 \times 10^{12}\ D/cm^2}$).

3. A steel wire has a length of $\boldsymbol{5\ m}$ and diameter of $\boldsymbol{0.72\ mm}$ fixed at its upper end, and a mass of $\boldsymbol{6\ kg}$ is attached to its lower end, the density of the mass is **7.5 g/cm^3** this mass is immersed in liquid has a density of $\boldsymbol{1.2\ g/cm^3}$. Find the extension ($\Delta \boldsymbol{L}$). ($\boldsymbol{Y = 2 \times 10^{12}\ D/cm^2}$ *and* $\boldsymbol{g = 980\ cm/s^2}$).

4. Two wires are made of the same material, but wire ($\boldsymbol{A}$) is twice as long and has twice the diameter of wire ($\boldsymbol{B}$). Find the elongation of wire ($\boldsymbol{B}$) relative to that of wire ($\boldsymbol{A}$) when both are subjected to the same load?

5. Calculate the elastic energy stored per unit volume of steel wire of cross-sectional area $\boldsymbol{4\ mm^2}$ when it is stretched by a force of $\boldsymbol{98\ N}$ within the elastic limit.

6. A cube of aluminum $\boldsymbol{4\ cm}$ length is subjected to a shearing force of $\boldsymbol{100\ kg}$. The top face of the cube is displaced $\boldsymbol{0.012\ cm}$ with respect to the bottom; calculate the shearing stress, the shearing strain, and the shearing modulus?

7. A solid copper sphere of $\boldsymbol{0.5\ m^3}$ volume is placed $\boldsymbol{100\ ft}$ below the ocean surface where the pressure is $\boldsymbol{3 \times 10^5\ N/m^2}$. What is the change in volume of the sphere? The bulk modulus for copper is $\boldsymbol{14 \times 10^{10}\ N/m^2}$.

8. A solid has a volume of $\boldsymbol{2.5\ liters}$ when the external pressure is $\boldsymbol{1\ atm}$. The bulk modulus of the material is $\boldsymbol{2 \times 10^{12}\ D/cm^2}$. What is the change in volume when the body is subjected to a pressure of $\boldsymbol{16\ atm}$? What additional energy per unit volume is now stored in the material?

9. Determine the fractional change in volume as the pressure of the atmosphere $\boldsymbol{10^5\ Pa}$ around a metal block is reduced to zero by placing the block in vacuum. (Bulk modulus for the metal is $\boldsymbol{125\ GPa}$).

CHAPTER (3)
FLUID STATICS

3.1 Introduction

In everyday life, we recognize three states of matter: Solid, Liquid and Gas. Although they different in many respects, liquids and gases have a common characteristic in which they differ from solids: they are fluids, lacking the ability of solids to offer a permanent resistance to a deforming force. The term "fluid statics" is applied to *the study of fluids at rest.*

3.2 Density

The density (ρ) (Greek letter rho) of any substance is defined as *its mass per unit volume.* The SI unit of density is one kilogram per cubic meter (kg/m^3) while the cgs unit is one gram per cubic centimeter (g/cm^3). The density value can be calculated using equation (3-1). **Table** 3-1 displays typical values of density at room temperature.

$$\rho = \frac{m}{V} \qquad (3-1)$$

The conversion of the density unit from cgs system to SI system can be done as illustrated below.

$$1\ g/cm^3 = 1000\ kg/m^3$$

Table 3-1: Densities of some selected materials

Material	**Density (g/cm^3)**	**Material**	**Density (g/cm^3)**
Air	0.00129	Helium gas	0.179
Aluminum	2.7	Lead	11.3
Brass	8.6	Silver	10.5
Copper	8.9	Steel	7.8
Gold	19.3	Mercury	13.6
Ice	0.92	Water	1.0
Iron	7.8	Sea water	1.03

Example (3.1):

Calculate the mass of a solid iron sphere that has a diameter of $\boldsymbol{3\ cm}$ if the density of iron is $\boldsymbol{7.8\ g/cm^3}$.

Solution:

$$M = \rho V = \rho\left(\frac{4}{3}\pi r^3\right) = (7800kg/m^3)\left(\frac{4}{3}\pi\left(\frac{3\times10^{-2}}{2}m\right)^3\right) = 0.011\ kg$$

3.3 Pressure in a fluid

A static fluid exerts a force on any surface with which it comes in contact. The direction of the force is perpendicular to the surface as illustrated in figure 3.1. The average pressure of fluid at points on a planar surface is given by equation (3-2).

$$P_{av} = \frac{F}{A} \qquad (3-2)$$

The SI unit for pressure is the newton per square meter (N/m^2), which is named Pascal (Pa). Another commonly used unit of pressure is atmosphere (atm). One atmosphere is the average air pressure at sea level. The conversion between atmosphere and pascal is:

$$\boldsymbol{1\ atm = 1.013\times10^5\ Pa}$$

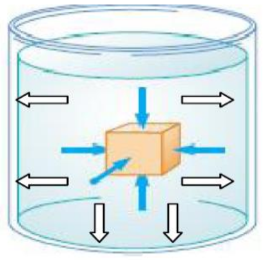


Figure 3.1: Forces due to a static fluid acting on the walls of the container and on a submerged object.

Example (3.2):

A $\boldsymbol{50-kg}$ woman balances on one heel of a pair of high heeled shoes. If the heel is circular and has a radius of $\boldsymbol{0.5\ cm}$, what pressure does she exert on the floor?

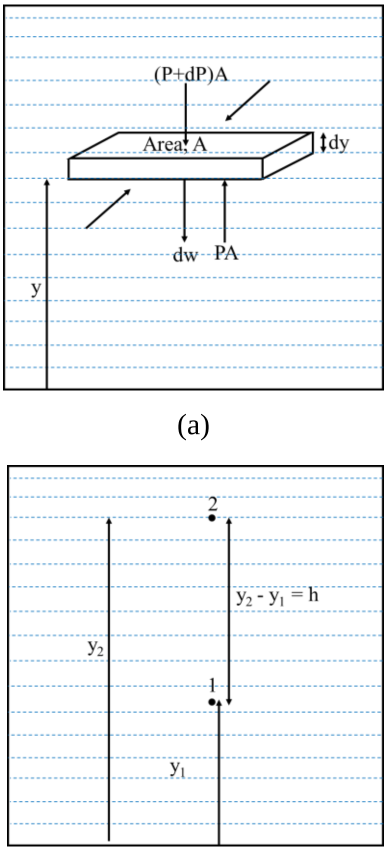
Solution:

$$P = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(50\ kg)(9.81m/s^2)}{\pi(0.5\times10^{-2}\ m)^2} = 6.24\times10^6\ N/m^2$$

3.3.1 Atmospheric pressure

On the surface of the earth, we live at the bottom of an ocean of fluid called air. The force exerted by air on our bodies and on the surfaces of other objects may be surprisingly large. The atmospheric pressure is greater at sea level than at mountain altitudes while the pressure in lake or in the ocean increases with increasing depth below the surface.

3.3.2 Variation of Pressure with Depth



(a)

(b)

Figure 3.2: Forces on a slab in equilibrium state

Consider an element in the form of a slab; shown in figure 3.2a whose thickness (dy) and its faces have an area (A). If (ρ) is the density of the fluid, the mass of the element is $(\rho A dy)$ and its weight dw is $(\rho g A dy)$. The force exerted on the element by the surrounding fluid is everywhere normal to its surface. The resultant horizontal force on its vertical sides is zero. The upward force on its lower face is (PA), and the downward force on its upper face is $(P + dP)A$. since it is in equilibrium,

$$\sum F_y = 0$$

$$(P + dP)\,A + \, dw = P\,A$$

$$(P + dP)\,A + \, \rho g A dy = P\,A$$

$$P\,A + dP\,A + \, \rho g A dy = P\,A$$

$$dP\,A \; = -\rho g A dy$$

$$dP \; = -\rho g dy$$

$$\frac{dP}{dy} = -\rho g$$

This means that an increase of elevation is accompanied by a decrease in pressure.

We can calculate **the pressure at a point in a fluid** as the following: *If* (P_1) *and* (P_2) *are the pressures at elevations* (y_1) *and* (y_2) *as shown in figure 3.2b, then:*

$$dP \; = -\rho g dy$$

$$\int_{P_1}^{P_2} dP = -\rho g \int_{y_1}^{y_2} dy$$

$$P_2 - P_1 \; = -\rho g (y_2 - y_1)$$

Let (P) *represent the pressure at point* (1) *and* (P_a) *represent the pressure at point* (2)*, then*

$$P \; = P_a + \rho g h \qquad (3-3)$$

Example (3.3):

(a) Calculate the absolute pressure at an ocean depth of $\mathbf{1000\ m}$. Assume the density of seawater is $\mathbf{1024\ kg/m^3}$ and that the air above exerts a pressure of $\mathbf{101.3\ kPa}$. **(b)** At this depth, what force must the frame around a circular submarine porthole having a diameter of $\mathbf{30\ cm}$ exert to counterbalance the force exerted by the water?

Solution:

$$(\boldsymbol{a})\ p\ = p_a + \rho g h = (101.3 \times 10^3\ \ N/m^2) + (1024\ kg/m^3)(9.81\ m/s^2)(1000\ m)$$

$$= 1.01 \times 10^7 N/m^2$$

$$P_{gauge} = p - p_a = (1.01 \times 10^7 N/m^2) - (1 \times 10^5\ N/m^2) = 1 \times 10^7 N/m^2$$

$$(\boldsymbol{b})\ F = P_{gauge} A = P_{gauge}(\pi r^2) = (1 \times 10^7 N/m^2)(\pi\ (15 \times 10^{-2}\ m)^2) = 7.1 \times 10^5\ N$$

Pascal's law: *a change in the pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and to the walls of the container.*

Because the pressure in a fluid depends on depth and on the value of (P), any increase in pressure at the surface must be transmitted to every other point in the fluid. An important application of Pascal's law is the hydraulic press illustrated in figure 3.3. The pressure is transmitted through an incompressible liquid to a larger piston of surface area (A_2). Because the pressure must be the same on both sides, $(P = F_1/A_1 = F_2/A_2)$. Therefore, the force (F_2) is greater than the force (F_1) by a factor of (A_2/A_1). By designing a hydraulic press with appropriate areas A_1 and A_2, a large output force can be applied by means of a small input force. *Hydraulic brakes*, *car lifts*, *hydraulic jacks*, and *forklifts* all make use of this principle.

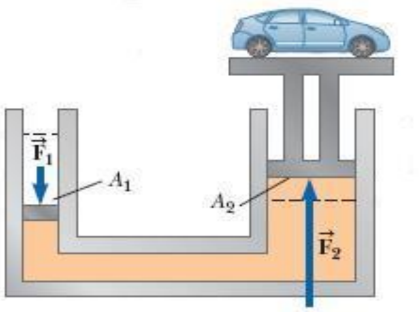


Figure 3.3: Diagram of hydraulic press

Example (3.4):

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section and a radius of $\boldsymbol{5\ cm}$. This pressure is transmitted by a liquid to a piston that has a radius of $\boldsymbol{15\ cm}$. What force must the compressed air exert to lift a car weighing $\boldsymbol{13300\ N}$? What air pressure produces this force?

Solution:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_1 = \left(\frac{A_1}{A_2}\right) F_2 = \left(\frac{\pi(5\ \times 10^{-2}\ m)^2}{\pi(15 \times 10^{-2}\ m)^2}\right)(13300\ N) = 1477.8\ N$$

$$P = \frac{F_1}{A_1} = \frac{(1477.8\ N)}{\pi\ (5\ \times 10^{-2}\ m)^2} = 0.0019\ N/m^2$$

3.3.3 Pressure Measurements

A ***barometer*** is a device for measuring atmospheric pressure. A simple barometer consists of a tube more than $30\ inch$ $(76\ cm)$ long inserted in an open container of mercury with a closed and evacuated end at the top and open tube end at the bottom as shown in figure 3.4. The closed end of the tube is nearly a vacuum, so the pressure at the top of the mercury column can be taken as zero. The pressure at point A, due to the column of mercury, must equal the pressure at point B, due to the atmosphere. If that were not the case, there would be a net force that would move mercury from one point to the other until equilibrium is established. Therefore,

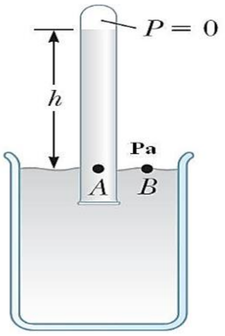


Figure 3.4: Barometer

$$P_A = P_B$$

$$\rho_{Hg} g h = P_a$$

$$P_a = 13600 \times 9.8 \times 0.76 = 1.013 \times 10^5\ N/m^2$$

where (ρ_{Hg}) is the density of the mercury and (h) is the height of the mercury column. One atmosphere of pressure is defined to be the pressure equivalent of a column of mercury that is exactly $(76\ cm)$ in height at $(0\ ^\circ C)$.

A device for measuring the pressure of a gas contained in a vessel is the open tube ***manometer*** illustrated in figure 3.5. A manometer consists of a U-tube. One end of a U-shaped tube contains one or more fluids such as mercury, water, alcohol, or oil and the other end is connected to a container of gas at pressure (P). Heavy fluids such as mercury are used if large pressure differences are anticipated. In an equilibrium situation, the pressures at points (A) and (B) must be the same. Therefore, the pressure (P) can be calculated using equation (3-3). The pressure (P) is called the absolute pressure, and the difference $(P - P_a)$ is called the ***gauge pressure.***

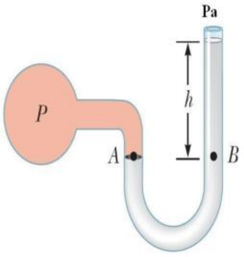


Figure 3.5: The open tube manometer.

$$P_A = P_B$$

$$P = P_a + \rho g h$$

<u>**Example (3.5):**</u>

Blaise Pascal duplicated Torricelli's barometer using a red Bordeaux wine, of density $\mathbf{984\ kg/m^3}$, as the working liquid. What was the height h of the wine column for normal atmospheric pressure? Would you expect the vacuum above the column to be as good as for mercury?

Solution:

$$P = \rho g h \Rightarrow h = \frac{P}{\rho g} = \frac{(1.013 \times 10^5\ N/m^2)}{(984\ kg/m^3) \times (9.81\ m/s^2)} = 10.5\ m$$

No. Some alcohol and water will evaporate. The equilibrium vapor pressures of alcohol and water are higher than the vapor pressure of mercury.

3.4 Archimedes' Principle

Buoyancy is a familiar phenomenon; a body immersed in water seems to have less weight than when immersed in air. When an object is immersed in a fluid, the pressure on the lower surface of the object is higher than the pressure on the upper surface. The difference in the pressures leads to an upward net force acting on the object due to the fluid pressure. If you try to push a beach ball underwater as illustrated in figure 3.6, you feel the effects of the buoyant force pushing the ball back up.

Figure 3.6: A swimmer pushes a beach ball under water.

Archimedes' principle: *A fluid exerts an upward buoyant force on a submerged object equal in magnitude to the weight of the volume of the fluid displaced by the object. It acts upward through the centroid of the displaced volume.* (F_B = W)

There are three possible cases as illustrated in figure 3.7;

1. $\rho_{body} < \rho\ fluid$: Floating body
2. $\rho_{body} = \rho_{fluid}$: Neutrally buoyant
3. $\rho_{body} > \rho\ fluid$: Sinking Body

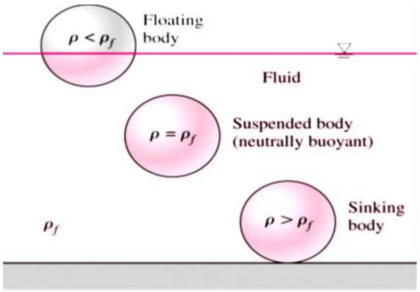


Figure 3.7: Cases of a body when immersed in a fluid.

Consider a rectangular solid immersed in a fluid of uniform density (ρ) as illustrated in figure 3.8. Let the top and the bottom surfaces have area (A). The force on the lower face of the block is ($F_2 = P_2 A$) and the force on the upper face is ($F_1 = P_1 A$). The total force on the block due to the fluid called the buoyant force (F_B).

$$F_B = (P_2 - P_1)A$$

Since, ($P_2 - P_1 = \rho g d$), the magnitude of the buoyant force can be written as:

$$F_B = \rho g d A = \rho g V$$

where, ($V = Ad$) is the volume of the block.

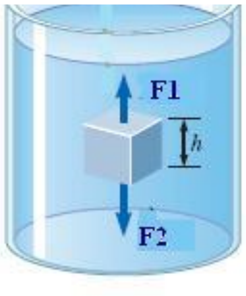


Figure 3.8: Forces due to fluid pressure on the top and bottom of an immersed rectangular solid.

Example (3.6):

A Ping-Pong ball has a diameter of $\mathbf{3.80\ cm}$ and average density of $\mathbf{0.084\ g/cm^3}$. What force is required to hold it completely submerged under water?

Solution:

$$\sum F = 0$$

$$F_{app} + mg = F_B$$

$$F_{app} = F_B - mg = \rho_w\, V\, g - \rho_{ball}\, V\, g = Vg(\rho_w - \rho_{ball})$$

$$= \left(\frac{4}{3}\,\pi\, r^3\right)(g)(\rho_w - \rho_{ball}) = (4/3\,\pi\,(1.9\times10^{-2}\ m)^3)(9.81\ m/s^2)(1000\ kg/m^3 - 84\ kg/m^3) = 0.258\ N$$

3.5 Forces Against a Dam

Suppose water stands at a depth (H) behind the vertical upstream face of a dam as illustrated in figure 3.9. Water exerts a certain resultant horizontal force on the dam, tending to slide it along its foundation, and exerts a certain torque, to overturn in front of the dam about the point (O). we wish to find the total horizontal force. The width of the dam is (L).

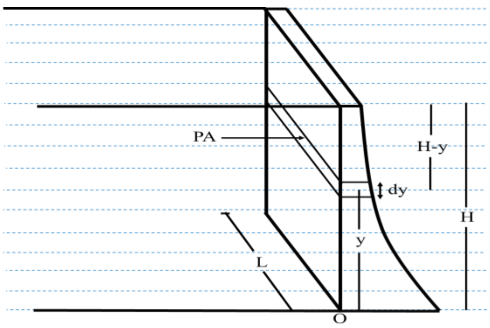


Figure 3.9: Water is filled behind a

The pressure at a certain elevation is

$$p = \rho\, g\,(H - y)$$

Atmospheric pressure can be **omitted**, since it also acts upstream against the other face of the dam. The force (dF) against the shaded strip is **F= PA= P Ldy**

$$\boldsymbol{F} = \int dF = \int_0^H \rho g L\,(H - y)dy = \rho g L \int_0^H (H - y)dy$$

$$F = \rho g L[\int_0^H H dy - \int_0^H y dy] = \rho g L[H\int_0^H dy - \int_0^H y dy]$$

$$F = \rho g L\left[H(y)_0^H - \left(\frac{y^2}{2}\right)_0^H\right] = \rho g L\left[H(H-0) - \left(\frac{H^2}{2} - \frac{0}{2}\right)\right] = \rho g L\left[H(H) - \frac{H^2}{2}\right] = \rho g L\left[H^2 - \frac{H^2}{2}\right]$$

$$= \rho g L\frac{H^2}{2}$$

and the total force is

$$\boldsymbol{F} = \int dF = \int_0^H \rho g L\,(H-y)dy = \rho g L\left[\frac{(y-H)^2}{2}\right]_0^H = \rho g L\left(\frac{(H-H)^2}{2} - \frac{(0-H)^2}{2}\right)$$

$$= \rho g L\left(0 + \frac{H^2}{2}\right) = \frac{1}{2}\rho g L H^2$$

The moment $(d\Gamma)$ *of the force* (dF) *about an axis through* (O) *is*

*عزم/الازدواج Torque = moment =Γ= F.y القوة * ذراعها*

$$d\Gamma = dF.y = \rho g L y\,(H-y)dy$$

The total torque about (O) is

$$\Gamma = \int d\Gamma = \int_0^H \rho g L y\,(H-y)\,dy = \int_0^H \rho g L y H\,dy - \int_0^H \rho g L y^2\,dy$$

$$= \rho g L H\int_0^H y\,dy - \rho g L\int_0^H y^2\,dy = \rho g L H\left[\frac{y^2}{2}\right]_0^H - \rho g L\left[\frac{y^3}{3}\right]_0^H$$

$$= \rho g L H\left(\frac{H^2}{2} - \frac{0}{2}\right) - \rho g L\left(\frac{H^3}{3} - \frac{0}{3}\right) = \rho g L\frac{H^3}{2} - \rho g L\frac{H^3}{3} = \boldsymbol{\frac{1}{6}\,\rho g L H^3}$$

$$\Gamma = F\,\hat{\text{H}}$$

$$\frac{1}{6}\rho g L\,H^3 = \frac{1}{2}\rho g L H^2 \times \hat{\text{H}}$$

$$\hat{\text{H}} = \boldsymbol{\frac{1}{3}\,H}$$

where, $(\hat{\text{H}})$ *is the height above* (O) *at which the total force* (F) *would have to act to produce this torque*

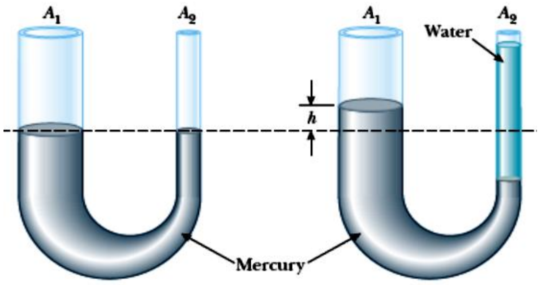
PROBLEMS

1. A woman's systolic blood pressure when resting is $\mathbf{160\,mmHg}$. What is this pressure in (a) Pa, (b) atm and (c) torr?

2. (a) Find the total weight of water on top of nuclear submarine at a depth of $\mathbf{200\ m}$. assuming that its (horizontal cross-section) hull area is $\mathbf{3000\ m^2}$. (b) What water pressure would a diver experience at this depth? Assume the density of the sea water is $\mathbf{1.03\ g/cm^3}$.

3. The small piston of a hydraulic lift has a cross-sectional area of $\mathbf{3\ cm^2}$, and its large piston has a cross-sectional area of $\mathbf{200\ cm^2}$. What force must be applied to the small piston for the lift to raise a load of $\mathbf{15\ kN}$?

4. Mercury is poured into a U-tube. The left arm of the tube has cross-sectional area A$_1$ of $\mathbf{10\ cm^2}$and the right arm has a cross-sectional area A$_2$ of $\mathbf{5\ cm^2}$ One hundred grams of water are then poured into the right arm as in the following figure. (a) Determine the length of the water column in the right arm of the U-tube. (b) Given that the density of mercury is $\mathbf{13.6\ g/cm^3}$, what distance $\boldsymbol{h}$ does the mercury rise in the left arm?



5. A piece of aluminum with mass $\mathbf{1\ kg}$ and density $\mathbf{2700\ kg/m^3}$ is suspended from a string and then completely immersed in a container of water. Calculate the tension in the string (a) before and (b) after the metal is immersed.

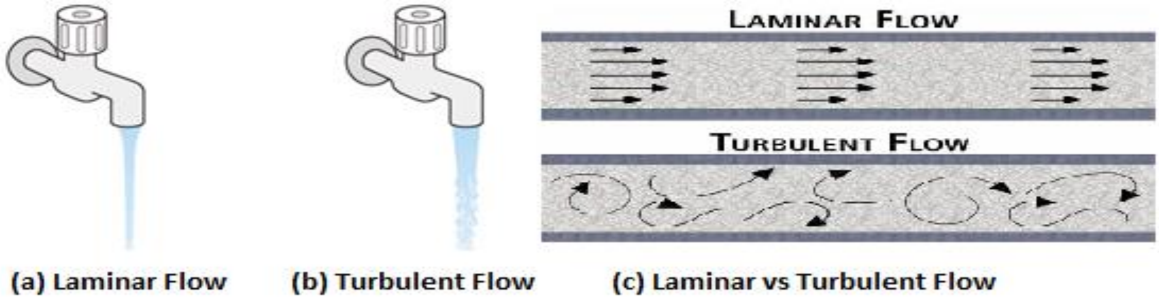
6. About **one third** of the body of a physicist swimming in the Dead Sea will be above the water line. Assuming that the human body density is $\mathbf{0.98\ g/m^3}$. Find the density of the water in the Dead Sea.

7. A block of wood floats in water with **two thirds** of its volume submerged. In oil the block floats with $\mathbf{0.9}$ of its volume submerged. Find the density of (a) the wood and (b) the oil.

CHAPTER 4
FLUID DYNAMICS

4.1 Introduction

Fluid dynamics is the study of fluids in motion. When fluid is in motion, its flow can be characterized as being one of two main types. The flow is said to be *steady*, or *laminar*, if each particle of the fluid follows a smooth path such that the paths of different particles never cross each other as shown in figure 4.1a. In steady flow, every fluid particle arriving at a given point has the same velocity. Above a certain critical speed, fluid flow becomes *turbulent*. Turbulent flow is irregular flow characterized by small whirlpool-like regions as shown in figure 4.1b.



(a) Laminar Flow (b) Turbulent Flow (c) Laminar vs Turbulent Flow

Figure 4.1: Difference between the laminar and turbulent flow.

The term *viscosity* is commonly used in the description of fluid flow to characterize the degree of internal friction in the fluid. This internal friction, or viscous force, is associated with the resistance that two adjacent layers of fluid have to moving relative to each other. Viscosity causes part of the fluid's kinetic energy to be converted to internal energy. Because the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In our model of *ideal fluid,* we make the following four assumptions:

1. **The fluid is non-viscous**. In a non-viscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.
2. **The flow is steady**. In steady (laminar) flow, all particles passing through a point have the same velocity.
3. **The fluid is incompressible**. The density of an incompressible fluid is constant.
4. **The flow is irrotational**. In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, the flow is irrotational.

4.2 The Continuity Equation

In a fluid has a steady flow, the mass passing through one section of pipe in time interval (Δt) must pass through any other section of the pipe in the same interval. Suppose an incompressible fluid flows into a pipe of nonuniform cross-sectional area under conditions of steady flow as illustrated in figure 4.2. The fluid on the left moves at speed (υ_1). During a time (Δt), the fluid travels a distance;

$$x_1 = \upsilon_1 \Delta t$$

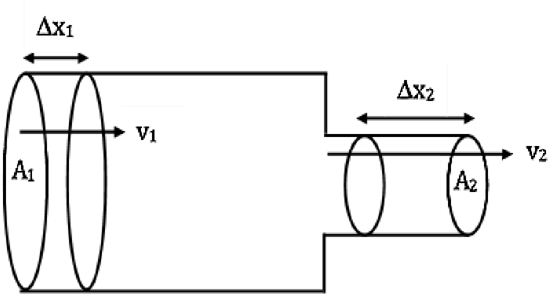


Figure 4.2: An incompressible fluid flowing horizontally through a nonuniform pipe

If (A_1) is the cross-sectional area of this section of pipe, then the mass of water moving past point 1in time (Δt) is

$$\Delta m_1 = \rho \Delta V_1 = \rho A_1 \Delta x_1 = \rho A_1 \upsilon_1 \Delta t$$

During this same time interval, the mass of fluid moving past point (2) is

$$\Delta m_2 = \rho \Delta V_2 = \rho A_2 \Delta x_2 = \rho A_2 \upsilon_2 \Delta t$$

$$\Delta m_1 = \Delta m_2$$

$$\rho A_1 \upsilon_1 \Delta t = \rho A_2 \upsilon_2 \Delta t$$

The continuity equation for incompressible fluid:

$$A_1 \upsilon_1 = A_2 \upsilon_2 \qquad (4-1)$$

<u>Example (4.1):</u>

The heart pumps blood into the aorta, which has an inner radius of **1 cm**. the aorta feeds **32** major arteries. If blood in the aorta travels at a speed of **28 cm/s**, at approximately what average speed does it travel in the arteries? Assume that blood can be treated as an ideal fluid and that the arteries each have an inner radius of **0.21 cm**.

<u>Solution:</u>

We start by finding the cross-sectional area of the aorta $A_1 = \pi r_{aorta}^2$

And then the total cross-sectional area of the arteries $A_2 = 32\pi r_{artery}^2$

Now we apply the continuity equation and solve for the unknown speed. $A_1 \upsilon_1 = A_2 \upsilon_2$

$$\upsilon_2 = \upsilon_1 \frac{A_1}{A_2}$$

$$\upsilon_2 = (0.28\, m/s) \frac{\pi \times (1 \times 10^{-2}\, m)^2}{32\pi \times (0.21 \times 10^{-2}\, m)^2}$$

$$\upsilon_2 = 0.2\, m/s$$

4.3 Bernoulli's Equation

As a fluid moves through a pipe of varying cross section and elevation, the pressure changes along the pipe. In 1738 the Swiss physicist Daniel Bernoulli (1700–1782) derived *an expression that relates the pressure of a fluid to its speed and elevation.* Bernoulli's equation is not a freestanding law of physics; rather, it's a consequence of energy conservation as applied to an ideal fluid. In deriving Bernoulli's equation, we again assume that the fluid is incompressible, non-viscous, and flows in a non-turbulent, steady-state manner.

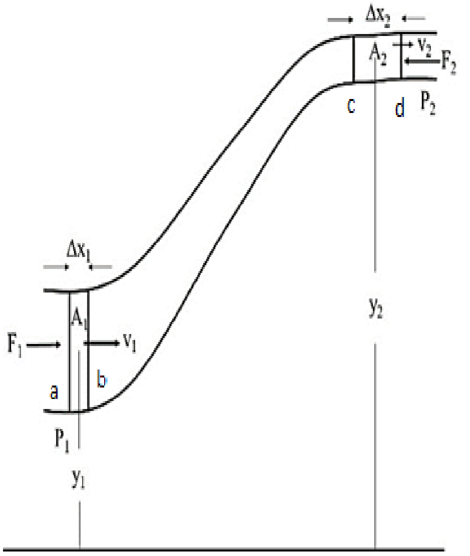


Figure 4.3: Applying conservation of energy to the flow of an ideal fluid.

When an incompressible fluid flows along a horizontal flow tube of varying cross-section as illustrated in figure 4.3, its velocity changes according to the continuity equation. A force is required to produce this acceleration and for this force to be caused by the fluid surrounding a particular element of fluid, the pressure must be different in different regions. If the pressure were the same everywhere, the net force on any fluid element would be zero. Thus when the cross-section of a flow tubes varies, the pressure must be varying along the tube, even when there is no difference in elevation. If the elevation also changes, there is an additional pressure difference. Because of the continuity relation, the volume of fluid (ΔV) passing any cross-section in time (Δt) is $\Delta V = A_1 \Delta x_1 = A_2 \Delta x_2$. Where A_1 and A_1 are the cross-sectional areas at the points (a) and (c) respectively.

To derive the Bernoulli's equation, we apply the work-energy theorem to the fluid in a section of a flow tube. The work done is equal to the total change in kinetic and gravitational potential energy.

$$\Delta W = \Delta K + \Delta U$$

We can compute the work done on this fluid during (Δt). The force on the cross-section at $(1\ )$is $(P_1 A_1)$ and that at (2) is $(P_2 A_2)$, where (P_1) and (P_2) are the pressures at the two ends. The net work done on the element.

$$\Delta W = W_1 - W_2 = F_1 \Delta X_1 - F_2 \Delta X_2$$

$$= P_1 A_1 \Delta X_1 - P_2 A_2 \Delta X_2 = P_1 \Delta V - P_2 \Delta V$$

$$\Delta W = (P_1 - P_2)\Delta V \qquad (4-2)$$

$$\Delta K = (K.E)_2 - (K.E)_1 = [½\, m(v_2^2 - v_1^2)]$$

$$\Delta K = [½\, \rho\, \Delta V(v_2^2 - v_1^2)]$$

$$\Delta U = U_2 - U_1 = [m\,g(y_2 - y_1)]$$

$$\Delta U = [\,\rho\,\Delta V g\,(y_2 - y_1)]$$

$$\Delta K + \Delta U = [½\,\rho\,\Delta V(v_2^2 - \,v_1^2)] + [\,\rho\,\Delta V g\,(y_2 - y_1)] \qquad (4-3)$$

From (4-2) and (4-3), we get:

$$(P_1 - \,P_2)\Delta V = ½\,\rho\,\Delta V(v_2^2 - \,v_1^2) + \,\rho\,\Delta V g\,(y_2 - y_1)$$

$$P_1 - \,P_2 = ½\,\rho\,(v_2^2 - \,v_1^2) + \,\rho\,g\,(y_2 - y_1)$$

or

$$P_1 + \,½\,\rho\,v_1^2 \,+\,\rho\,g\,y_1 = \,P_2 + \,½\,\rho\,v_2^2 \,+\,\rho\,g\,y_2 \qquad (4-4)$$

Bernoulli's equation states that: "*The sum of the pressure* (P)*, the kinetic energy per unit volume* ($1/2\,\rho v^2$) *and the potential energy per unit volume* ($\rho g y$)*, has the same value at all points along a streamline*".

Example (4.2):

A horizontal pipe $\mathbf{10\ cm}$ in diameter has a smooth reduction to a pipe $\mathbf{5\ cm}$ in diameter. If the pressure of the water in the larger pipe is $\mathbf{8\,\times\,10^{\,4} Pa}$ and the pressure in the smaller pipe is $\mathbf{6\,\times\,10^{\,4}\,Pa}$, Find the velocities on each section of the pipes?

Solution:

$$r_1 = 2r_2 \Rightarrow v_2 = 4v_1 \quad since, v \propto r^2$$

$$horizontal\ pip \Leftrightarrow y_1 = \ y_2$$

$$P_1 + \,½\,\rho\,v_1^2 \,+\,\rho\,g\,y_1 = \,P_2 + \,½\,\rho\,v_2^2 \,+\,\rho\,g\,y_2$$

$$P_1 + \,½\,\rho\,v_1^2 \,=\,P_2 + \,½\,\rho\,v_2^2$$

$$(8 \times 10^4\ Pa) \,+\,½\,(1000\ kgm^{-3})\,v^2 = \,(6 \times 10^4\ Pa) \,+\,½\,(1000\ kg/m^3)\,\,(4v)^2$$

$$(2 \times\,104) \,=\,½\,(1000)\,15\,v2$$

$$v \,=\,1.63\,m/s$$

$$v_1 \,=\,1.63\,m/s \quad and \quad v_2 \,=\,4v_1 \,=\,4(1.63\,m/s) = 6.52\,m/s$$

Example (4.3):

Water is moving with a speed of $\mathbf{5\ m/s}$ through a pipe with a cross-sectional area of $\mathbf{4\ cm^2}$. The water gradually descends $\mathbf{10\ m}$ as the pipe increases in area to $\mathbf{8\ cm^2}$. . (a) What is the speed of flow at the lower level? (b) If the pressure at the upper level is $\mathbf{1.5\,\times\,10^5\ Pa}$, what is the pressure at the lower level?

Solution:

$$(a)\ A_1 v_1 = A_2 v_2 \Longrightarrow v_2 = \frac{A_1 v_1}{A_2} = \frac{(4 \times 10^{-4}\ m^2)(5\ m/s)}{(8 \times 10^{-4}\ m^2)} = 2.5\ m/s$$

$$(b)\ P_1 - P_2 = ½\,\rho\,(v_2^2 - v_1^2) + \rho\,g\,(y_2 - y_1)$$

$$\Rightarrow P_2 = P_1 + ½\,\rho\,(v_1^2 - v_2^2) + \rho\,g\,(y_1 - y_2)$$

$$= [1.5 \times 10^5\ Pa] + \left[\frac{1}{2}(1000\ kg/\ m^3)((25\ m/s)^2 - (6.25\ m/s)^2)\right]$$

$$+ [(1000\ kg/\ m^3)(9.81\ m/s^2\ )(10\ m)] = 2.57 \times 10^5\ Pa$$

4.3.1 Applications of Bernoulli's equation

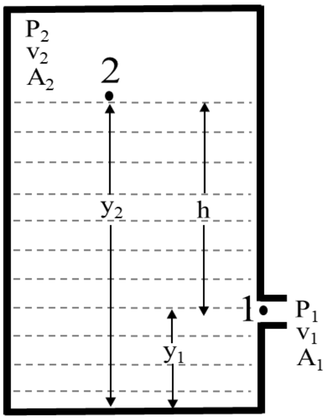


Figure 4.4: A liquid leaves a hole in a closed tank at speed

1- The equation of hydrostatics is special case of Bernoulli's equation. When the velocity υ_1 and υ_2 are zero equation (4-4) are reduces to

$$P_1 - P_2 = \rho g\,(y_2 - y_1)$$

2- Torricelli's Law; an *enclosed tank* containing a liquid of density ρ has a hole in its side at a distance y_1 from the tank's bottom as illustrated in figure 4.4. The hole is open to the atmosphere, and its diameter is much smaller than the diameter of the tank. The air above the liquid is maintained at a pressure P. let (v_1) and (v_2) be the speed at points (1) and (2). The quantity υ_1 is called the speed of efflux. The pressure at point 1 is atmospheric (P_a). applying Bernoulli's equation to points (1) and (2) and taking the bottom of the tank as our reference level, we get

$$P_1 - P_2 = ½\,\rho\,(v_2^2 - v_1^2) + \rho\,g\,(y_2 - y_1)$$

$$[P_1 = P_a], [P_2 = P], [v_2 = 0]\ and\ [y_2 - y_1 = h]$$

$$P_a - P = ½\,\rho\,(0 - v_1^2) + \rho\,g\,h$$

$$½\,\rho v_1^2 = (P - P_a) + \rho\,g\,h$$

$$v_1^2 = \frac{2(p - p_a)}{\rho} + 2gh$$

$$v_1 = \sqrt{\frac{2(p - p_a)}{\rho} + 2gh} \qquad (4-5)$$

Suppose the *tank is open* to the atmosphere as illustrated in figure 4.5, so the pressure at point (1) and point (2) is atmospheric pressure. Hence, the speed of efflux at point (1) is

$$p = p_a$$

$$v_1 = \sqrt{\frac{2(p_a - p_a)}{\rho} + 2gh}$$

$$v_1 = \sqrt{2gh} \qquad (4-6)$$

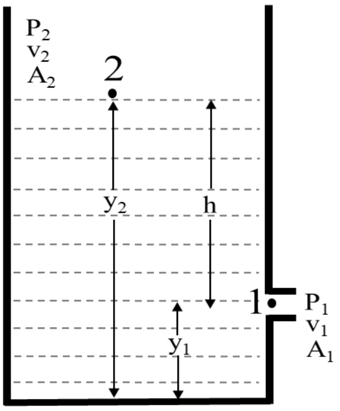


Figure 4.5: A liquid leaves a hole in an open tank at speed υ_1.

Example (4.4):

A large storage tank, open at the top and filled with water, develops a small hole in its side at a point **16 m** below the water level. If the rate of flow from the leak is equal to **2.5×10^3 m^3/min**, determine **(a)** the speed at which the water leaves the hole and **(b)** the diameter of the hole.

Solution:

$$v_2 = \sqrt{2gh} = \sqrt{2(9.8\,ms^{-2})(16\,m)} = 17.7\,m/s$$

$$Flow\ rate\ = Av \Rightarrow \left(\frac{2.5 \times 10^3}{60}\ m^3/s\right) = \left(\pi\left(\frac{d}{2}\right)^2\right)(17.7\,m/s) \Rightarrow d = 1.73 \times 10^{-3}\,m$$

4.4 Viscosity

The viscosity of a fluid is the measure of its resistance to gradual deformation by shear stress or tensile stress (figure 4.6).

Viscosity is the internal friction of a fluid. Because of viscosity, a force must be exerted to cause one layer of a fluid to slide past another or to cause one surface to slide past another if there is a layer of fluid between the surfaces. Both liquid and gases exhibit viscosity, although liquids are much more viscous than gases.

Viscosities of all fluids are markedly dependent on the temperature, increasing for gases and decreasing for liquids as the temperature is increased. If (A) is the area of the fluid over which these forces are applied, the ratio (F/A) is the shear stress exerted on the fluid. The shear strain increases without limit so long as the stress is applied and the stress is found by experiment to depend not on the shear strain, but on its rate of change.

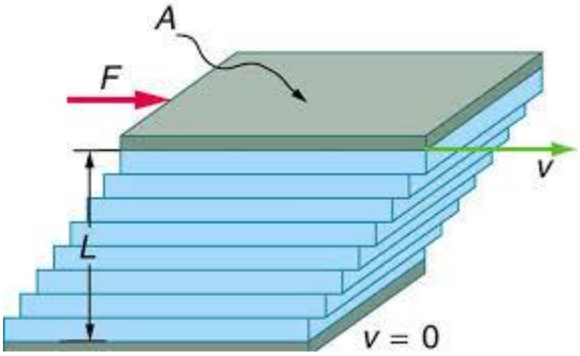


Figure 4.6: Variation of velocity with height

- *Rate of change of shear strain = v/l (change of velocity with length)*
- *The rate of change of shear strain is also referred to simply as the strain rate. The coefficient of viscosity (η) of the fluid is defined as the ratio of the shear stress to the rate of change of shear strain.*

$$\eta = \frac{shear\ stress}{rate\ of\ change\ of\ shear\ strain} = \frac{F/A}{v/l} \qquad (4-7)$$

The unit of viscosity in the SI units is $1N.s/m^2$, while the corresponding cgs unit is 1 $dyne.s/cm^2$. The common unit of viscosity is $(1\,poise)$. $\mathbf{1\,poise\ =\ 1\,dyne.s/cm^2\ =\ 10^{-1}\,N.s/m^2}$

Fluids which obey the Newton's law of viscosity are called as Newtonian fluids. All gases and most liquids which have simpler molecular formula and low molecular weight such as water, benzene, ethyl alcohol, hexane and most solutions of simple molecules are Newtonian fluids.

- *Fluids that are suspensions or dispersion are often non-Newtonian in their viscous behavior such as slurries, pastes, gels, polymer solutions.* A few typical values of viscosity are illustrated in **table** 4-1.

Table 4-1: Typical values of viscosity

Temperature ˚C	Viscosity caster oil, poise	Viscosity of water, centipoise	Viscosity of air, micropoise
0	53	1.792	171
20	9.86	1.005	181
40	2.31	0.656	190
60	0.8	0.469	200
80	0.3	0.357	209
100	0.17	0.284	218

4.5 Poiseuille's law

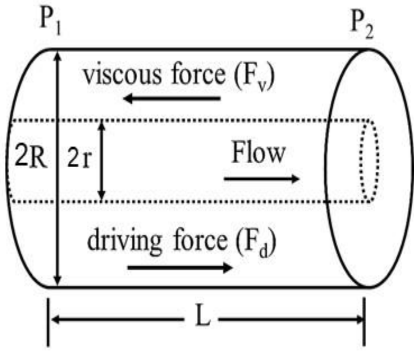


Figure 4.7: Forces on a cylindrical element of a viscous flow.

When a viscous fluid flows in a tube, the flow velocity is different at different points of a cross-section. The outermost layer of fluid clings to the walls of the tube and its velocity is zero. The tube walls exert a backward drag on this layer, which in turn drags backward on the next layer beyond it, and so on. If the velocity is not too great, the flow is laminar, with a velocity that is greatest at the center of the tube and decreases to zero at the walls. Let us consider the variation of velocity with radius for a cylindrical pipe of inner radius (R). we consider the flow of a cylindrical element of fluid coaxial with the pipe of radius (r) and length (L) as shown in figure 4.7.

The driving force on the cylinder due to the pressure difference

$$F_d = (P_1 - P_2)A_{top} = (P_1 - P_2)\pi r^2 \qquad (4-8)$$

The viscous drag force opposing motion depends on the surface area of the cylinder (length L and radius r)

$$F_v = -\eta A_{side}\frac{dv}{dr} = -\eta 2\pi r L\frac{dv}{dr} \qquad (4-9)$$

In an equilibrium condition of constant speed, where the net force goes to zero

$$F_v = F_d$$

$$-\eta 2\pi r L \frac{dv}{dr} = (P_1 - P_2)\pi r^2$$

$$-\frac{dv}{dr} = \left(\frac{P_1 - P_2}{2\eta L}\right) r$$

$$\int dv = -\left(\frac{P_1 - P_2}{2\eta L}\right) \int r\,dr$$

$$v = -\left(\frac{P_1 - P_2}{4\eta L}\right) r^2 + C \qquad (4-10)$$

Use the condition of $(r = R \Leftrightarrow v = 0)$*, then substitute in equation (4-10), we get,*

$$0 = -\left(\frac{P_1 - P_2}{4\eta L}\right) R^2 + C$$

$$C = \left(\frac{P_1 - P_2}{4\eta L}\right) R^2 \qquad (4-11)$$

If we substitute in equation (4-10), we will get,

$$v = -\left(\frac{P_1 - P_2}{4\eta L}\right) r^2 + \left(\frac{P_1 - P_2}{4\eta L}\right) R^2$$

$$v = \left(\frac{P_1 - P_2}{4\eta L}\right) (R^2 - r^2) \qquad (4-12)$$

Use the condition of $(r = 0 \Leftrightarrow v = v_{max})$*, then substitute in equation (4-12), we get,*

$$v_{max} = \left(\frac{P_1 - P_2}{4\eta L}\right) R^2$$

Now the equation of continuity giving the volume flux for a variable speed is:

$$\frac{dV}{dt} = \int v\,dA$$

$$But: v = \left(\frac{P_1 - P_2}{4\eta L}\right) (R^2 - r^2) \ and \ dA = d(\pi r^2) = 2\pi r dr$$

$$Then, \quad \frac{dV}{dt} = \int v\,dA = \int \left(\frac{P_1 - P_2}{4\eta L}\right) (R^2 - r^2)(2\pi r dr) = \left(\frac{\pi(P_1 - P_2)}{2\eta L}\right) \int_0^R (R^2 - r^2)(r dr)$$

$$= \left(\frac{\pi(P_1 - P_2)}{2\eta L}\right) \int_0^R (rR^2 dr - r^3 dr) = \left(\frac{\pi(P_1 - P_2)}{2\eta L}\right) \left[\frac{r^2 R^2}{2} - \frac{r^4}{4}\right]_0^R = \left(\frac{\pi(P_1 - P_2)}{2\eta L}\right) \left(\frac{R^4}{2} - \frac{R^4}{4}\right)$$

$$= \frac{\pi(P_1 - P_2) R^4}{8\eta L} \qquad (4-13)$$

This relation is called **Poiseuille's law**.

Example (4.5):

Water at ***20 ℃*** flows through a pipe of radius ***1 cm***. if the maximum velocity ***10 cm/s***. Find the pressure drop a long a ***2 m*** section of pipe due to viscosity.

Solution:

$$\Delta P = \frac{4\eta L v_{max}}{R^2} = \frac{4\,(1.005\times10^{-2}\ kg\ m/s)(2\ m)(10\times10^{-2}\ m/s)}{(1\times10^{-2}\ m)^2} = 80.4\ Pa$$

4.6 Stokes' law

When a sphere moves through a stationary fluid, the streamlines form a perfectly symmetrical pattern around the sphere. If the fluid has viscosity, there will be a viscous drag on the sphere. Suppose a sphere of radius (r) falls in a stationary fluid in a vertical glass tube with velocity (v) as illustrated in figure 4.8. The falling sphere in a viscous fluid reaches a terminal velocity (v_T) at which the viscous retarding force (F_d) plus the buoyant force (F_b) equals the weight of the sphere (F_w). The force of viscosity on a small sphere moving through a viscous fluid is given by

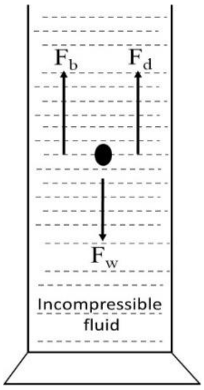


Figure 4.8: A metal sphere falls through a liquid.

$$F_d = 6\pi r\eta v_T \qquad (4-14)$$

Let (ρ_s) be the density of the sphere and (ρ_l) the density of the fluid. The weight force of the sphere is then $\left(F_w = {}^4\!/_3\,\pi r^3\rho_s g\right)$ while, the buoyant force is given by $\left(F_b = {}^4\!/_3\,\pi r^3\rho_l g\right)$.

For the velocity to be steady,

$$F_w = F_b + F_d$$

$$V\rho_s g = V\rho_L g + 6\pi r\eta v_T$$

$$\left(\frac{4}{3}\pi r^3\right)\rho_s g = \left(\frac{4}{3}\pi r^3\right)\rho_L g + 6\pi r\eta v_T$$

$$\left(\frac{4}{3}\pi r^3\right)g(\rho_s - \rho_L) = 6\pi r\eta v_T$$

$$v_T = \frac{2}{9}\frac{(\rho_s - \rho_l)gr^2}{\eta} \qquad (4-15)$$

where (v_T) is the terminal velocity and (η) is the coefficient of viscosity of the viscous liquid.

Example (4.6):

An aluminum ball of radius ***1 cm*** falls through water at ***20 ˚C***. What is the terminal velocity, assuming laminar flow and including buoyancy?

Solution:

$$\upsilon_T = \frac{2}{9}\,\frac{r^2 g\,(\rho_s - \rho_L^{\prime})}{\eta} = \frac{2}{9}\,\frac{(1\times 10^{-2}\,m)^2(9.81\,m/s^2)\,(2700\,kg/m^3 - 1000\,kg/m^3)}{(10^{-3}\,P.s)} = 37.1\,m/s$$

PROBLEMS

1. A garden hose pipe of inner radius **1 cm** carries water at **2 m/s**. the nozzle at the end has radius **0.2 cm**. how fast dose the water moves through the nozzle?

2. A horizontal segment of pipe tapers from a cross-section area **50 cm²** to **0.5 cm²**. The pressure at the larger end of the pipe is $\mathbf{1.2 \times 10^5\ Pa}$ and the speed is **0.04 m/s**. what is the pressure at the narrow end of the segment? Suppose the liquid is water.

3. Water enters a house through a pipe $\mathbf{2\ cm}$ in inside diameter, at an absolute pressure of $\mathbf{4 \times 10^5\ Pa}$. The pipe leading to the second floor bathroom $\mathbf{5\ m}$ above is $\mathbf{1\ cm}$ in diameter. When the flow velocity at the inlet pipe is $\mathbf{4\ m/s}$, find the flow velocity and pressure in the bathroom.

4. A sniper fires rifle bullet into a gasoline tank, making a hole $\mathbf{50\ m}$ below the surface of the gasoline. The tank was sealed and is under $\mathbf{3\ atm}$ absolute pressures. The stored gasoline has a density of $\mathbf{660\ kg/m^3}$. At what speed does the gasoline begin to shoot out of the hole?

5. A nozzle is connected to a horizontal hose. The nozzle shoots out water moving at $\mathbf{25\ m/s}$. What is the gauge pressure of the water in the hose? Assuming the diameter of the nozzle is much smaller than the inner diameter of the hose.

6. What is the pressure difference required to make water flow through a tube of inner radius $\mathbf{2\ mm}$ and length $\mathbf{0.2\ m}$ at speed of $\mathbf{6\ cm/s}$? If the viscosity coefficient of water at $\mathbf{20\ ^\circ C}$ is $\mathbf{1.005\ cp}$, calculate the total volume of water flow per unit time?

7. Oil at $\mathbf{20\ ^\circ C}$ flows through a tube of inner radius $\mathbf{20\ cm}$ with coefficient of viscosity $\boldsymbol{\eta} = \mathbf{9.86\ poise}$. if the pressure drop along a $\mathbf{4\ m}$ section of pipe is $\mathbf{1200\ Pa}$. Find the velocity of the oil flow at radius $\mathbf{10\ cm}$.

8. A sphere of radius $\mathbf{1\ cm}$ is dropped into a glass cylinder filled with a viscous liquid. The mass of the sphere is $\mathbf{12\ g}$ and the density of the liquid is $\mathbf{1200\ kg/m^3}$. The sphere reaches a terminal speed of $\mathbf{0.15\ m/s}$. What is the viscosity of the liquid?

9. (a) With what terminal velocity will an air bubble $\mathbf{1\ mm}$ in diameter rise in a liquid of viscosity $\mathbf{150\ cp}$ and density $\mathbf{0.9\ g/cm^3}$. . **(b)** What is the terminal velocity of the same bubble in water?

CHAPTER (5)
SIMPLE HARMONIC MOTION

5.1 Introduction

A very special kind of motion occurs when the force acting on a body is proportional to the displacement of the body from some equilibrium position. If this force is always directed toward the equilibrium position, repetitive back and forth motion occurs about this position. Such motion is called *periodic motion*, *harmonic motion*, *oscillation*, or *vibration* (the four terms are completely equivalent). *Periodic motion* is motion of an object that regularly repeats the object returns to a given position after a fixed time interval. With a little thought, we can identify several types of periodic motion in everyday life. Your car returns to the driveway each afternoon. You return to the dinner table each night to eat. The Earth returns to the same position in its orbit around the Sun each year, resulting in the variation among the four seasons. The Moon returns to the same relationship with the Earth and the Sun, resulting in a full Moon approximately once a month.

5.2 Oscillation

You are most likely familiar with several examples of periodic motion, such as the oscillations of a block attached to a spring, the swinging of a child on a playground swing, the motion of a pendulum, and the vibrations of a stringed musical instrument. In addition to these everyday examples, numerous other systems exhibit periodic motion. For example, the molecules in a solid oscillate about their equilibrium positions; electromagnetic waves, such as light waves, radar, and radio waves, are characterized by oscillating electric and magnetic field vectors; and in alternating-current electrical circuits, voltage, current, and electrical charge vary periodically with time.

Periodic motion, from masses on springs to vibrations of atoms, is one of the most important kinds of physical behavior. In this chapter we take a more detailed look at Hooke’s law, where the force is proportional to the displacement, tending to restore objects to some equilibrium position. A large number of physical systems can be successfully modeled with this simple idea, including the vibrations of strings, the swinging of a pendulum, and the propagation of waves of all kinds. All these physical phenomena involve periodic motion. Periodic vibrations can cause disturbances that move through a medium in the form of waves. Many kinds of waves occur in nature, such as sound waves, water waves, seismic waves, and electromagnetic waves.

5.3 Simple Harmonic Motion

A special kind of periodic motion occurs in mechanical systems when the force acting on an object is proportional to the position of the object relative to some equilibrium position. If this force is always directed toward the equilibrium position, the motion is called *simple harmonic motion,* which means any motion that repeat itself at regular intervals and its acceleration is proportional to its displacement from its equilibrium position.

A particle moving along the $x-axis$ exhibits simple harmonic motion when (X), the particle's displacement from equilibrium, varies in time according to the relationship.

$$x = A\cos(\omega t + \varphi) \qquad (5-1)$$

where (A), (ω), and (φ) are constants. To give physical significance to these constants, we have labeled a plot of (x) as a function of (t) in figure 5.1a.

The amplitude (A) of the motion is the maximum displacement of the particle in either the positive or negative (x) direction. The constant (ω) is called the angular frequency of the motion and has units of radians per second. The constant angle (φ), called the phase constant (or phase angle), is determined by the initial displacement and velocity of the particle. If the particle is at its maximum position, $(x = A\ at\ t = 0)$, then $(\varphi = 0)$ and the curve of (x) versus $t$is as shown in figure 5.1b. If the particle is at some other position at $(t = 0)$ the constants (φ) and (A) tell us what the position was at time $(t = 0)$. The quantity $(\omega t + \varphi)$ is called the phase of the motion and is useful in comparing the motions of two oscillators.

From last equation the trigonometric function (x) is *periodic* and repeats itself every time (ωt) increases by $(2\pi\ rad)$. The period $(T)$of the motion is the time it takes for the particle to go through one full cycle. We say that the particle has made *one oscillation.* This definition of (T) tells us that the value of (x) at time t equals the value of (x) at time $(t + T)$. We can show that $(T = 2\pi/\omega)$ by using the preceding observation that the phase $(\omega t + \varphi)$ increases by (2π) rad in a time (T):

$$\omega t + \varphi + 2\pi = \omega(t + T) + \varphi$$

Hence, $\omega t = 2\pi,$ or $\omega = 2\pi/t$

The inverse of the period is called the frequency $f$of the motion. The frequency represents the number of oscillations that the particle makes per unit time:

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

The unit of (f) are cycles per second (s^{-1}), or hertz (Hz). Thus,

$$\omega = \frac{2\pi}{T} = 2\pi f$$

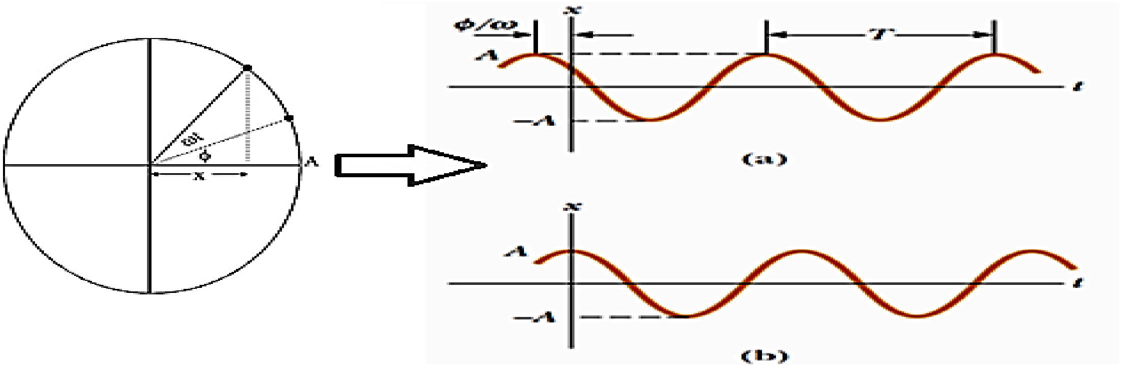


Figure 5.1: (a) An $(x-t)$curve for a particle undergoing simple harmonic motion. The amplitude of the motion is (A), the period is (T), and the phase constant is (φ).(b) The $(x-t)$ curve in the special case in which $(x=A)$ at $T=0$ and hence $(\varphi\ =\ 0)$.

Velocity and acceleration of simple harmonic motion:

We can obtain the linear velocity of a particle undergoing simple harmonic motion by differentiating equation (5-1) with respect to time:

$$v=\frac{dx}{dt}=\frac{d}{dt}(A\cos(\omega t+\varphi))=-A\,\omega\sin(\omega t+\varphi) \qquad (5-2)$$

The acceleration of the particle is

$$a=\frac{dv}{dt}=\frac{d}{dt}(-A\,\omega\sin(\omega t+\varphi))=-A\,\omega^2\cos(\omega t+\varphi)=-\,\omega^2(A\cos(\omega t+\varphi)) \qquad (5-3)$$

Because, x $=A\cos\,(\omega t+\varphi)$, we can express an expression for a in the form:

$$a=-\omega^2 x \qquad (5-4)$$

And from equation (5-2), we see that, because the sine function oscillates between (± 1), the extreme values of (v) are $(\pm\,\omega A)$. Because the cosine function also oscillates between (± 1), equation (5-3) tells us that the extreme values of a are $(\pm\ \omega^2 A)$. Therefore, the maximum speed and the magnitude of the maximum acceleration of a particle moving in simple harmonic motion are

$$\boldsymbol{v_{max}=A\omega}$$

$$\boldsymbol{a_{max}=A\omega^2}$$

Example (5.1):

An object oscillates with simple harmonic motion along the $x-axis$. Its displacement from the origin varies with time according to the equation

$$\boldsymbol{x=(4.00\ m)\cos(\pi t+\frac{\pi}{4})}$$

where (t) is in seconds and the angles in the parentheses are in radians.

(a) Determine the amplitude, frequency, and period of the motion. **(b)** Determine the maximum speed and maximum acceleration of the object.

Solution:

$$(a)\quad A = 4.00\,m,\ \ \omega = \pi\frac{rad}{s} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{(\pi\frac{rad}{s})} = 2\,s,\ \ f = \frac{1}{T} = \frac{1}{(2\,s)} = 0.5\,s$$

$$(b) v_{max} = \omega A = \left(\pi\frac{rad}{s}\right)(4\,m) = 12.6\,ms^{-1}$$

$$a_{max} = \omega^2 A = \left(\pi\frac{rad}{s}\right)^2 (4\,m) = 39.5\,ms^{-2}$$

5.3.1 The Force Law

As a model for simple harmonic motion, consider a block of mass m attached to the end of a spring, with the block free to move on a horizontal, frictionless surface (figure 5.2). When the spring is neither stretched nor compressed, the block is at the position called the equilibrium position of the system, which we identify as $(x = 0)$.

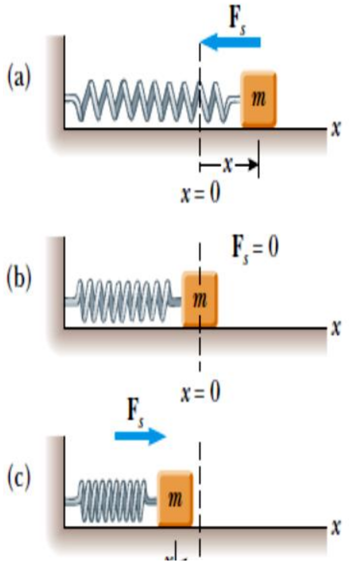


Figure 5.2: A block attached to a spring moving on a frictionless surface. (a) When the block is displaced to the right of equilibrium $(x > 0)$, the force exerted by the spring acts to the left. (b) When the block is at its equilibrium position $(x = 0)$, the force exerted by the spring is zero. (c) When the block is displaced to the left of equilibrium $(x < 0)$, the force exerted by the spring acts to the right.

We can understand the motion in figure 5.2 qualitatively by first recalling that when the block is displaced to a position (x), the spring exerts on the block a force that is proportional to the position and given by Hooke's law

$$F = -kx$$

where (k) is the spring constant and $x$is the displacement of the object from its equilibrium position. This force law for springs was discovered by Robert Hooke in 1678 and is known as **Hooke's law**. The value of (k) is a measure of the stiffness of the spring. Stiff springs have large (k) values, and soft springs have small (k) values. The negative sign means that the force exerted by the spring is always directed *opposite* the displacement of the object. We call this a restoring force because it is always directed toward the equilibrium position and therefore *opposite* the displacement from equilibrium. That is, when the block is displaced to the right of $(x = 0)$ in figure 5.2, then the position is positive and the restoring force is directed to the left. When the block is displaced to the left of $(x = 0)$, then the position is negative and the restoring force is directed to the right.

Applying Newton's second law to the motion of the block,

$$F = ma \quad \text{"}Newton's\ second\ law\text{"}$$

$$a = \omega^2 x \quad \text{"}S.H.M\text{"}$$

$$\Rightarrow F = m\omega^2 x$$

$$F = -kx \quad \text{"}Hook's\ law\text{"}$$

$$\Rightarrow k = m\omega^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

To express the period and frequency of the motion for the particle–spring system in terms of the characteristics (m) and (k) of the system as

$$\Rightarrow \boldsymbol{T = 2\pi\sqrt{\frac{m}{k}}} \qquad \Rightarrow \boldsymbol{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

That is, the period and frequency depend *only* on the mass of the particle and the force constant of the spring, and *not* on the parameters of the motion, such as (A) or (φ). As we might expect, the frequency is larger for a stiffer spring (larger value of (k)) and decreases with increasing mass of the particle.

Simple harmonic motion occurs when the net force along the direction of motion obeys Hooke's law, when the net force is proportional to the displacement from the equilibrium point and is always directed toward the equilibrium point.

Example (5.2):

A car with a mass of $\boldsymbol{1300\ kg}$ is constructed so that its frame is supported by four springs. Each spring has a force constant of $\boldsymbol{20000\ N/m}$. If two people riding in the car have a combined mass of $\boldsymbol{160\ kg}$, find the frequency of vibration of the car after it is driven over a pothole in the road.

Solution:

We assume that the mass is evenly distributed. Thus, each spring supports one fourth of the load. The total mass is 1460 (1300+160) kg, and therefore each spring supports 365 kg.

Hence, the frequency of vibration

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{(2000\ Nm^{-1})}{((1300+160)/4)}} = 1.18\ s^{-1}$$

Example (5.3):

A block with a mass of $\mathbf{200\ g}$ is connected to a light spring for which the force constant is $\mathbf{5\ N/m}$ and is free to oscillate on a horizontal, frictionless surface. The block is displaced $\mathbf{5\ cm}$ from equilibrium and released from rest (a) Find the period of its motion.(b) Determine the maximum speed of the block.(c) What is the maximum acceleration of the block?

Solution:

$$(a)\ T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{(\ 200\times 10^{-3}\ kg)}{(5\ Nm^{-1})}} = 1.26\ s$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(1.26\ s)} = 5\ rad/s$$

$$(b) v_{max} = \omega A = \left(5\frac{rad}{s}\right)(5\times 10^{-2}\ m) = 0.25\ ms^{-1}$$

$$(c) a_{max} = \omega^2 A = \left(5\frac{rad}{s}\right)^2(5\times 10^{-2}\ m) = 1.25\ ms^{-2}$$

5.3.2 Simple Pendulum

A simple pendulum is another mechanical system that exhibits periodic motion. It consists of a small bob of mass m suspended by a light string of length (L) fixed at its upper end, as in figure 5.3. (By a light string, we mean that the string's mass is assumed to be very small compared with the mass of the bob and hence can be ignored.) When released, the bob swings to and fro over the same path, but is its motion simple harmonic?

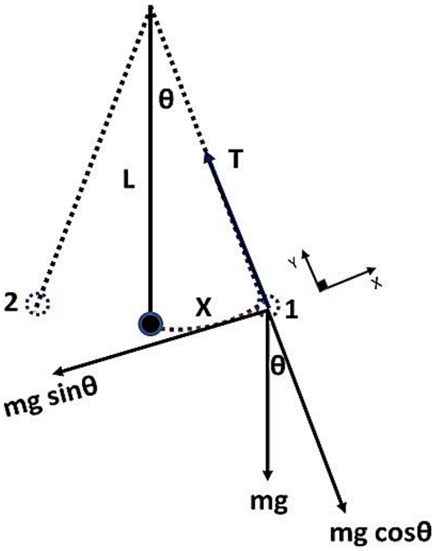


Figure 5.3: A simple pendulum consists of a bob of mass $m$suspended by a light string of length (L). ($L$is the distance from the pivot to the center of mass of the bob.) The restoring force that causes the pendulum to undergo simple harmonic motion is the component of gravitational force tangent to the path of motion, $(mg\ sin\ \theta)$.

Answering this question requires examining the restoring force, the force of gravity that acts on the pendulum. The pendulum bob moves along a circular arc, rather than back and forth in a straight line. When the oscillations are small, however, the motion of the bob is nearly straight, so Hooke's law may apply approximately.

In figure 5.3, (x) is the displacement of the bob from equilibrium along the arc. Hooke's law is $(F = -kx)$, so we are looking for a similar expression involving F_t. $(F_t = -kx)$, where (F_t) is the force acting in a direction tangent to the circular arc. From the figure, the restoring force is

$$F_t\ =\ -\ m\ g\ sin\ \theta$$

This expression isn't of the form ($F = -kx$), so in general, the motion of a pendulum is *not* simple harmonic. If we restrict the motion to small angles, the approximation ($\sin\theta \approx \theta$) is valid and substituting ($\theta = x/L$), we obtain the restoring force as

$small\,\theta \Rightarrow \sin\theta \approx \theta \approx \dfrac{x}{L}$

$$F_t = -\frac{mg}{L}x = -kx$$

$$\Rightarrow k = \frac{mg}{L}$$

Recall that for the object spring system, the angular frequency is given by equation

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg/L}{m}} = \sqrt{\frac{g}{L}}$$

$$\frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

The period (T) and frequency (f) of the motion are

$$\Rightarrow \boldsymbol{T = 2\pi\sqrt{\frac{L}{g}}} \qquad\qquad \Rightarrow \boldsymbol{f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}}$$

In other words, the period and frequency of a simple pendulum depends only on the length of the string and the acceleration due to gravity. Because the period is independent of the mass, we conclude that all simple pendulums that are of equal length and are at the same location (so that (g) is constant) oscillate with the same period.

The simple pendulum can be used as a time keeper because its period depends only on its length and the local value of (g). It is also a convenient device for making precise measurements of the free-fall acceleration. Such measurements are important because variations in local values of (g) can provide information on the location of oil and of other valuable underground resources.

Example (5.4):

Using a small pendulum of length $\mathbf{0.171\ m}$, a geophysicist counts **72** complete swings in a time of **60 s**. What is the value of (g) in this location?

Solution:

Calculate the period by dividing the total elapsed time by the number of complete oscillations:

$$T = \frac{total\ time}{total\ number\ of\ oscillations} = \frac{(60\ s)}{(72\ osscilations)} = 8.33\ s$$

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(0.171\ m)}{(8.33\ s)^2} = 9.711\ ms^{-2}$$

PROBLEMS

1. A simple harmonic oscillator takes $\mathbf{12\ s}$ to undergo five complete vibrations. Find (a) the period of its motion, (b) the frequency in hertz, and (c) the angular frequency in radians per second.

2. A $\mathbf{0.6\ kg}$ block attached to a spring with force constant $\mathbf{130\ N/m}$ is free to move on a frictionless, horizontal surface. The block is released from rest after the spring is stretched $\mathbf{0.13\ m}$. At that instant, find (a) the force on the block and (b) its acceleration.

3. When a $\mathbf{4.25\ kg}$ object is placed on top of a vertical spring, the spring compresses a distance of $\mathbf{2.62\ cm}$. What is the force constant of the spring?

4. At an outdoor market, a bunch of bananas is set into oscillatory motion with amplitude of $\mathbf{20\ cm}$ on a spring with a force constant of $\mathbf{16\ N/m}$. It is observed that the maximum speed of the bunch of bananas is $\mathbf{40\ cm/s}$. What is the weight of the bananas in Newton's?

5. The period of motion of an object spring system is $\mathbf{0.223\ s}$ when a $\mathbf{35.4\ g}$ object is attached to the spring. What is the force constant of the spring?

6. A man enters a tall tower, needing to know its height. He notes that a long pendulum extends from the ceiling almost to the floor and that its period is $\mathbf{15.5\ s}$. (a) How tall is the tower? (b) If this pendulum is taken to the Moon, where the free-fall acceleration is $\mathbf{1.67 m/s^2}$, what is the period there?

7. A simple pendulum makes **120** complete oscillations in $\mathbf{3\ min}$ at a location where $\mathbf{g\ = 9.8\ m/s^2}$. Find (a) the period of the pendulum and (b) its length.

8. A “seconds” pendulum is one that moves through its equilibrium position once each second. (The period of the pendulum is $\mathbf{2\ s}$.) The length of a second's pendulum is $\mathbf{0.9927\ m}$ at Tokyo and $\mathbf{0.9942\ m}$ at Cambridge, England. What is the ratio of the free-fall accelerations at these two locations?

CHAPTER (6)
WAVES

6.1 Introduction

When you drop a pebble into a pool of water, the disturbance produces water waves, which move away from the point where the pebble entered the water. A leaf floating near the disturbance moves up and down and back and forth about its original position but doesn't undergo any net displacement attributable to the disturbance. This means that the water wave (or disturbance) moves from one place to another, *but the water isn't carried with it*. When we observe a water wave, we see a rearrangement of the water's surface. Without the water, there wouldn't be a wave. Similarly, a wave traveling on a string wouldn't exist without the string. Sound waves travel through air as a result of pressure variations from point to point. Therefore, we can consider a wave to be *the motion of a disturbance* as in figure 6.1.

Figure 6.1: Wave as motion of a disturbance.

The world is full of waves, the two main types being *mechanical* waves and *electromagnetic* waves. In the case of mechanical waves, some physical medium is being disturbed in our pebble and beach ball example, elements of water are disturbed. Electromagnetic waves do not require a medium to propagate; some examples of electromagnetic waves are visible light, radio waves, television signals, and x-rays as in figure 6.2.

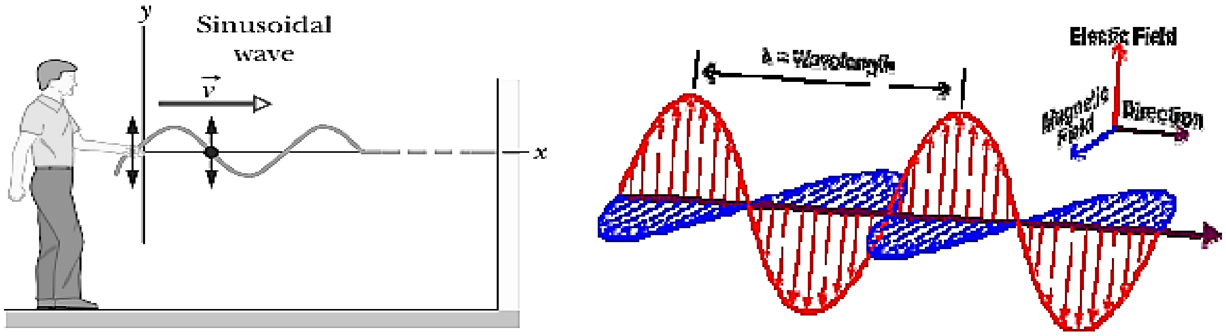


Figure 6.2: Types of waves.

Here, in this part of the book, we study only mechanical waves. The wave concept is abstract. When we observe what we call a water wave, what we see is a rearrangement of the water's surface. Without the water, there would be no wave. A wave traveling on a string would not exist without the string. Sound waves could not travel from one point to another if there were no air molecules between the two points.

With mechanical waves, what we interpret as a wave corresponds to the propagation of a disturbance through a medium. Considering further the beach ball floating on the water, note that we have caused the ball to move at one point in the water by dropping a pebble at another location. The ball has gained kinetic energy from our action, so energy must have transferred from the point at which we drop the pebble to the position of the ball. This is a central feature of wave motion energy is transferred over a distance, but matter is not.

All waves carry energy, but the amount of energy transmitted through a medium and the mechanism responsible for that transport of energy differ from case to case. For instance, the power of ocean waves during a storm is much greater than the power of sound waves generated by a single human voice.

6.2 Types of Mechanical Waves

All mechanical waves require (1) some source of disturbance, (2) a medium that can be disturbed, and (3) some physical mechanism through which elements of the medium can influence each other. There are two types of mechanical waves as illustrated in figure 6.3.

1- Transverse waves

- When the particles of the medium vibrate upwards and downwards, the wave transfers to the medium in the form of crests and troughs
- The direction of vibration is perpendicular to the direction of wave propagation.
- The work done by the vibrating source is stored in the form of:
  1. Potential energy
  2. Kinetic energy

Transverse waves are defined as *the wave in which the vibration of the medium particles is perpendicular to the direction of the wave propagation.*

2- Longitudinal Waves

- When the particles of the medium vibrate to right and left, the wave transfers to the medium in the form of compressions and rarefactions.
- The direction of vibration is same to the direction of wave propagation.
- The work done by the vibrating source is stored in the form of:
  1. Potential energy.
  2. Kinetic energy.

Longitudinal waves are defined as *the wave in which the vibration of the medium particles is same to the direction of the wave Propagation.*

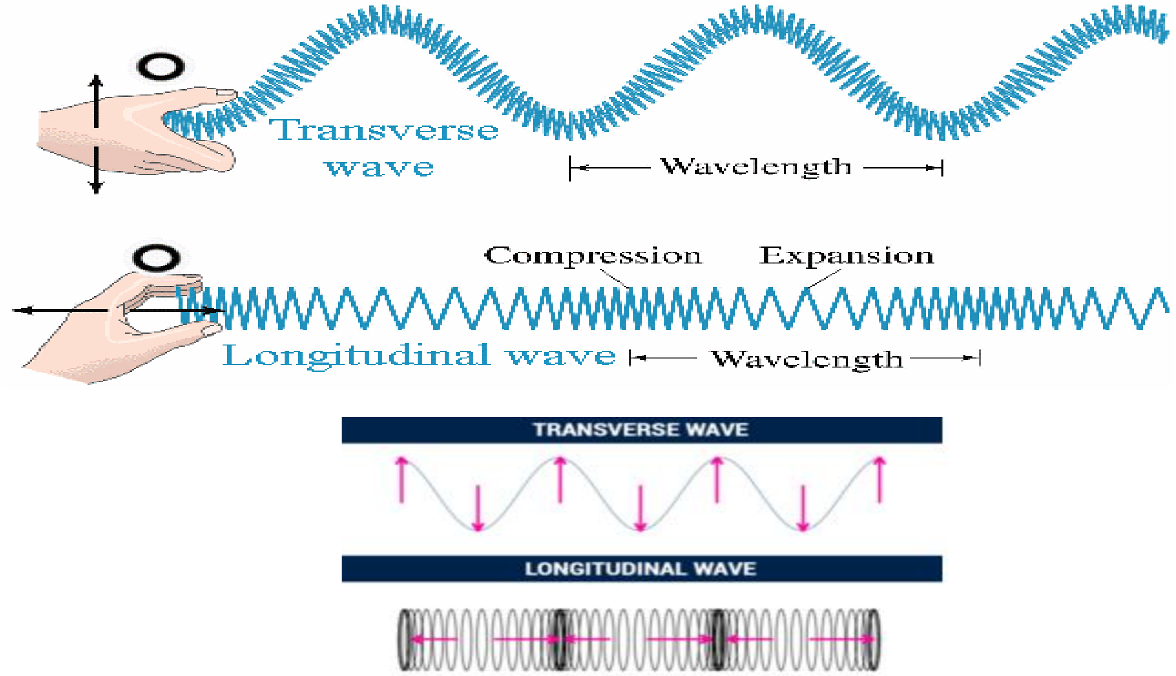


Figure 6.3: Types of mechanical waves.

6.3 Waves in a Stretched String

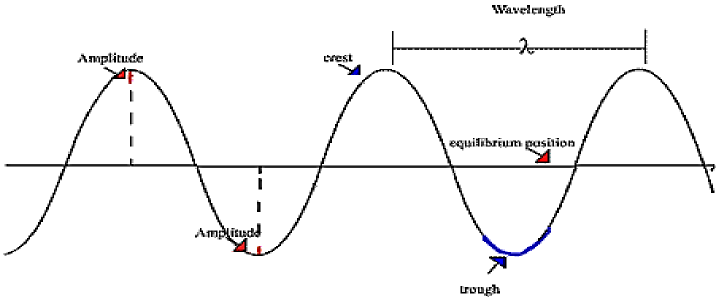


Figure 6.4: Wave in a stretched string.

Figure 6.4 shows a snapshot of a wave moving through a medium the point at which the displacement of the element from its normal position is highest is called the crest of the wave and the point at which the displacement of the element from its normal position is lowest is called the trough of the wave. The distance from one crest (trough) to the next is called the **wavelength** λ (Greek lambda). More generally, the wavelength is the minimum distance between any two identical points on adjacent waves, as shown in figure 6.4. If you count the number of seconds between the arrivals of two adjacent crests at a given point in space, you are measuring the **period** T of the waves. In general, the period is the time interval required for two identical points of adjacent waves to pass by a point. The period of the wave is the same as the period of the simple harmonic oscillation of one element of the medium. The same information is more often given by the inverse of the period, which is called the **frequency** (f). In general, the frequency of a periodic wave is the numbers of crests (or troughs, or any other point on the wave) that pass a given point in a unit time interval. The frequency of a sinusoidal wave is related to the period by the expression:

$$f = \frac{1}{T}$$

The frequency of the wave is the same as the frequency of the simple harmonic oscillation of one element of the medium. The most common unit for frequency is (s^{-1}), or $hertz$ (Hz). The corresponding unit for (T) is seconds. The maximum displacement from equilibrium of an element of the medium is called the amplitude $(Y_m$)of the wave.

We can use these definitions to derive an expression for the speed of a wave. We start with the defining equation for the wave speed (v):

$$v = {}^{\Delta x}\!/_{\Delta t}$$

The wave speed is the speed at which a particular part of the wave say, a crest moves through the medium. A wave advances a distance of one wavelength in a time interval equal to one period of the vibration. Taking $(\Delta x \ = \lambda)$ and $\Delta(t \ = T)$, we see that:

$$v = {}^{\lambda}\!/_{T}$$

Because the frequency is the reciprocal of the period, we have:

$$v = \lambda f$$

This important general equation applies to many different types of waves, such as sound waves and electromagnetic waves.

6.4 Wave Equation

The wave equation is periodic displacement function $Y\ (x,t)$ in the time (t) and position of oscillating particle (x) throughout the wave propagation direction. The mathematical formula of wave equation can be represented as:

$$Y(x,t) = Y_m\, sin(kx - \omega t) \qquad (6-1)$$

This equation can represent the motion (vibration) of stretched string, where (Y_m) is the amplitude, (k) is the angular wave number, (ω) is the angular frequency. The sign between the terms in the phase of equation determine the direction of wave propagation as following:

$i)$ $(k\,x -\ \omega\ t)$ means that the wave propagates from left to right direction $(+ve\ x - direction)$.

$ii)$ $(k\,x\ +\ \omega\ t)$ means that the wave propagates from right to left direction $(-\ ve\ x - direction)$.

$iii)$ (k) is the angular wave number, $(k\ = 2\pi/\lambda)$

$iv)$ $(k\ = 2\pi K), \Rightarrow (K)$ is the wave number, $(K\ = 1/\lambda)$

$iv)$ (ω) is the angular frequency, $(\omega\ = 2\ \pi f), \Longrightarrow (f)$ is the frequency

Transverse velocity and transverse acceleration of vibration string

- *Speed* $v\ (x,t)$, taking the derivative of the wave equation with respect to (t) while (x) is a constant

$$v(x,t)=\frac{d}{dt}Y(x,t)$$

$$=\frac{d}{dt}Y_m\,sin(kx-\omega t)$$

$$\therefore v(x,t)=-\omega Y_m\,cos(kx-\omega t) \qquad (6-2)$$

- The negative sign indicates that, the velocity decrease with increase the wave displacement
- To obtain the maximum transverse velocity substitute by maximum value of $\cos(kx-\omega t)$ which is equal one ($cos\,0=1$).

$$v_{max}=-\omega Y_m$$

- *Acceleration* $a\ (x,t)$, taking the derivative of above equation with respect to time

$$a(x,t)=\frac{d}{dt}v(x,t)$$

$$=\frac{d}{dt}[-\omega Y_m\,cos(kx-\omega t)]$$

$$\therefore a(x,t)=-\omega^2Y_m\,sin(kx-\omega t) \qquad (6-3)$$

- The negative sign indicates that, the acceleration decrease with increase the wave displacement (this equation of simple harmonic motion).
- To obtain the maximum transverse velocity substitute by maximum value of which is equal one ($sin\,0=1$).

$$a_{max}=-\omega^2Y_m \qquad (6-4)$$

<u>Example (6.1):</u>

Sinusoidal wave traveling along a string is described by:

$$\boldsymbol{Y(x,t)=0.00327\,sin(72.1\,x-2.72\,t)}$$

in which the numerical constants are in SI units. (a) What is the amplitude of this wave? (b) What are the wavelength and the period of this wave? (c) What are the wave number and the frequency of this wave (d) What is the speed of this wave.

<u>Solution:</u>

$(a)\ Y_m=\ 0.00327\,m$

$(b)\ \lambda=\frac{2\pi}{k}=\frac{2\pi}{(72.1\frac{rad}{m})}=0.0871\,m, \qquad T=\frac{2\pi}{\omega}=\frac{2\pi}{(2.72\,rad/s)}=2.31\,s$

$$(c)\ \ K=\frac{1}{\lambda}=\frac{1}{(0.0871\,m)}=11.5\,m^{-1},\qquad f=\frac{1}{T}=\frac{1}{(2.31\,s)}=0.433\,Hz$$

$$(d)\ \ v=\frac{\omega}{k}=\frac{(2.72\,rad/s)}{(72.1\,\frac{rad}{m})}=0.0377\,m/s$$

Example (6.2):

Sinusoidal wave traveling along a string is described by:

$$\boldsymbol{Y(x,t)=0.00327\,sin(72.1\,x-2.72\,t)}$$

What is the displacement Y at $\boldsymbol{x\ =\ 22.5}$ **cm** and $\boldsymbol{t\ =\ 18.9\ s}$?

Solution:

$$Y(x,t)=(0.00327\,m)\,sin((72.1\frac{rad}{m})\,x-(2.72\frac{rad}{s})\,t)$$

$$=(0.00327\,m)\,sin((72.1\frac{rad}{m})\,((22.5\times10^{-2}\,m))-(2.72\frac{rad}{s})\,(18.9\,s))$$

$$=0.00192\,m$$

Example (6.3):

In Example (6.2), the displacement $\boldsymbol{Y\ =\ 0.00192\ m}$ at $\boldsymbol{x\ =\ 22.5\ cm}$ and $\boldsymbol{t\ =\ 18.9\ s}$? (a) What is the transverse speed of the same element of the string? (b) What is the transverse acceleration of the same element of the string?

Solution:

$$u=-\omega y_m\,cos(kx-\omega t)$$

$$=-(2.72\,rad/s)(0.00327\,m)\,cos((72.1\,rad/m)(22.5\times10^{-2}\,m)$$

$$-((2.72\,rad/s)(18.9\,s))=0.0072\,m/s$$

$$a=-\omega^2Y_m\,sin(kx-\omega t)\ \ =-(2.72\,rad/s)^2(0.00327\,m)\,sin(72.1\,rad/m)(22.5\times10^{-2}\,m)$$

$$-(2.72\,rad/s)(18.9\,s)=0.0142\,m/s$$

6.5 Wave Speed on a Stretched String

For a vibrating string, there are two speeds to consider. One is the speed of the physical string that vibrates up and down, transverse to the string, in the $y-\ direction$. The other is the *wave speed*, which is the rate at which the disturbance propagates along the length of the string in the $x-direction$. We wish to find an expression for the wave speed.

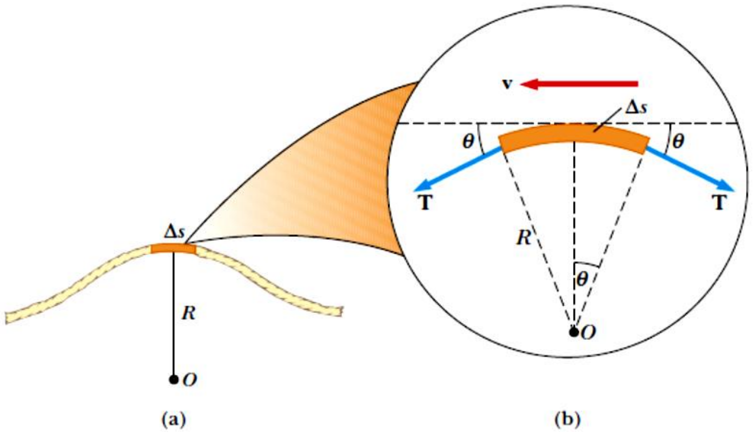


Figure 6.5: (a) To obtain the speed v of a wave on a stretched string, it is convenient to describe the motion of a small element of the string in a moving frame of reference. (b) In the moving frame of reference, the small element of length Δs moves to the left with speed v. The net force on the element is in the radial direction because the horizontal components of the tension force cancel.

Derivation by using Newton's Second Law

Consider a small string element of length (ΔL) and its mass (Δm). It forms a single pulse such as that in figure an arc of a circle of radius (R) and making an angle (2θ). This element oscillates under the action of two forces,

(1) Tangential tension force (τ) acts in both side of element.

- The horizontal components (x-axis) of tension forces cancel

$$\tau cos\theta - \tau cos\theta = 0$$

- The vertical components (y-axis) add to form a radial restoring force.

$$F_\tau = \tau sin\theta + \tau sin\theta = 2\tau sin\theta$$

(2) Centripetal force, (F_c) (imagine the element as part of circle of radius (R)

$$F_c = \Delta m \frac{v^2}{R}$$

$$F_t = F_c$$

$$2\tau \sin\theta = \Delta m \frac{v^2}{R}$$

From figure 6.5; $sin\theta = \frac{\Delta l}{R}$, $\mu = \frac{\Delta m}{\Delta l}$

$$2\tau\theta = \mu \Delta L \frac{v^2}{R}$$

$$\tau(2\theta) = \mu \Delta L \frac{v^2}{R}$$

$$\tau(\frac{\Delta L}{R}) = \mu \Delta L \frac{v^2}{R}$$

$$\tau = \mu v^2 \Rightarrow v = \sqrt{\frac{\tau}{\mu}} \qquad (6-5)$$

where τ the tension in the string and (μ) is the mass of the string per unit length, called the *linear density*. From equation (6-5), it's clear that a larger tension τ results in a larger wave speed, whereas a larger linear density (μ) gives a slower wave speed, as expected. According to equation (6-5), the propagation speed of a mechanical wave, such as a wave on a string, depends only on the properties of the string through which the disturbance travels. It doesn't depend on the amplitude of the vibration. This turns out to be generally true of waves in various media.

The dimensional analysis can easily verify that the expression is dimensionally correct. Note that the dimensions of (τ) are ML/T^2, and the dimensions of (μ) are (M/L). The dimensions of $(\tau$ / μ) are therefore (L^2/T^2), so those of $\sqrt{\tau / \mu}$ are L/T, giving the dimensions of speed. No other combination of (τ) and (μ) is dimensionally correct, so in the case in which the tension and mass density are the only relevant physical factors, we have verified equation (6-5) up to an overall constant. According to equation (6-5), we can increase the speed of a wave on a stretched string by increasing the tension in the string. Increasing the mass per unit length, on the other hand, decreases the wave speed. These physical facts lie behind the metallic windings on the bass strings of pianos and guitars. The windings increase the mass per unit length, m, decreasing the wave speed and hence the frequency, resulting in a lower tone. Tuning a string to a desired frequency is a simple matter of changing the tension in the string.

Example (6.4):

A uniform cord has a mass of $\mathbf{0.3\ kg}$ and a length of $\mathbf{6\ m}$. The cord passes over a pulley and supports a $\mathbf{2\ kg}$ object. Find the speed of a pulse traveling along this cord.

Solution:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{mg}{M/L}} = \sqrt{\frac{(0.3\ kg)(9.81\ m/s^2)}{(2\ kg)/(6\ m)}} = 19.8\ m/s$$

6.6 Rate of Energy Transfer by Sinusoidal Waves on Strings

Waves transport energy as kinetic and elastic potential energy when they propagate through a medium. Consider a sinusoidal wave traveling on a string (figure 6.6). The source of the energy is some external agent at the left end of the string, which does work in producing the oscillations. We can consider the string to be a non-isolated system. As the external agent performs work on the end of the string, moving it up and down, energy enters the system of the string and propagates along its length. Let us focus our attention on an element of the string of length, (Δx) and mass, (Δm). All elements have the same angular frequency (ω) and the same amplitude (A). The kinetic energy (K) associated with a moving particle is $(K = 1/2\, mv^2)$. If we apply this equation to an element of length (Δx) and mass $\Delta(m)$, we see that the kinetic energy (ΔK) of this element is

$$K = 1/2\,(\Delta m) v^2$$

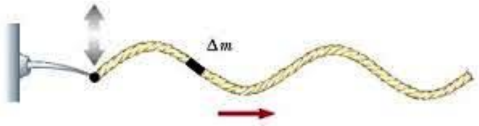


Figure 6.6: A sinusoidal wave traveling along the $x - axis$ on a stretched string. Every element moves vertically, and every element has the same total energy.

where (v) is the transverse speed of the element. If (μ) is the mass per unit length of the string, then the mass (Δm) of the element of length (Δx) is equal to $(\mu\, \Delta x)$. Hence, we can express the kinetic energy of an element of the string as

$$\Delta K = \frac{1}{2}(\mu\, \Delta x)\, v^2$$

As the length of the element of the string shrinks to zero, this becomes a differential relationship:

$$dK = \frac{1}{2}(dm)\, v^2$$

$$dK = \frac{1}{2}(\mu\, dx)\, v^2$$

We substitute for the general transverse speed of a simple harmonic oscillator:

$$dK = \frac{1}{2}(\mu\, dx)\,(-\omega y_m \cos(kx - \omega t))^2$$

$$dK = \frac{1}{2}(\mu\, dx)\,\omega^2 {y_m}^2 \cos^2(kx - \omega t)$$

Dividing the previous equation by (dt) gives the rate at which kinetic energy passes through a string element, and (dx/dt) equals the speed of wave (v).

$$\frac{dK}{dt} = \frac{1}{2}(\mu\, \frac{dx}{dt})\,\omega^2 {y_m}^2 \cos^2(kx - \omega t)$$

$$\frac{dK}{dt} = \frac{1}{2}\mu\, v\, \omega^2 {y_m}^2 \cos^2(kx - \omega t)$$

The average rate at which kinetic energy is transported is

$$\overline{\frac{dK}{dt}} = \frac{1}{2}\,\mu\, v\, \omega^2 {y_m}^2\, \overline{\cos^2(kx - \omega t)}$$

$$\overline{\cos^2(kx - \omega t)} = \frac{1}{2}$$

$$\overline{\frac{dK}{dt}} = \frac{1}{2}\,\mu\, v\, \omega^2 {y_m}^2\, (\frac{1}{2})$$

$$\overline{\frac{dK}{dt}} = \frac{1}{4}\,\mu\, v\, \omega^2 {y_m}^2$$

In addition to kinetic energy, each element of the string has potential energy associated with it due to its displacement from the equilibrium position and the restoring forces from neighboring elements. A similar analysis to that above for the total potential energy (U) in one wavelength will give exactly the same result:

$$\overline{\frac{dU}{dt}} = \frac{1}{4}\,\mu\, v\, \omega^2 {y_m}^2$$

$$\bar{P} = \overline{\frac{dK}{dt}} + \overline{\frac{dU}{dt}}$$

$$= \frac{1}{4}\,\mu\, v\, \omega^2 {y_m}^2 + \frac{1}{4}\,\mu\, v\, \omega^2 {y_m}^2$$

$$\therefore \boldsymbol{\bar{P} = \frac{1}{2}\,\mu\, v\, \omega^2 {y_m}^2} \qquad (6-6)$$

This expression shows that the rate of energy transfer by a sinusoidal wave on a string is proportional to (a) the square of the frequency, (b) the square of the amplitude, and (c) the wave speed. In fact: the rate of energy transfer in any sinusoidal wave is proportional to the square of the angular frequency and to the square of the amplitude.

Example (6.5):

A string has linear density $\mu = \mathbf{525\ g/m}$ and is under tension $\boldsymbol{\tau = 45\ N}$. We send a sinusoidal wave with frequency $\boldsymbol{f = 120\ Hz}$ and amplitude $\boldsymbol{y_m = 8.5\ \mathrm{mm}}$ along the string. At what average rate does the wave transport energy?

Solution:

$$\omega = 2\pi f = 2(3.14)(120\ Hz) = 754\ rad/s$$

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{(45\,N)}{(525 \times 10^{-3}\,kg/m)}} = 9.26\,m/s$$

$$P_{av} = \frac{1}{2}\mu v \omega^2 y_m^2 = \frac{1}{2}\left(525 \times 10^{-3}\frac{kg}{m}\right)\left(9.26\frac{m}{s}\right)\left(754\frac{rad}{s}\right)^2 (8.5 \times 10^{-3}\,m) = 100W$$

6.7 Interference of Waves

It is overlap of two or more waves to produce a new wave has a new intensity. The waves must have same amplitude, form, frequency or wavelength and same propagation direction to be interfere. The phase or path difference between two waves is constant with time. Interference is very important in the application of communication science.

To derive an expression for interference of wave;

- Let two wave traveling along x-axis with constant phase difference between them (φ), their equations given by

$$Y_1 = Y_m\, sin(kx - \omega t)$$
$$Y_2 = Y_m\, sin(kx - \omega t + \phi)$$

- From the principle of superposition, the resultant wave is the algebraic sum of the two interfering waves and has displacement

$$Y = Y_1 + Y_2$$
$$= Y_m\, sin(kx - \omega t) + Y_m\, sin(kx - \omega t + \phi)$$
$$= Y_m\, [sin(kx - \omega t) + sin(kx + \omega t + \phi)]$$
$$= Y_m\, [sin(kx - \omega t) + sin(kx + \omega t + \phi)]$$

- We can write the sum of the sines of two angles (α) and (β) as;

$$sin\,\alpha + sin\,\beta = 2\,sin\left[\frac{\alpha+\beta}{2}\right]cos\left[\frac{\alpha-\beta}{2}\right]$$

$$= 2Y_m\,sin\left[\frac{(kx - \omega t) + (kx - \omega t + \phi)}{2}\right]cos\left[\frac{(kx - \omega t) - (kx - \omega t + \phi)}{2}\right]$$

$$\therefore \boldsymbol{Y = [2Y_m\,cos(\frac{\phi}{2})]\,sin\left(kx - \omega t + \frac{\phi}{2}\right)} \qquad (6-7)$$

The above equation is the resultant wave of interference; it is sinusoidal wave traveling in the direction of increasing (x). The interference wave equation dependent on phase constant is φ

The amplitude of interference equation is $(2Y_m Cos\ \varphi\ /2)$.

- If $(\varphi\ =\ 0)$, the two interfering waves are in phase and is

$$= 2\mathrm{Y_m}\,\mathrm{sin(kx - \omega t)} \Rightarrow \textbf{Fully constructive interference}$$

- If ($\varphi = \pi\, rad\ (or\ 180°)$), the interfering waves are out of phase as in figure 6.7. Then $\cos\varphi$ becomes $\cos(\pi/2) = 0$, the amplitude of the resultant wave is zero.

$$Y = [2Y_m \cos(\tfrac{\pi}{2})] \sin\left(kx - \omega t + \tfrac{\pi}{2}\right) = 0 \Rightarrow \textbf{Fully destructive interference}$$

From these equations we can get that the combination of separate waves in the same region of space to produce a resultant wave is called interference. For the two pulses shown in figure 6.7a, the displacement of the medium is in the positive y direction for both pulses, and the resultant wave (created when the pulses overlap) exhibits a displacement greater than that of either individual pulse. Because the displacements caused by the two pulses are in the same direction, we refer to their superposition as **constructive interference**.

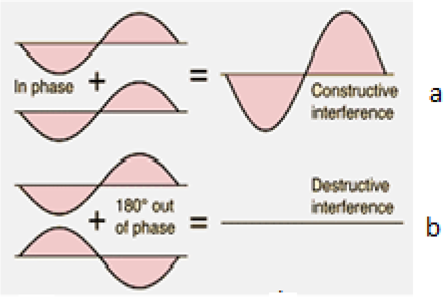


Figure 6.7: constructive and destructive interference.

Now consider two pulses traveling in opposite directions on a taut string where one pulse is inverted relative to the other, as illustrated in figure 6.7b. In this case, when the pulses begin to overlap, the resultant wave is given by but the values of the function y_2 are negative. Again, the two pulses pass through each other; however, because the displacements caused by the two pulses are in opposite directions, we refer to their superposition as **destructive interference**.

6.8 Standing Waves

If two sinusoidal waves of the same amplitude and wavelength *travel in opposite directions* along a stretched string, their interference with each other produces a standing wave. Standing waves are defined as the interference of two identical sinusoidal waves moving in opposite directions produces standing waves. To analyze a standing wave, we represent the two combining waves traveling in opposite direction of $x - axis$ with the equations:

$$Y_1 = Y_m \sin(kx - \omega t)$$

$$Y_2 = Y_m \sin(kx + \omega t)$$

From the principle of superposition, the resultant wave is the algebraic sum of the two waves and has displacement

$$Y = Y_1 + Y_2$$

$$= Y_m \sin(kx - \omega t) + Y_m \sin(kx + \omega t)$$

We can write the sum of the sines of two angles (α) and (β) as;

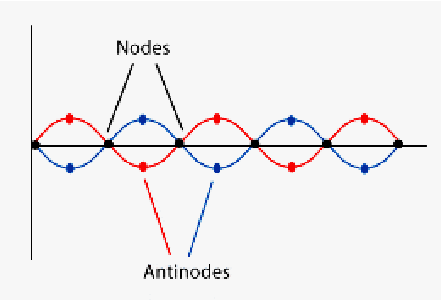


Figure 6.8: Standing wave pattern.

$$sin\,\alpha + sin\,\beta = 2\,sin\left[\frac{\alpha+\beta}{2}\right]cos\left[\frac{\alpha-\beta}{2}\right]$$

$$= 2Y_m\,sin\left[\frac{(kx-\omega t)+(kx+\omega t)}{2}\right]cos\left[\frac{(kx-\omega t)-(kx+\omega t)}{2}\right]$$

$$\therefore \boldsymbol{Y = [2Y_m\,sin(kx)]\,cos(\omega t)} \qquad (6-8)$$

This equation does not describe a traveling wave, it describes a standing wave at position (x). The quantity $[2Y_m\ Sin\ kx]$ can be viewed as the amplitude of oscillation of the standing wave that is located at position (x).

To derive the position of nodes and antinodes of standing waves shown as in figure 6.8. in terms of wavelength;

Positions of nodes:

$$nodes \Leftrightarrow [sin(kx)] = 0$$

$$[sin(kx)] = 0 \Leftrightarrow kx = n\pi, n = 0, 1, 2, 3, \dots$$

$$\frac{2\pi}{\lambda}x = n\pi, \quad x = n\frac{\lambda}{2} \qquad n = 0, 1, 2, 3, \dots$$

Positions of antinodes:

$$antinodes \Leftrightarrow [sin(kx)] = 1$$

$$[sin(kx)] = 1 \Leftrightarrow kx = (n + 1/2)\pi, n = 0, 1, 2, 3, \dots$$

$$\frac{2\pi}{\lambda}x = (n + 1/2)\pi, n = 0, 1, 2, 3, \dots$$

$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2} \qquad n = 0, 1, 2, 3, \dots$$

We note the following important features of the locations of nodes and antinodes:

- The distance between adjacent antinodes is equal to $(\lambda/2)$.
- The distance between adjacent nodes is equal to $(\lambda/2)$.
- The distance between a node and an adjacent antinode is $(\lambda/4)$.

Wave patterns of the elements of the medium produced at various times by two waves traveling in opposite directions are shown in figure 6.9. The (y_1) and (y_2) curves are the wave patterns for the individual traveling waves, and the y curves are the wave patterns for the resultant standing wave. At $(t = 0)$ (figure 6.9a), the two traveling waves are in phase, giving a wave pattern in which, each element of the medium is experiencing its maximum displacement from equilibrium. One quarter of a period later, at $(t = T/4)$ (figure 6.9b), the traveling waves have moved one quarter of a wavelength (one to the right and the other to the left). At this time, the traveling waves are out of phase, and each element of the medium is passing through the equilibrium position in its simple harmonic motion. The result is zero displacement for elements at all values of x, that is, the wave pattern is a straight line. At $(t = T/2)$ (figure 6.9c), the traveling waves are again in phase, producing a wave pattern that is inverted relative to the $(t = 0)$ pattern. In the standing wave, the elements of the medium alternate in time between the extremes shown in figure 6.9a and c.

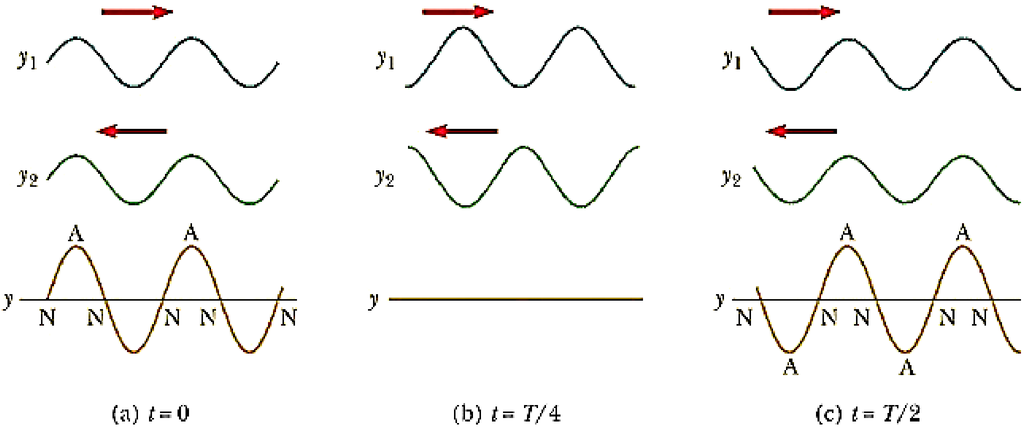


Figure 6.9: Standing-wave patterns produced at various times by two waves of equal amplitude traveling in opposite directions. For the resultant wave y, the nodes (N) are points of zero displacement, and the antinodes (A) are points of maximum displacement.

6.9 Standing Wave and Resonance

Consider a string is stretched between two clamps. Suppose we send a continuous sinusoidal wave of a certain frequency along the string. For certain frequencies (*resonant frequencies*), the interference produces a standing wave pattern (or oscillation mode) with nodes and large antinodes like those in Figure, let a string be stretched between two clamps separated by a fixed distance (L). The simplest pattern of vibrating string with different modes shows in figure 6.10 and illustrated in **table** 6-1.

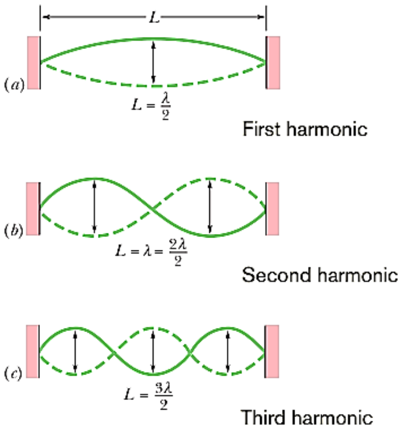


Figure 6.10: A string, stretched between two clamps, is made to oscillate in standing wave patterns.

Table 6-1: Standing waves in a stretched string.

	Number of loops string makes	**The wavelength of vibration**	**The frequency of vibration**
First harmonic mode **Or** **Fundamental mode**	One loop	$\lambda_1 = 2L$	$f_1 = \frac{v_1}{\lambda_1}$ $= \frac{1}{2L}\sqrt{\frac{\tau}{\mu}}$
Second harmonic mode	Two loops	$\lambda_2 = L$	$f_2 = \frac{1}{L}\sqrt{\frac{\tau}{\mu}}$
Third harmonic mode	Three loop	$\lambda_3 = \frac{2}{3}L$	$f_3 = \frac{3}{2L}\sqrt{\frac{\tau}{\mu}}$
Generally, for (n) harmonic mode	(n) loop	$\lambda_n = \frac{2}{n}L$	$f_n = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}$

PROBLEMS

1. By rocking a boat produces surface water waves on a previously quite lake. He observes that the boat performs $\mathbf{12}$ oscillations in $\mathbf{20\ s}$; each oscillation producing a wave's crests $\mathbf{15\ cm}$ above undisturbed surface of the lake. He further observes that a given waves crest reaches shore, $\mathbf{12\ m}$ away in $\mathbf{6\ s}$. What are (a) the period, (b) the speed, (c) the wave length, and (d) the amplitude of this wave?

2. The equation of a transverse wave traveling in a string is given by:

$$\mathbf{y(x,t) = (2\ mm)\,sin((20\ m^{-1})x - (600\ s^{-1})t)}$$

(i) Find the amplitude, frequency, velocity and wavelength of the wave. (ii) Find the maximum transverse speed of a particle in the string.

3. What is the speed of a transverse wave in a rope of length $\mathbf{2\ m}$ and mass $\mathbf{60\ g}$ under a tension of $\mathbf{500\ N}$?

4. A piano string having a mass per unit length $\mathbf{5 \times 10^{-3}\ kg/m}$ is under a tension of $\mathbf{1350\ N}$. Find the speed with which a wave travels on this string.

5. Transverse pulses travel with a speed of $\mathbf{200\ m/s}$ along a taut copper wire whose diameter is $\mathbf{1.5\ mm}$. What is the tension in the wire? (The density of copper is $\mathbf{8.92\ g/cm^3}$.)

6. The speed of a wave on a string is $\mathbf{170\ m/s}$ when the tension is $\mathbf{120\ N}$. To what value must the tension be increased in order to raise the wave speed $\mathbf{180\ m/s}$?

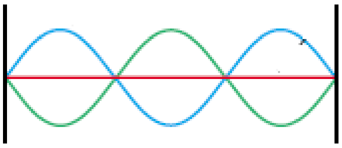
7. Power $(\boldsymbol{P_1})$ is transmitted by a wave of frequency $(\boldsymbol{f_1})$ on a string with tension $(\boldsymbol{T_1})$. What is the transmitted power $(\boldsymbol{P_2})$ in terms of $(\boldsymbol{P_1})$: (a) if, instead, the tension of the string is increased to $(\boldsymbol{T_2}) = \boldsymbol{4T_1}$, and (b) if the frequency is decreased to $\left(\boldsymbol{f_2} = \boldsymbol{f_1}/\mathbf{2}\right)$?

8. A string $\mathbf{2.7\ m}$ long has a mass of $\mathbf{260\ g}$. The tension in the string is $\mathbf{36\ N}$. What must be the frequency of traveling waves of amplitude $\mathbf{7.7\ mm}$ in order to that the average transmitted power is $\mathbf{85\ W}$?

9. A taut rope has a mass of $\mathbf{0.18\ kg}$ and a length of $\mathbf{3.6\ m}$. What power must be supplied to the rope to generate sinusoidal waves having amplitude of $\mathbf{0.1\ m}$ and a wavelength of $\mathbf{0.5\ m}$ and traveling with a speed of $\mathbf{30\ m/s}$?

10. A string fixed at both ends is $\boldsymbol{8.4\ m}$ long and has a mass of $\boldsymbol{0.12\ kg}$, it is subjected to a tension of $\boldsymbol{96\ N}$ and stets to oscillating. (a) What is the speed of the waves on the string? (b) What is the longest possible wavelength for the standing waves? (c) Give the frequency of the wave waves on the string.

11. A nylon guitar string has a linear density of $\boldsymbol{7.2\ g/m}$ and is under a tension of $\boldsymbol{150\ N}$. The fixed supports are $\boldsymbol{90\ cm}$ apart. The string is oscillating in the standing wave pattern as in the corresponding figure. Calculate the (a) speed, (b) wavelength, and frequency of the waves whose superposition gives this standing wave.



12. A cello A-string vibrates in its first normal mode with a frequency of $\boldsymbol{220\ Hz}$. The vibrating segment is $\boldsymbol{70\ cm}$ long and has a mass of $\boldsymbol{1.2\ g}$. (a) Find the tension in the string. (b) Determine the frequency of vibration when the vibrating segment is tripled.

13. A taut rope has a mass of $\boldsymbol{0.18\ kg}$ and a length of $\boldsymbol{3.6\ m}$. What power must be supplied to the rope to generate sinusoidal waves having amplitude of $\boldsymbol{0.1\ m}$ and a wavelength of $\boldsymbol{0.5\ m}$ and traveling with a speed of $\boldsymbol{30\ m/s}$?

14. A $\boldsymbol{1.5\ cm}$ wire has a mass $\boldsymbol{8.7\ g}$ and is held under a tension of $\boldsymbol{120\ N}$. The wire is held rigidly at both ends and set into vibration. Calculate

(a) The velocity of waves on the wire.

(b) The wavelength of the waves that produce one and two loop standing waves on the string.

(c) The frequency of the waves that produce one and two loop standing waves.

CHAPTER (7)
SOUND WAVES

7.1 Introduction

Sound waves are the most important example of longitudinal waves. They can travel through any material medium with a speed that depends on the properties of the medium. As the waves travel, the particles in the medium vibrate to produce changes in density and pressure along the direction of motion of the wave. These changes result in a series of high-pressure and low-pressure regions. If the source of the sound waves vibrates sinusoidally, the pressure variations are also sinusoidal. We shall find that the mathematical description of sinusoidal sound waves is identical to that of sinusoidal string waves, which was discussed in the previous chapter.

Sound waves are divided into three categories that cover different frequency ranges.

(1) **Audible waves** are waves that lie within the range of sensitivity of the human ear. They can be generated in a variety of ways, such as by musical instruments, human vocal cords, and loudspeakers.

(2) **Infrasonic waves** are waves having frequencies below the audible range. Elephants can use infrasonic waves to communicate with each other, even when separated by many kilometers.

(3) **Ultrasonic waves** are waves having frequencies above the audible range. You may have used a “silent” whistle to retrieve your dog. The ultrasonic sound it emits is easily heard by dogs, although humans cannot detect it at all. Ultrasonic waves are also used in medical imaging.

7.2 The Speed of Sound

The speed of a sound wave in a fluid depends on the fluid’s compressibility and density of medium. If the medium is a gas or a liquid and has a bulk modulus (B) and an equilibrium density (ρ), the speed of sound in that medium is

$$v = \sqrt{\frac{B}{\rho}} \qquad (7-1)$$

where bulk modulus is defined as the ratio of the change in pressure, $(\Delta P,)$ to the resulting fractional change in volume, $(\Delta V/V)$*:*

$$B = -\frac{\Delta P}{\Delta V/V} \qquad (7-2)$$

(B) is always positive because an increase in pressure (positive (ΔP)) results in a decrease in volume. Hence, the ratio $(\Delta P/\Delta V)$ is always negative.

It’s interesting to compare this expression with equation for the speed of transverse waves on a string, $\left(v = \sqrt{\tau/\mu}\right)$. In both cases the wave speed depends on an elastic property of the medium ((B) or (τ)) and on an inertial property of the medium ((ρ) or (μ)). In fact, the speed of all mechanical waves follows an expression of the general form

$$v = \sqrt{\frac{elastic\ properity}{ineratial\ properity}}$$

For longitudinal sound waves in a solid rod of material, the speed of sound depends on Young's modulus (Y) and the density (ρ) as follows

$$v = \sqrt{\frac{Y}{\rho}} \tag{7-3}$$

In general, sound travels faster through solids than liquids and faster through liquids than gases, although there are exceptions.

The speed of sound also depends on the temperature of the medium. For sound traveling through air, the relationship between the speed of sound and temperature is

$$v = \left(331\,\frac{m}{s}\right)\sqrt{\frac{T}{273\ K}} \tag{7-4}$$

where $(331\ m/s)$ is the speed of sound in air at $(0\ ℃)$ and (T) is the absolute $(Kelvin)$ temperature. Using this equation, the speed of sound in air at ( )(a typical room temperature) is approximately $(343\ m/s)$.

7.3 Traveling Sound Waves Equations

Traveling sound waves are represented by two opposite simple harmonic equations as shown in figure 7.1.

(i) 1st equation describes the oscillation of ***displacement of particles***

$$S = S_m\,cos(kx - \omega t) \tag{7-5}$$

where (S_m) is the displacement amplitude, $(k = 2\pi/\lambda)$, and $(\omega = 2\pi f)$.

(ii) 2nd equation describes the ***changes of transmitted pressure*** in medium which causes the particles displacement of medium.

$$\Delta P = \Delta P_m\,sin(kx - \omega t) \tag{7-6}$$

where (ΔP_m) is pressure amplitude.

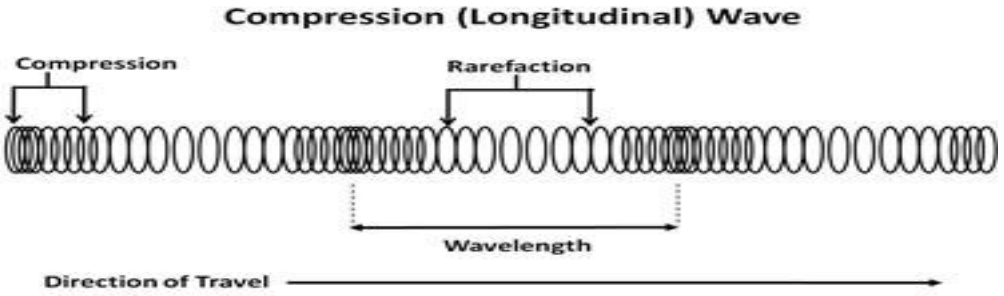


Figure 7.1: Longitudinal sound waves.

Relation between (ΔP_m) and (S_m)

Consider a thin disk-shaped element of gas whose circular cross section is parallel to the piston in figure 7.2. This element will undergo changes in position, pressure, and density as a sound wave propagates through the gas. From the definition of bulk modulus, the pressure variation in the gas is

$$\Delta P = -B\frac{\Delta V}{V_i}$$

The element has a thickness $\Delta(x)$ in the horizontal direction and a cross-sectional area (A), so its volume is ($V_i = \mathrm{A}\,\Delta \mathrm{x}$). The change in volume ΔV accompanying the pressure change is equal to ($A\Delta s$), where (Δs) is the difference between the value of s at ($x + \Delta x$) and the value of (s) at (x). Hence, we can express (ΔP) as

$$\Delta \boldsymbol{P} = -B\frac{\Delta V}{V_i} = -B\frac{A\Delta s}{A\Delta x} = -B\frac{\Delta s}{\Delta x}$$

As (Δx) approaches zero, the ratio ($\Delta s/\Delta x$) becomes $\left(\frac{\partial s}{\partial x}\right)$. (The partial derivative indicates that we are interested in the variation of (s) with position at a fixed time. Therefore,

$$\Delta P = -B\frac{\partial s}{\partial x}$$

If the position function is the simple sinusoidal function given by equation (7-5), we find that

$$\Delta P = -B\frac{\partial}{\partial x}[S_m\cos(kx-\omega t)] = BS_m k\sin(kx-\omega t)$$

Because the bulk modulus is given by ($B = \rho v^2$) (see equation (7-1)), and we can write $= \omega/v$; hence, (ΔP) can be expressed as

$$\Delta P = \rho v^2 S_m\sin(kx-\omega t) = \rho v\omega S_m\sin(kx-\omega t) = \Delta P_m\sin(kx-\omega t)$$

$$\therefore \Delta P_m = \rho v\omega S_m \qquad (7-7)$$

where the speed of sound wave is $(v) = 343\ m/s$, $(\rho_{air} = 1.21\ kg/m^3)$ and $(\omega = 2\pi f)$ depend on frequency of sound source).

Example (7.1):

The maximum pressure amplitude that the human ear can tolerate in loud sounds is about $\boldsymbol{28\ Pa}$. What is the displacement amplitude $\boldsymbol{S_m}$ for such a sound in air of density $\boldsymbol{\rho = 1.21\ kg/m^3}$, at a frequency of $\boldsymbol{1000\ Hz}$ and a speed of $\boldsymbol{343\ m/s}$?

Solution:

$$S_m = \frac{\Delta P_m}{v\rho\omega} = \frac{\Delta P_m}{v\rho(2\pi f)} = \frac{(28\ Pa)}{(343\ m/s)(1.21\ kg/m^3)(2\pi(1000\ Hz))} = 1.1 \times 10^{-9}\ m$$

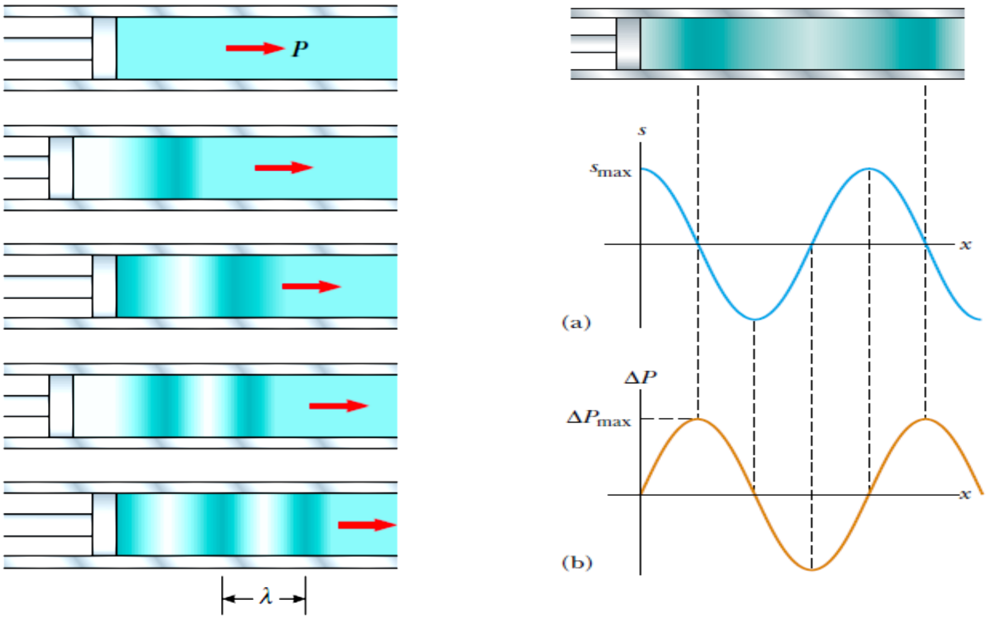


Figure 7.2: A longitudinal wave propagating through a gas filled tube. The source of the wave is an oscillating piston at the left. (a) Displacement amplitude and (b) pressure amplitude versus position for a sinusoidal longitudinal wave.

7.4 Intensity of Sound Waves (I)

The intensity (I) of a wave is defined as the average rate of energy transmitted (power) per unit area, and can be defined as the rate at which the energy being transported by the wave flows through a unit area (A) perpendicular to the direction of travel of the wave.

$$\boldsymbol{I = \frac{Power}{area}}$$

The intensity (I) at a distance (r) from a point source that emits sound waves of power (P) is given by

$$I = \frac{P}{A_{sphere}} = \frac{P}{4\pi r^2}$$

where (P) is the sound power passing through the surface, measured in watts, and the intensity again has units of watts per square meter.

The intensity (I) is related to the displacement amplitude (S_m) of the sound wave by

$$I = \frac{1}{2}\rho v \omega^2 S_m^2 = \frac{\Delta P_{max}^2}{2\rho v} \qquad (7-8)$$

Thus, we see that the intensity of a periodic sound wave is proportional to the square of the displacement amplitude and to the square of the angular frequency (as in the case of a periodic string wave). This can also be written in terms of the pressure amplitude (ΔP_{max})

$$I = \frac{\Delta P_{max}^2}{2\rho v} \qquad (7-9)$$

7.5 Sound Level

The human ear can detect a wide range of intensities**.** Because this range is so wide, it is convenient to use a logarithmic scale, where the sound level (β) (Greek beta) is defined by the equation

$$\beta = (10\ dB)\log\left(\frac{I}{I_o}\right) \qquad (7-10)$$

where the constant (I_o) is the *reference intensity*, taken to be at the threshold of hearing $(I_o = 1 \times 10^{-12}\ W/m^2)$, and I is the intensity in watts per square meter to which the sound level (β) corresponds, where (β) is measured in decibels (dB). **Table** 7-1 gives some typical sound levels.

Table 7-1: Sound levels

Sources of Sounds	β (dB)
Nearby jet airplane	150
Jackhammer; machine gun	130
Siren; rock concert	120
Subway; power mower	100
Busy traffic	80
Vacuum cleaner	70
Normal conversation	50
Mosquito buzzing	40
Whisper	30
Rustling leaves	10
Threshold of hearing	0

Example (7.2):

A point source emits sound waves with an average power output of $\mathbf{80\ W}$. (a) Find the intensity $\mathbf{3\ m}$ from the source. (B) Find the distance at which the intensity of the sound is $\mathbf{1\ \times 10^{-8}\ W/m^2}$.

Solution:

$$(a)\ I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{(80\ W)}{4\pi(3\ m)^2} = 0.707\ W/m^2$$

$$(b)\ r = \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{(80\ W)}{4\pi(1\times 10^{-8}\ W/m^2)}}$$

Example (7.3):

Two identical machines are positioned the same distance from a worker. The intensity of sound delivered by each machine at the location of the worker is $\mathbf{2\ \times 10^{-7}\ W/m^2}$. Find the sound level heard by the worker (a) when one machine is operating and (b) when both machines are operating.

Solution:

(a) The sound level at the location of the worker with one machine operating

$$\beta = (10\ dB) log\frac{I}{I_o} = (10\ dB) log\frac{(2\times 10^{-12}\ W/m^2)}{(1\times 10^{-12}\ W/m^2)} = 53\ dB$$

(b) When both machines are operating, the intensity is doubled to $4\ \times\ 10^{-7}\ W/m^2$; therefore, the sound level now is

$$\beta = (10\ dB) log\frac{I}{I_o} = (10\ dB) log\frac{(4\times 10^{-12}\ W/m^2)}{(1\times 10^{-12}\ W/m^2)} = 56\ dB$$

From these results, we see that when the intensity is doubled, the sound level increases by only 3 dB.

7.6 Sources of Musical Sound

Standing waves can be set up in a tube of air, such as that inside an organ pipe, as the result of interference between longitudinal sound waves traveling in opposite directions. The phase relationship between the incident wave and the wave reflected from one end of the pipe depends on whether that end is open or closed. This relationship is analogous to the phase relationships between incident and reflected transverse waves at the end of a string when the end is either fixed or free to move.

As shown in figure 7.3. a pipe open at both ends, the natural frequencies of oscillation form a harmonic series that includes all integral multiples of the fundamental frequency. Because all harmonics are present, and because the fundamental frequency is given by the same expression as that for a string, we can express the natural frequencies of oscillation as

$$f_n = \frac{nv}{2L} \qquad n = 1, 2, 53, \ldots$$

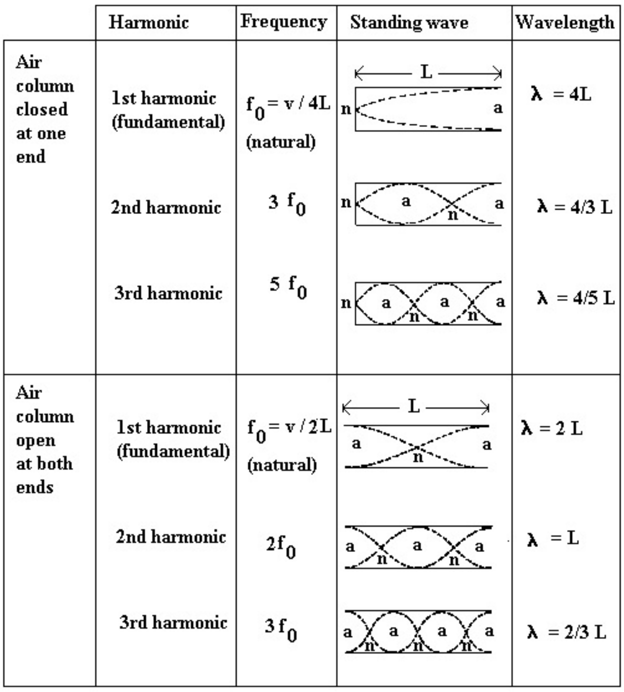
where (v) is the sound speed in air and (L) is the length of the pipe.

In a pipe closed at one end, the natural frequencies of oscillation form a harmonic series that includes only odd integral multiples of the fundamental frequency. Because all harmonics are present, and because the fundamental frequency is given by the same expression as that for a string, we can express the natural frequencies of oscillation as

$$f_n = \frac{nv}{4L} \qquad n = 1, 3, 5, \dots$$

where, (v) is the sound speed in air and (L) is the length of the pipe.

Musical instruments based on air columns are generally excited by resonance. The air column is presented with a sound wave that is rich in many frequencies. The air column then responds with a large-amplitude oscillation to the frequencies that match the quantized frequencies in its set of harmonics. In many woodwind instruments, the initial rich sound is provided by a vibrating reed. In the brasses, this excitation is provided by the sound coming from the vibration of the player's lips. In a flute, the initial excitation comes from blowing over an edge at the mouthpiece of the instrument. This is similar to blowing across the opening of a bottle with a narrow neck. The sound of the air rushing across the edge has many frequencies, including one that sets the air cavity in the bottle into resonance.



	Harmonic	Frequency	Standing wave	Wavelength
Air column closed at one end	1st harmonic (fundamental)	$f_0 = v/4L$ (natural)	L n a	$\lambda = 4L$
	2nd harmonic	$3f_0$	n a n a	$\lambda = 4/3\ L$
	3rd harmonic	$5f_0$	n a n a n a	$\lambda = 4/5\ L$
Air column open at both ends	1st harmonic (fundamental)	$f_0 = v/2L$ (natural)	L a n a	$\lambda = 2L$
	2nd harmonic	$2f_0$	a n a n a	$\lambda = L$
	3rd harmonic	$3f_0$	a n a n a n a	$\lambda = 2/3\ L$

n = node, a = antinode

Figure 7.3: Standing waves in air columns.

7.7 The Doppler Effect

If a car or truck is moving while its horn is blowing, the frequency of the sound you hear is higher as the vehicle approaches you and lower as it moves away from you. This phenomenon is one example of the *Doppler effect*, named for Austrian physicist Christian Doppler (1803–1853), who discovered it. The same effect is heard if you're on a motorcycle and the horn is stationary: the frequency is higher as you approach the source and lower as you move away. Although the Doppler effect is most often associated with sound, it's common to all waves, including light. In deriving the Doppler effect, we assume the air is stationary and that all speed measurements are made relative to this stationary medium. The speed (v_o) is the speed of the observer, (v_s) is the speed of the source, and (v) is the speed of sound as shown in figure 7.5.

- **Case 1:** The Observer Is Moving Relative to a Stationary Source:

An observer is moving with a speed of (v_o) toward the source (considered a point source) as shown in figure 7.4, which is at rest $(v_s = 0)$, we can get

(i) Detector move away from source;

$$\acute{f} = f\frac{v - v_o}{v}$$

(ii) Detector move toward the source;

$$\acute{f} = f\frac{v + v_o}{v}$$

where $(\acute{f})$ is the beat frequency which is the difference between two combined frequencies and (f) is the source frequency.

- **Case 2:** The source is moving relative to a stationary observer:

A source is moving toward an observer at rest $(v_o = 0)$ as shown in figure 7.5., We obtained:

(i) Source move away from detector;

$$\acute{f} = f\frac{v}{v + v_s}$$

(ii) Source move toward the detector;

$$\acute{f} = f\frac{v}{v - v_s}$$

- **General Case:** When both the source and the observer are in motion relative to Earth, we obtained:

$$\acute{f} = f\frac{v \pm v_o}{v \mp v_s}$$

the signs are chosen such that $(\acute{f})$, tends to be greater for relative motion toward (one of the objects moves toward the other) and less for motion away.

Choosing incorrect signs is the most common mistake made in working a Doppler effect problem. The following rules may be helpful: The word *toward* is associated with an *increase* in the observed frequency; the words *away from* are associated with a *decrease* in the observed frequency.

Although the Doppler effect is most typically experienced with sound waves, it is a phenomenon that is common to all waves. For example, the relative motion of source and observer produces a frequency shift in light waves. The Doppler effect is used in police radar systems to measure the speeds of motor vehicles. Likewise, astronomers use the effect to determine the speeds of stars, galaxies, and other celestial objects relative to the Earth.

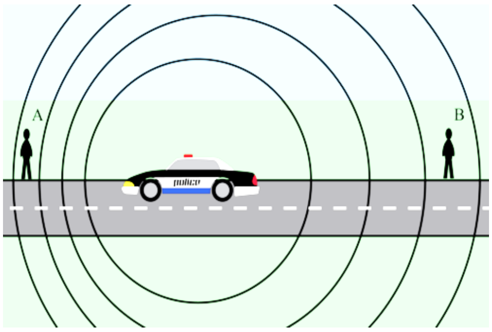


Figure 7.5: A source (S) moving with speed (v_s) toward a stationary observer (A) and away from a stationary observer (B). Observer (A) hears an increased frequency, and observer (B) hears a decreased frequency.

Example (7.4):

As an ambulance travels east down a highway at a speed of $\mathbf{33.5\ m/s}$, its siren emits sound at a frequency of $\mathbf{400\ Hz}$. What frequency is heard by a person in a car traveling west at $\mathbf{24.6\ m/s}$ (a) as the car approaches the ambulance and (b) as the car moves away from the ambulance?

Figure 7.4: An observer (O) (the cyclist) moves with a speed (v_o) toward a stationary point source (S), the horn of a parked truck. The observer hears a frequency $(\acute{f})$ that is greater than the source frequency.

Solution:

(a) From the general case equation, taking the speed of sound in air to be 343 m/s. As the ambulance and car approach each other, the person in the car hears the frequency

$$\grave{f} = f\frac{v + v_o}{v - v_s} = (400\text{ Hz})\left(\frac{343\text{ m/s} + 24.6\text{ m/s}}{343\text{ m/s} - 33.5\text{ m/s}}\right) = 475\text{ Hz}$$

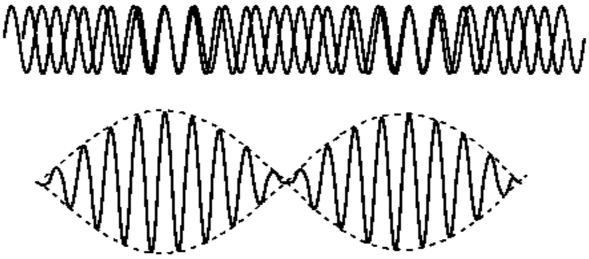
(b) As the vehicles recede from each other, the person hears the frequency

$$\grave{f} = f\frac{v - v_o}{v + v_s} = (400\text{ Hz})\left(\frac{343\text{ m/s} - 24.6\text{ m/s}}{343\text{ m/s} + 33.5\text{ m/s}}\right) = 338\text{ Hz}$$

7.7.1 Applications on Doppler effect

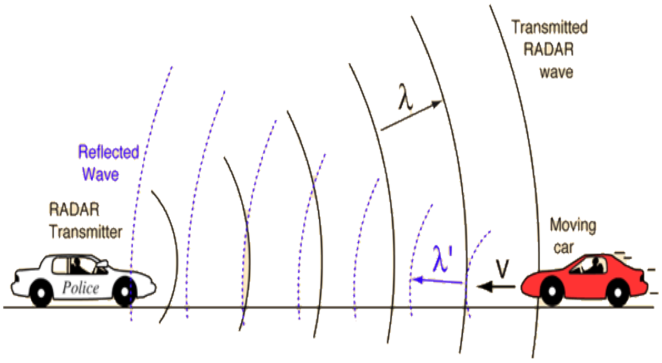
I) Subjective tones

When two single-frequency tones are present in the air at the same time, they will interfere with each other and produce a beat frequency. The beat frequency is equal to the difference between the frequencies of the two tones and if it is in the mid-frequency region, the human ear will perceive it as a third tone, called a "subjective tone" or "difference tone".



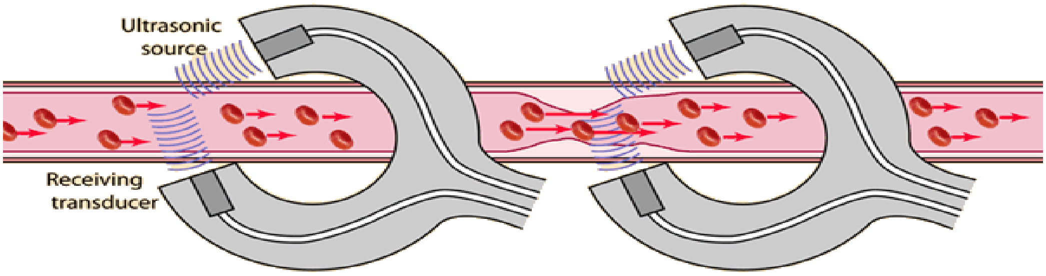
II) Police RADAR

RADAR speed detectors reflected waves from moving cars. These waves are shifted in frequency by the Doppler Effect, and the frequency between the directed and reflected waves provides a measure of the car speed.



III) Doppler Pulse Detection

The Doppler Effect in an ultrasonic pulse probe detects the reflected sound from moving blood. The frequency of the reflected sound is different, and the beat frequency between the direct and reflected sounds can be amplified and used in earphones to hear the pulse sound.



The surges in blood speed with the pumping action of the heart cause detectable changes in the beat frequency.

The increase in blood speed caused by a constriction or obstruction in an artery can be detected as a change in beat frequency.

7.8 Shock Waves

What happens when the source speed (v_s) *exceeds* the wave velocity (v)? Figure 7.5. describes this situation graphically. The circles represent spherical wave fronts emitted by the source at various times during its motion. At $(t = 0)$, the source is at point (S_o), and at some later time t, the source is at point (S_n). In the interval (t), the wave front centered at (S_o) reaches a radius of (vt). In this same interval, the source travels to (S) a distance of $(v_s t)$. At the instant the source is at (S_n), the waves just beginning to be generated at this point have wave fronts of zero radius. The line drawn from (S_n) to the wave front centered on (S_o) is tangent to all other wave fronts generated at intermediate times. All such tangent lines lie on the surface of a cone. The angle (θ) between one of these tangent lines and the direction of travel is given by

$$\sin\theta = v/v_s$$

The ratio (v_s/v) is called the **Mach number**. The conical wave front produced when $v_s > v$ (supersonic speeds) is known as a **shock wave**.

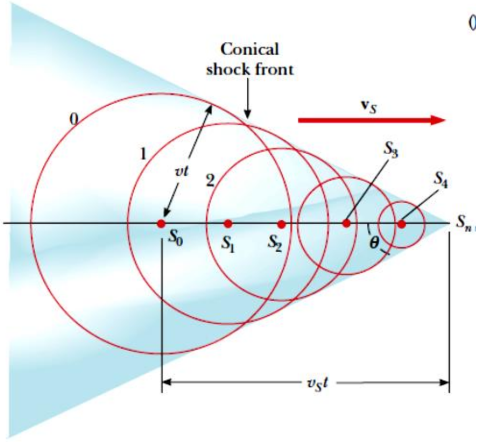


Figure 7.6: (a) A representation of a shock wave produced when a source moves from (S_o) to (S_n) with a speed (v_s), which is greater than the wave speed (v) in the medium. The envelope of the wave fronts forms a cone whose apex half-angle is given by $(\sin\theta = v/v_s)$.

Jet aircraft and space shuttles traveling at supersonic speeds produce shock waves that are responsible for the ***loud explosion***, or ***sonic boom***, heard on the ground. A shock wave carries a great deal of energy concentrated on the surface of the cone, with correspondingly great pressure variations. Shock waves are unpleasant to hear and can damage buildings when aircraft fly supersonically at low altitudes. In fact, an airplane flying at supersonic speeds produces a double boom because two shock waves are formed: one from the nose of the plane and one from the tail.

PROBLEMS

1. Find the speed of sound in mercury, which has a bulk modulus of approximately $\mathbf{2.8\ x\ 10^{10}\ N/m^2}$ and a density of $\mathbf{13600\ kg/m^3}$.

2. You are at a large outdoor concert seated $\mathbf{300\ m}$ from the speaker system. The concert is also being broadcast live via satellite. Consider a listener $\mathbf{5000\ km}$ away. Who hears the music first, you or the listener, and by what time difference?

3. As a certain sound wave travels through the air, it produces pressure variations (above and below atmospheric pressure) given by $\Delta\mathbf{P} = \mathbf{1.27\ sin\ (\pi x - 340\pi t)}$ in SI units. Find (a) the amplitude of the pressure variations, (b) the frequency, (c) the wavelength in air, and (d) the speed of the sound wave.

4. The maximum pressure variation that the ear tolerates in loud sounds is about $\mathbf{28\ Pa}$. What is the displacement amplitude for such a sound in air of density $\mathbf{1.21\ Kg/m^3,}$ at a frequency of $\mathbf{1000\ Hz}$?

5. A certain loudspeaker produces a sound with frequency $\mathbf{2000\ Hz}$ and an intensity of $\mathbf{0.96\ W/m^2}$ at a distance of $\mathbf{6.1\ m}$. Assume that there is no reflection and that the loudspeaker emits the power in all direction. (a) What is intensity at $\mathbf{30\ m}$? (b) What is the displacement amplitude at $\mathbf{6.1\ m}$? (c) What is the pressure amplitude at $\mathbf{6.1\ m}$? where $\boldsymbol{\rho_{air}} = \mathbf{1.21\ Kg/m^3.}$

6. The source of a sound wave has a power of $\mathbf{1\ pW}$. If it is a point source. (a) What is the intensity at $\mathbf{3\ m}$ away? (b)What is the sound level in decibels at that distance?

7. A vacuum cleaner produces sound with a measured sound level of $\mathbf{70\ dB}$. (a) What is the intensity of this sound in $\mathbf{W/m^2}$? (b) What is the pressure amplitude of the sound?

8. The intensity of a sound wave at a fixed distance from a speaker vibrating at $\mathbf{1\ kHz}$ is $\mathbf{0.6\ W/m^2}$. (a) Determine the intensity if the frequency is increased to $\mathbf{2.5\ kHz}$ while a constant displacement amplitude is maintained. (b) Calculate the intensity if the frequency is reduced to $\mathbf{0.5\ kHz}$ and the displacement amplitude is doubled.

9. A sound wave of frequency $\mathbf{1000\ Hz}$ propagating through air has pressure amplitude $\mathbf{10\ Pa}$. What are: (a) Wavelength (b) particle displacement amplitude (c) An organ pipe open at both ends has this frequency as fundamental. How long is the pipe? Density of air $\mathbf{1.2\ kg/\ m^3}$.

10. (a) Find the speed of waves on an $\mathbf{800\ mg}$ violin string $\mathbf{22\ cm}$ long if the fundamental frequency is $\mathbf{920\ Hz}$ (b) What is the tension in the string for the fundamental (c) What is the wavelength of the waves on the string and (d) The sound waves emitted by the string?

11. A pipe open at both ends has a fundamental frequency of $\mathbf{300\ Hz}$. (a) What is the length of the pipe? (b) What is the fundamental frequency?

12. Calculate the length of a pipe that has a fundamental frequency of $\mathbf{240\ Hz}$ if the pipe is (a) closed at one end and (b) open at both ends.

13. Two identical tuning forks can oscillate at $\mathbf{440\ Hz}$. A person is located somewhere on the line between them. Calculate the beat frequency as measured by this individual if (a) she is standing still and the tuning forks both move to the right, say at $\mathbf{30\ m/s}$ and (b) the tuning forks are stationary and the listener moves to the right at $\mathbf{3\ m/s}$?

14. Acoustic burglar alarm consists of a source emitting waves of frequency $\mathbf{28\ kHz}$. What will the beat frequency of the waves reflected from an intruder walking at an average speed of $\mathbf{0.95\ m/s}$ directly away from the alarm?

15. An airplane traveling at half the speed of sound ($\boldsymbol{v = 172\ m/s}$) emits a sound of frequency $\mathbf{5\ kHz}$. At what frequency does a stationary listener hear the sound (a) as the plane approaches? (b) After it passes?

CHAPTER (8)
TEMPERATURE AND THERMODYNAMICS

8.1 Introduction

Thermodynamics is *the study which involves situations in which the temperature or state (solid, liquid, gas) of a system changes due to energy transfers.* Thermodynamics is very successful in explaining the bulk properties of matter and the correlation between these properties and the mechanics of atoms and molecules.

Thermodynamics also addresses more practical questions. Have you ever wondered how a refrigerator is able to cool its contents, or what types of transformations occur in a power plant or in the engine of your automobile, or what happens to the kinetic energy of a moving object when the object comes to rest? The laws of thermodynamics can be used to provide explanations for these and other phenomena.

Description of thermal phenomena requires definitions of such important terms as temperature, heat, and internal energy. So this chapter begins with a discussion of temperature.

8.2 Temperature

Temperature is one of the seven SI basic quantities. Physicists measure temperature on the Kelvin scale, which is marked in units called kelvins. Although the temperature of a body apparently has no upper limit, it does have a lower limit; this limiting low temperature is taken as the zero of the Kelvin temperature scale. Room temperature is about $290\ kelvins$, or $290\ K$ as we write it, above this absolute zero.

The concept of temperature often associates with how hot or cold an object feels when we touch it. In this way, our senses provide us with a qualitative indication of temperature. Our senses, however, are unreliable and often mislead us. Your skin “measures” the rate of energy transfers by heat rather than the actual temperature. What we need is a reliable and reproducible method for measuring the relative hotness or coldness of objects rather than the rate of energy transfer. Scientists have developed a variety of thermometers for making such quantitative measurements.

To understand the concept of temperature, it is useful to define two often used phrases: thermal contact and thermal equilibrium. To grasp the meaning of thermal contact, let us imagine that two objects are placed in an insulated container such that they interact with each other but not with the rest of the world. If the objects are at different temperatures, energy is exchanged between them, even if they are initially not in physical contact with each other. We always assume that two objects are in thermal contact with each other *if energy can be exchanged between them.* Thermal equilibrium *is a situation in which two objects in thermal contact with each other cease to exchange energy by the process of heat.*

8.3 The Zeroth Law of Thermodynamics

Let us consider two objects (A) and (B), which are not in thermal contact, and a third object (C), which is our thermometer. We wish to determine whether (A) and (B) are in thermal equilibrium with each other. The thermometer (object C) is first placed in thermal contact with object (A) until thermal equilibrium is reached. From that moment on, the thermometer's reading remains constant, and we record this reading. The thermometer is then removed from object (A) and placed in thermal contact with object (B). The reading is again recorded after thermal equilibrium is reached. If the two readings are the same, then object (A) and object (B) are in thermal equilibrium with each other. We can summarize these results in a statement known as the zeroth law of thermodynamics (the law of equilibrium), which states that: "*If objects (A) and (B) are separately in thermal equilibrium with a third object (C), then (A) and (B) are in thermal equilibrium with each other*".

This statement can easily be proved experimentally and is very important because it enables us to define temperature. We can think of temperature as *the property that determines whether an object is in thermal equilibrium with other objects.* Two objects in thermal equilibrium with each other are at the same temperature. Conversely, if two objects have different temperatures, then they are not in thermal equilibrium with each other.

8.4 Thermometers

Thermometers *are devices used to measure the temperature of a system.* All thermometers are based on the principle that *some physical property of a system changes as the system's temperature changes.* Some physical properties that change with temperature are (1) the volume of a liquid, (2) the dimensions of a solid, (3) the pressure of a gas at constant volume, (4) the volume of a gas at constant pressure, (5) the electric resistance of a conductor, and (6) the color of an object.

A common thermometer in everyday use consists of a mass of liquid usually mercury or alcohol that expands into a glass capillary tube when heated. In this case, the physical property that changes is the volume of a liquid. Any temperature change in the range of the thermometer can be defined as being proportional to the change in length of the liquid column. The thermometer can be calibrated by placing it in thermal contact with a natural system that remains at constant temperature. One such system is a mixture of water and ice in thermal equilibrium at atmospheric pressure. On the **Celsius temperature scale,** this mixture is defined to have a temperature of zero degrees Celsius, which is written as $0\ ^\circ\mathrm{C}$ this temperature is called the ice point of water. Another commonly used system is a mixture of water and steam in thermal equilibrium at atmospheric pressure; its temperature is defined as $100\ ℃$, which is the steam point of water.

Once the liquid levels in the thermometer have been established at these two points, the length of the liquid column between the two points is divided into 100 equal segments to create the Celsius scale. Therefore, each segment denotes a change in temperature of one Celsius degree. A practical problem of any thermometer is the limited range of temperatures over which it can be used. A mercury thermometer, for example, cannot be used below the freezing point of mercury, which is $239\ ℃$ and an alcohol thermometer is not useful for measuring temperatures above $85\ ℃$ the boiling point of alcohol. To surmount this problem, we need a universal thermometer whose readings are independent of the substance used in it. The gas thermometer, discussed in the next section, approaches this requirement.

8.5 The Constant-Volume Gas Thermometer

The temperature readings given by a gas thermometer are nearly independent of the substance used in the thermometer. One version of a gas thermometer is the constant-volume apparatus shown in figure 8.1. The physical change exploited in this device is the variation of pressure of a fixed volume of gas with temperature. The flask is immersed in an ice-water bath, and mercury reservoir (B) is raised or lowered until the top of the mercury in column (A) is at the zero point on the scale. The height (h), the difference between the mercury levels in reservoir B and column A, indicates the pressure in the flask at $(0\ ℃)$ by means of equation,

$$P = P_o + \rho g h$$

The flask is then immersed in water at the steam point. Reservoir (B) is readjusted until the top of the mercury in column (A) is again at zero on the scale, which ensures that the gas's volume is the same as it was when the flask was in the ice bath. This adjustment of reservoir (B) gives a value for the gas pressure at $100\ ℃$. These two pressure and temperature values are then plotted as shown in figure 8.2. The line connecting the two points serves as a calibration curve for unknown temperatures. To measure the temperature of a substance, the gas flask of figure 8.1 is placed in thermal contact with the substance and the height of reservoir (B) is adjusted until the top of the mercury column in (A) is at zero on the scale. The height of the mercury column in (B) indicates the pressure of the gas; knowing the pressure, the temperature of the substance is found using the graph in figure 8.2.

Now suppose temperatures of different gases at different initial pressures are measured with gas thermometers. Experiments show that the thermometer readings are nearly independent of the type of gas used as long as the gas pressure is low and the temperature is well above the point at which the gas liquefies (figure 8.4). The agreement among thermometers using various gases improves as the pressure is reduced.

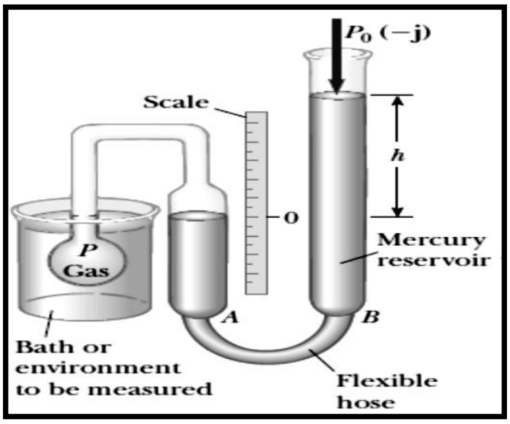


Figure 8.1: A constant-volume gas thermometer measures the pressure of

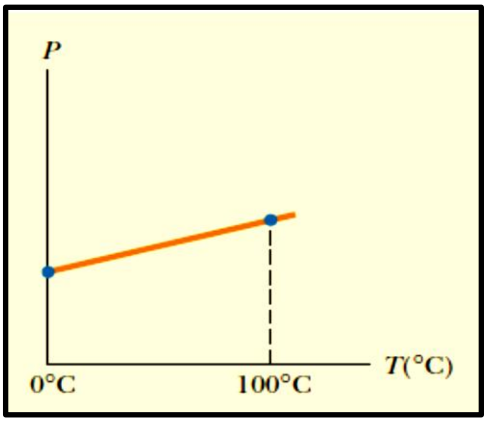


Figure 8.2: A typical graph of pressure versus temperature taken with a

8.6 The Absolute Temperature Scale

In figure 8.3 if we extend the straight lines toward negative temperatures, we find a remarkable result: in every case, the pressure is zero when the temperature is $-273.15\ ℃$ This finding suggests some special role that this temperature used as the basis for the absolute temperature scale, which sets $-273.15\ ℃$ as its zero point. This temperature is often referred to as absolute zero. Therefore, the conversion between these temperatures is

$$T_c = T - 273.15 \qquad (8-1)$$

where (T_c) is the Celsius temperature and (T) is the absolute temperature.

An absolute temperature scale based on two new fixed points was adopted in 1954 by the International Committee on Weights and Measures. The first point is absolute zero. The second reference temperature for this new scale was chosen as the triple point of water, which is the single combination of temperature and pressure at which liquid water, gaseous water, and ice (solid water) coexist in equilibrium. This triple point occurs at a temperature of $(0.01\ ℃)$ or $(273.16\ K)$ and a pressure of $(4.58\ mm)$ of mercury. This choice was made so that the old absolute temperature scale based on the ice and steam points would agree closely with the new scale based on the triple point. This new absolute temperature scale (also called the Kelvin scale) employs the SI unit of absolute temperature, the kelvin, which is defined to be $(1/273.16)$ of the difference between absolute zero and the temperature of the triple point of water. Figure 8.4 gives the absolute temperature for various physical processes and structures.

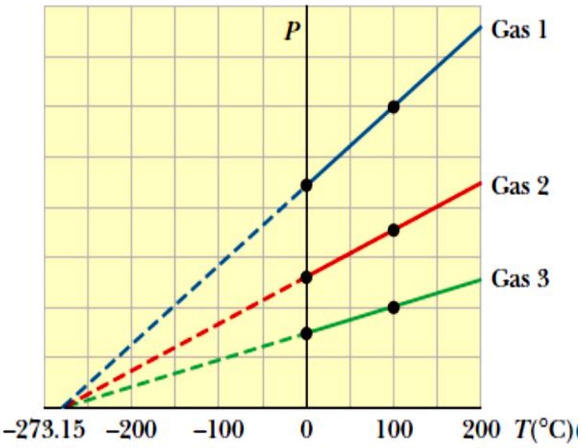


Figure 8.3: Pressure versus temperature for experimental trials in which gases have different pressures in a constant-volume gas thermometer.

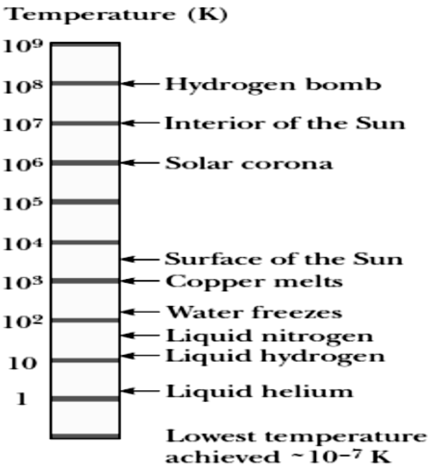


Figure 8.4: Absolute temperatures at which various physical processes occur.

8.7 The Celsius, Fahrenheit, Rankine and Kelvin Temperature Scales

The Celsius temperature (T_c) is shifted from the absolute (Kelvin) temperature T by $(273.15\ ^\circ\mathrm{C})$ (**table** 8-1). Because the size of one degree is the same on the two scales, a temperature difference of $(5\ ^\circ\mathrm{C})$ is equal to a temperature difference of $(5\ K)$.

A common temperature scale in everyday use in the United States is the Fahrenheit scale. This scale sets the temperature of the ice point at $(32\ ^\circ\mathrm{F})$ and the temperature of the steam point at $(212\ ^\circ\mathrm{F})$.

Rankine, it may be used in engineering systems where heat computations are done using degrees Fahrenheit.

We can use **tables** 8-1 and 8-2 to find a relationship between changes in temperature on the Celsius, Kelvin, Rankine and Fahrenheit scales:

Table 8-1: Temperature conversions (Celsius)

	from Celsius	**to Celsius**
Fahrenheit	$[^\circ F] = \frac{9}{5}[^\circ C] + 32$	$[^\circ C] = \frac{5}{9}([^\circ F] - 32)$
Kelvin	$[K] = [^\circ C] + 273.15$	$[^\circ C] = [K] - 273.15$
Rankine	$[^\circ R] = \frac{9}{5}([^\circ C] + 273.15)$	$[^\circ C] = \frac{5}{9}([^\circ R] - 491.67)$

Table 8-2: Temperature conversions (Kelvin)

	from Kelvin	to Kelvin
Celsius	$[°C] = [K] - 273.15$	$[K] = [°C] + 273.15$
Fahrenheit	$[°F] = \frac{9}{5}[K] - 459.67$	$[K] = \frac{5}{9}([°F] + 459.67)$
Rankine	$[°R] = \frac{9}{5}[K]$	$[K] = \frac{5}{9}[°R]$

Example 8.1

On a day when the temperature reaches $\mathbf{50\ °F}$, what is the temperature in degrees Celsius and in kelvins?

Solution:

Substitute the given temperature from **tables** 8-1 & 8-2:

$$T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(50 - 32) = 10\ °C$$

$$T = T_C + 273.15 = 10\ °C + 273.15 = 283\ K$$

A convenient set of weather-related temperature equivalents to keep in mind is that $(0\ °C)$ is (literally) freezing at $(32\ °F)$, $(10\ °C)$ is cool at $(50\ °F)$, $(20\ °C)$ is room temperature, $(30\ °C)$ is warm at $(56\ °F)$, and $(40\ °C)$ is a hot day at $(104\ °F)$.

8.8 Thermal Expansion of Solids and Liquids

Thermal expansion is a consequence of the change in the *average* separation between the atoms in an object. To understand this, imagine that the atoms are connected by stiff springs, as shown in figure 8.5. At ordinary temperatures, the atoms in a solid oscillate about their equilibrium positions with an amplitude of approximately $(10^{-11}\ m)$ and a frequency of approximately $(10^{13}\ Hz)$. The average spacing between the atoms is about $(10^{-10}\ m)$. As the temperature of the solid increases, the atoms oscillate with greater amplitudes; as a result, the average separation between them increases. Consequently, the object expands.

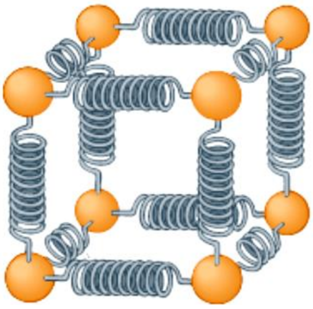


Figure 8.5: A mechanical model of the atomic configuration in a substance. The atoms (spheres) are imagined to be attached to each other by springs that reflect the elastic nature of the interatomic forces.

Suppose an object has an initial length (L_i) along some direction at some temperature and the length increases by an amount (ΔL) for a change in temperature (ΔT). Because it is convenient to consider the fractional change in length per degree of temperature change, we define the average coefficient of linear expansion as $\alpha = \frac{\Delta L/L}{\Delta T}$

Experiments show that α is constant for small changes in temperature. For purposes of calculation, this equation is usually rewritten as

$$\Delta L = \alpha L_i \Delta T \qquad (8-2)$$

or as

$$\left(L_f - L_i\right) = \alpha L_i\left(T_f - T_i\right) \qquad (8-3)$$

where, $\left(L_f\right)$ is the final length, (T_i) and $\left(T_f\right)$ are the initial and final temperatures, respectively, and the proportionality constant α is the average coefficient of linear expansion for a given material and has units of $(0\ ^\circ\mathrm{C})^{-1}$. Equation (8-2) can be used for both thermal expansion, when the temperature of the material increases, and thermal contraction, when its temperature decreases.

It may be helpful to think of thermal expansion as an effective magnification or as a photographic enlargement of an object. For example, as a metal washer is heated (figure 8.6), all dimensions, including the radius of the hole, increase according to equation (8-2). A cavity in a piece of material expands in the same way as if the cavity were filled with the material. **Table** 8-3 lists the average coefficients of linear expansion for various materials. For these materials, α is positive, indicating an increase in length with increasing temperature. Because the linear dimensions of an object change with temperature, it follows that surface area and volume change as well. The change in volume is proportional to the initial volume V_i and to the change in temperature according to the relationship

$$\Delta V \ = \ \beta V_i\, \Delta T \qquad (8-4)$$

where (β) is the average coefficient of volume expansion. To find the relationship between (b) and (a), assume the average coefficient of linear expansion of the solid is the same in all directions; that is, assume the material is *isotropic*. Consider a solid box of dimensions (l), (w), and (h). Its volume at some temperature (T_i) is $(V_i = lwh)$. If the temperature changes to $(T_i + \Delta T)$, its volume changes to $(V_i + \Delta V)$, where each dimension changes according to equation (8-2). Therefore,

$$\Delta V_i + \Delta V = (l + \Delta l)(w + \Delta w)(h + \Delta h)$$

$$= (l {+} \propto l\Delta T)(w {+} \propto w\Delta T)(h {+} \propto h\Delta T)$$

$$= lwh(1 {+} \propto \Delta T)^3$$

$$= [1 + 3 \propto \Delta T + 3(\propto \Delta T)^2 + (\propto \Delta T)^3]$$

Dividing both sides by (V_i) and isolating the term $(\Delta V/V_i)$, we obtain the fractional change in volume:

$$\frac{\Delta V}{V_t} = 3\alpha\Delta\mathrm{T} + 3(\alpha\Delta\mathrm{T})^2 + (\alpha\Delta\mathrm{T})^3$$

Because $\alpha\Delta\mathrm{T} << 1$ for typical values of $\Delta\mathrm{T}(< \sim 100\ ℃)$, we can neglect the terms $3(\alpha\Delta\mathrm{T})^2$ and $(\alpha\Delta\mathrm{T})^3$. Upon making this approximation, we see that

$$\frac{\Delta V}{V_t} = 3\alpha\Delta\mathrm{T} \;\; \rightarrow \;\; \Delta\mathrm{V} = (3\alpha)V_t\Delta\mathrm{T}$$

Comparing this expression to equation (8-4) shows that

$$\beta = 3\alpha$$

In a similar way, you can show that the change in area of a rectangular plate is given by $\Delta\mathrm{A} = 2\alpha A_i \Delta T$.

$$\gamma = 2\alpha$$

Table 8-3: Average Expansion Coefficients for Some Materials Near Room Temperature

Material (Solids)	**Average Linear Expansion Coefficient $(\alpha)(°C)^{-1}$**	**Material (Liquids and Gases)**	**Average Volume Expansion Coefficient $(\beta)(°C)^{-1}$**
Aluminum	24 x 10^{-6}	Acetone	1.5 x 10^{-4}
Brass and bronze	19 x 10^{-6}	Alcohol ethyl	1.12 x 10^{-4}
Concrete	12 x 10^{-6}	Benzene	1.24 x 10^{-4}
Copper	17 x 10^{-6}	Gasoline	9.6 x 10^{-4}
Glass (ordinary)	9 x 10^{-6}	Glycerin	4.85 x 10^{-4}
Glass (Pyrex)	3.2 x 10^{-6}	Mercury	1.82 x 10^{-4}
Invar (Ni–Fe alloy)	0.9 x 10^{-6}	Turpentine	9.0 x 10^{-4}
Lead	29 x 10^{-6}	Air at 0 °C	3.67 x 10^{-3}
Steel	11 x 10^{-6}	Helium	3.665 x 10^{-3}

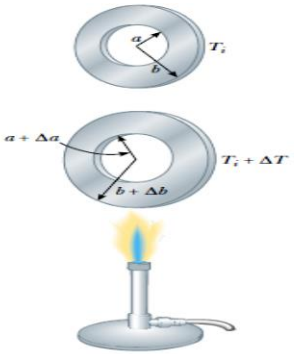


Figure 8.6: Thermal expansion of a homogeneous metal washer. (The expansions exaggerated in this figure.)

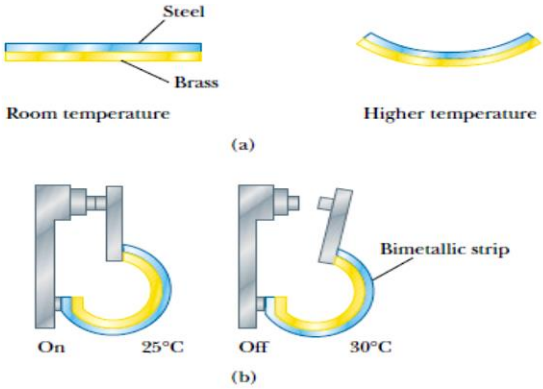


Figure 8.7: (a) A bimetallic strip bends as the temperature changes because the two metals have different expansion coefficients. (b) A bimetallic strip used in a thermostat to break or make electrical contact.

A simple mechanism called a *bimetallic strip,* found in practical devices such as thermostats, uses the difference in coefficients of expansion for different materials. It consists of two thin strips of dissimilar metals bonded together. As the temperature of the strip increases, the two metals expand by different amounts and the strip bends as shown in figure 8.7

8.9 Thermal Stress

Thermal stress is created by thermal expansion or contraction. Thermal stress can be destructive, such as when expanding gasoline ruptures a tank. It can also be useful, for example, when two parts are joined together by heating one in manufacturing, then slipping it over the other and allowing the combination to cool. Thermal stress can explain many phenomena, such as the weathering of rocks and pavement by the expansion of ice when it freezes.

Forces and pressures created by thermal stress can be quite large. Railroad tracks and roadways can buckle on hot days if they lack sufficient expansion joints. Power lines sag more in the summer than in the winter, and will snap in cold weather if there is insufficient slack. Cracks open and close in plaster walls as a house warms and cools. Glass cooking pans will crack if cooled rapidly or unevenly, because of differential contraction and the stresses it creates. (Pyrex is less susceptible because of its small coefficient of thermal expansion). Nuclear reactor pressure vessels are threatened by overly rapid cooling, and although none have failed, several have been cooled faster than considered desirable. Biological cells are ruptured when foods are frozen, detracting from their taste. Repeated thawing and freezing accentuates the damage. Even the oceans can be affected. A significant portion of the rise in sea level that is resulting from global warming is due to the thermal expansion of sea water.

Another example of thermal stress is found in the mouth. Dental fillings can expand differently from tooth enamel. It can give pain when eating ice cream or having a hot drink. Cracks might occur in the filling. Metal fillings (gold, silver, etc.) are being replaced by composite fillings (porcelain), which have smaller coefficients of expansion, and are closer to those of teeth.

It is a simple matter to compute the thermal stress set up in a rod that is not free to expand or contract. Suppose that a rod of length (L_o) and cross section area (A) has its ends rigidly fastened while the temperature reduced by an amount (ΔT).

The fractional change in length if the rod were free to contract would be $\frac{\Delta L}{L} = \alpha \Delta T$ where (ΔL) and (ΔT) are both negative. Since the rod is not free to contract, the tension by a sufficient amount to produce an equal and opposite fractional change in length. But from the definition of Young's modulus,

$$Y = \frac{F/A}{\Delta L/L}. \quad \frac{\Delta L}{L} = \frac{F}{AY}$$

Where this (ΔL) is positive. The tensile (F) is determined by the requirement that the total fractional change in length, thermal expansion plus elastic strain must be zero:

$$\alpha \Delta T + \frac{F}{AY} = 0$$

$$\Rightarrow F = AY\alpha\Delta T$$

Since (ΔT) represents a decrease in temperature, it is negative, so (F) is positive.

The tensile stress in the rod is:

$$\frac{F}{A} = -Y\alpha\Delta T \qquad (8-5)$$

If, instead, (ΔT) represents an increase in temperature, then (F) and (F/A) become negative, corresponding to compressive force and stress, respectively.

Similar phenomena occur with volume expansion. If a bottle is completely filled with water, titly capped and then heated, it will break because the thermal expansion coefficient for water is greater than that of glass. If a material is enclosed in a very rigid container so that its volume cannot be change, then arise in temperature (ΔT) is accompanied be an increase in pressure (ΔP). An analysis similar to that leading to equation (8-5) shows that the pressure increase is given by:

$$\Delta P = B\beta\Delta T$$

where (β) is the coefficient of volume expansion and (B) is the Bulk modulus.

Example (8.2):

A segment of steel railroad track has a length of $\boldsymbol{30\ m}$ when the temperature is **0** ℃. **(a)** What is its length when the temperature is **40** ℃ ?

Solution:

Use equation (8-4) and the value of the coefficient of linear expansion from **table** 8-1:

$$\Delta \mathrm{L} = \alpha L_i \Delta T = [11 \text{ x } 1026\ (°\mathrm{C}) - 1](30000\ \mathrm{m})(40\ °\mathrm{C}) = 0.013\ \mathrm{m}$$

Find the new length of the track: $L_f = 30\,m + 0.013\,m = 30.013\,m$

(b) Suppose the ends of the rail are rigidly clamped at 0 °C so that expansion is prevented. What is the thermal stress set up in the rail if its temperature is raised to 40 °C

Solution:

Using tensile stress equation:

$$\text{Tensile stress} = \frac{F}{A} = Y\frac{\Delta L}{L_i}$$

$$\frac{F}{A} = \left(20 \times 10^{10}\,\frac{N}{m^2}\right)\left(\frac{0.013\,m}{30\,m}\right) = 8.7 \times 10^{7} N/m^2$$

The expansion in part (**a**) is 1.3 cm. This expansion is indeed measurable as predicted in the Conceptualize step. The thermal stress in part (**b**) can be avoided by leaving small expansion gaps between the rails.

Example (8.3):

The Thermal Electrical Short is a poorly designed electronic device has two bolts attached to different parts of the device that almost touch each other in its interior as in figure 8.8. The steel and brass bolts are at different electric potentials, and if they touch, a short circuit will develop, damaging the device. The initial gap between the ends of the bolts is **5 μm** at **27** °C **.** At what temperature will the bolts touch? Assume the distance between the walls of the device is not affected by the temperature change.

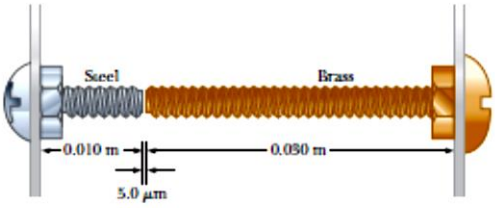


Figure 8.8: Two bolts attached to different parts of an electrical device are almost touching when the temperature is 27 °C. As the temperature increases, the ends of the bolts move toward each other.

Solution:

Set the sum of the length changes equal to the width of the gap:

$$\Delta L_{br} + \Delta L_{st} = \propto_{br} L_{ibr} \Delta T + \propto_{st} L_{ist} \Delta T = 5 \times 10^{-6} m$$

Solve for ΔT: $\Delta T = \frac{5\times10^{-6}\,m}{\propto_{br} L_{ibr}\Delta T + \propto_{st} L_{ist}\Delta T}$

$$\Delta T = \frac{5 \times 10^{-6}\,m}{[19 \times 10^{-6}(°\mathrm{C})^{-1}](0.030m) + [11 \times 10^{-6}(°\mathrm{C})^{-1}](0.010m)]} = 7.4\ °\mathrm{C}$$

Find the temperature at which the bolts touch: $T = 27\ °\mathrm{C} + 7.4\ °\mathrm{C} = 34\ °\mathrm{C}$

This temperature is possible if the air conditioning in the building housing the device fails for a long period on a very hot summer day.

PROBLEMS

1. Convert the following to equivalent temperatures on the Celsius and Kelvin scales: (a) the normal human body temperature, $\mathbf{98.6\ °F}$; (b) the air temperature on a cold day, $\mathbf{-5\ °F}$.

2. In a constant-volume gas thermometer, the pressure at $\mathbf{20\ °C}$ is $\mathbf{0.98\ atm}$. (a) What is the pressure at $\mathbf{45\ °C}$? (b) What is the temperature if the pressure is $\mathbf{0.5\ atm}$?

3. There is a temperature whose numerical value is the same on both the Celsius and Fahrenheit scales. What is this temperature?

4. A copper telephone wire has essentially no sag between poles $\mathbf{35\ m}$ apart on a winter day when the temperature is $\mathbf{-20\ °C}$. How much longer is the wire on a summer day when $\mathbf{T_c = 35\ °C}$?

5. The concrete sections of a certain superhighway are designed to have a length of $\mathbf{25\ m}$. The sections are poured and cured at $\mathbf{10\ °C}$. What minimum spacing should the engineer leave between the sections to eliminate buckling if the concrete is to reach a temperature of $\mathbf{50\ °C}$?

6. A brass ring with a diameter of $\mathbf{10\ cm}$ at $\mathbf{20\ °C}$ is heated and slipped over an aluminum rod with a diameter of $\mathbf{10.01\ cm}$ at $\mathbf{20\ °C}$. Assume that the average coefficients of linear expansion are constant. (a) To what temperature must this combination be cooled to separate them? Is this temperature attainable? (b) If the aluminum rod were $\mathbf{10.02\ cm}$ in diameter, what would be the required temperature?

7. The New River Gorge Bridge in West Virginia is a steel arch bridge $\mathbf{518\ m}$ in length. How much does its length change between temperature extremes of $\mathbf{-20°C}$ and $\mathbf{35\ °C}$?

8. The average coefficient of volume expansion for carbon tetrachloride is $\mathbf{5.81 \times 10^{-4}\ °C^{-1}}$. If a $\mathbf{50\ gal}$ steel container is filled completely with carbon tetrachloride when the temperature is $\mathbf{10\ °C}$, how much will spill over when the temperature rises to $\mathbf{30\ °C}$?

9. The active element of a certain laser is a glass rod $\mathbf{30\ cm}$ long by $\mathbf{1.5\ cm}$ in diameter. If the temperature of the rod increases by $\mathbf{65\ °C}$, what is the increase in (a) its length, (b) its diameter, and (c) its volume? (Let $\boldsymbol{\alpha} = \mathbf{9 \times 10^{-6}\ °C^{-1}}$).

10. A volumetric glass flask made of Pyrex is calibrated at $\mathbf{20\ °C}$. It is filled to the $\mathbf{100\ mL}$ mark with $\mathbf{35\ °C}$. (a) What is the volume of the acetone when it cools to $\mathbf{20\ °C}$? (b) How significant is the change in volume of the flask?

11. A concrete walk is poured on a day when the temperature is $\mathbf{20\ °C}$, in such a way that the ends are unable to move. (a) What is the stress in the cement on a hot day of $\mathbf{50\ °C}$? (b) Does the concrete fracture? Take Young's modulus for concrete to be $\mathbf{7 \times 10^{9}\ N/m^{2}}$ and the tensile strength to be $\mathbf{2 \times 10^{9}\ N/m^{2}}$.

12. A steel rod undergoes a stretching force of $\mathbf{500\ N}$. Its cross-sectional area is $\mathbf{2\ cm^{2}}$. Find the change in temperature that would elongate the rod by the same amount that the $\mathbf{500\ N}$ force does.

13. A steel rod $\mathbf{4\ cm}$ in diameter is heated so that its temperature increases by $\mathbf{70\ °C}$. It is then fastened between two rigid supports. The rod is allowed to cool to its original temperature. Assuming that Young's modulus for the steel is $\mathbf{20.6 \times 10^{10}\ N/m^{2}}$ and that its average coefficient of linear expansion is $\mathbf{11 \times 10^{-6}\ °C^{-1}}$, calculate the tension in the rod.

CHAPTER (9)
THE FIRST LAW OF THERMODYNAMICS

9.1 Introduction

This chapter focuses on the concept of internal energy, the first law of thermodynamics, and some important applications of the first law. The first law of thermodynamics describes systems in which the only energy change is that of internal energy and the transfers of energy are by heat and work.

9.2 Heat and Internal Energy

Internal energy is *all the energy of a system that is associated with its microscopic components atoms and molecules when viewed from a reference frame at rest with respect to the center of mass of the system.* Any bulk kinetic energy of the system due to its motion through space is not included in internal energy. Internal energy includes kinetic energy of random translational, rotational, and vibrational motion of molecules.

Heat is defined as *the transfer of energy across the boundary of a system due to a temperature difference between the system and its surroundings.* This is the case, for example, when you place a pan of cold water on a stove burner the burner is at a higher temperature than the water, and so the water gains energy. We shall also use the term heat to represent the amount of energy transferred by this method.

It is important to recognize that the internal energy of a system can be changed even when no energy is transferred by heat. For example, when a gas is compressed by a piston, the gas is warmed and its internal energy increases, but no transfer of energy by heat from the surroundings to the gas has occurred. If the gas then expands rapidly, it cools and its internal energy decreases, but no transfer of energy by heat from it to the surroundings has taken place. The temperature changes in the gas are due not to a difference in temperature between the gas and its surroundings but rather to the compression and the expansion. In each case, energy is transferred to or from the gas by work, and the energy change within the system is an increase or decrease of internal energy. The changes in internal energy in these examples are evidenced by corresponding changes in the temperature of the gas.

9.2.1 Units of Heat

Initial notions of heat were based on a fluid called *caloric* that flowed from one substance to another and caused changes in temperature. From the name of this mythical fluid came an energy unit related to thermal processes, the calorie (cal), which is defined as *the amount of energy transfer necessary to raise the temperature of* $(1\ g)$ *of water from* $(14.5\ °C)$ *to* $(15.5\ °C)$. (The “Calorie,” written with a capital

"C" and used in describing the energy content of foods, is actually a kilocalorie.) The unit of energy in the U.S. customary system is the British thermal unit (Btu), which is defined as the amount of energy transfer required to raise the temperature of $(1\ lb)$ of water from $(63\ °F)$ to $(64\ °F)$. The joule has already been defined as an *energy unit based on mechanical processes.* Scientists are increasingly turning away from the calorie and the Btu and are using the joule when describing thermal processes.

9.3 The Mechanical Equivalent of Heat

It was found that whenever friction is present in a mechanical system, some mechanical energy is lost. Various experiments show that this lost mechanical energy does not simply disappear but is transformed into internal energy. Joule established the equivalence of these two forms of energy.

A schematic diagram of Joule's most famous experiment is shown in figure 9.1. The system of interest is the Earth, the two blocks, and the water in a thermally insulated container. Work is done within the system on the water by a rotating paddle wheel, which is driven by heavy blocks falling at a constant speed. If the energy transformed in the bearings and the energy passing through the walls by heat are neglected, the decrease in potential energy of the system as the blocks fall equals the work done by the paddle wheel on the water and, in turn, the increase in internal energy of the water. If the two blocks fall through a distance (h), the decrease in potential energy is $(2mgh)$, where m is the mass of one block; this energy causes the temperature of the water to increase.

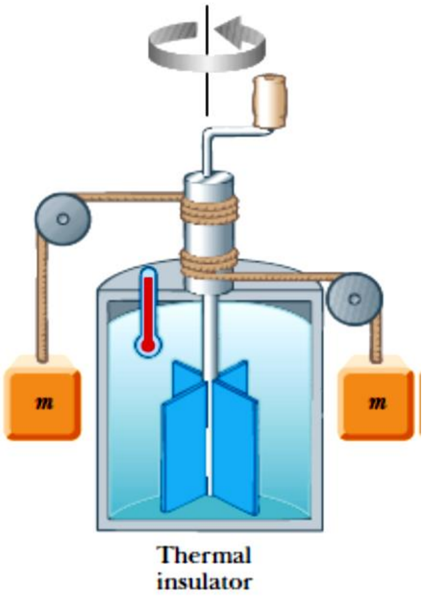


Figure 9.1: Joule's experiment for determining the mechanical equivalent of heat. The falling blocks rotate the paddles, causing the temperature of the water to increase.

By varying the conditions of the experiment, Joule found that the decrease in mechanical energy is proportional to the product of the mass of the water and the increase in water temperature. The proportionality constant was found to be approximately $(4.18\ J/g.°C)$. Hence, $(4.18\ J)$ of mechanical energy raises the temperature of $(1\ g)$ of water by $1°C$. More precise measurements taken later demonstrated the proportionality to be $(4.186\ J/g\ .°C)$ when the temperature of the water was raised from $(14.5\ °C)$ to $(15.5\ °C)$. We adopt this "$15 - degree\ calorie$" value:

$$1\ cal\ =\ 4.186\ J \qquad (9-1)$$

This equality is known, for purely historical reasons, as the mechanical equivalent of heat. A more proper name would be *equivalence between mechanical energy and internal energy,* but the historical name is well entrenched in our language, despite the incorrect use of the word *heat*.

Example (9.1):

A student eats a dinner rated at $\mathbf{2000\ Calories}$. He wishes to do an equivalent amount of work in the gymnasium by lifting a $\mathbf{50\ kg}$ barbell. How many times must he raise the barbell to expend this much energy? Assume he raises the barbell $\mathbf{2\ m}$ each time he lifts it and he regains no energy when he lowers the barbell.

Solution:

Imagine the student raising the barbell. He is doing work on the system of the barbell and the Earth, so energy is leaving his body. The total amount of work that the student must do is $2000\ Calories$.

Reduce the conservation of energy equation to the appropriate expression for the system of the barbell and the Earth:

$$\Delta U_{total} = W_{total} \quad (1)$$

Express the change in gravitational potential energy of the system after the barbell is raised once:

$$\Delta U = mgh$$

Express the total amount of energy that must be transferred into the system by work for lifting the barbell n times, assuming energy is not regained when the barbell is lowered:

$$\Delta U_{total} = nmgh \quad (2)$$

Substitute equation (2) into equation (1):

$$nmgh = W_{total}$$

Solve for (n): $$n = \frac{W_{total}}{mgh} = \frac{(2000 \times 10^3 \times 4.186 J)}{(50\ kg)(9.80\ m/s^2)(2\ m)} = 8.54 \times 10^3\ times$$

If the student is in good shape and lifts the barbell once every $(5\ s)$, it will take him about $(12\ h)$ to perform this feat. Clearly, it is much easier for this student to lose weight by dieting.

In reality, the human body is not 100% efficient. Therefore, not all the energy transformed within the body from the dinner transfers out of the body by work done on the barbell. Some of this energy is used to pump blood and perform other functions within the body. Therefore, the $(2000\ Calories)$ can be worked off in less time than $(12\ h)$ when these other energy processes are included.

9.4 Specific Heat and Calorimetry

When energy is added to a system and there is no change in the kinetic or potential energy of the system, the temperature of the system usually rises. (An exception to this statement is the case in which a system undergoes a change of state also called a *phase transition* as discussed in the next section.) If the system consists of a sample of a substance, we find that the quantity of energy required to raise the temperature of a given mass of the substance by some amount varies from one substance to another. For example, the quantity of energy required to raise the temperature of $(1\ kg)$ of water by $(1°C)$ is $(4186\ J)$, but the quantity of energy required to raise the temperature of $(1\ kg)$ of copper by $(1°C)$ is only $(387\ J)$.

The **heat capacity** (C) of a particular sample is defined as *the amount of energy needed to raise the temperature of that sample by* $(1°C)$. From this definition, we see that if energy (Q) produces a change (ΔT) in the temperature of a sample, then:

$$Q = C\,\Delta T \qquad (9-2)$$

The **specific heat** (c) of a substance is *the heat capacity per unit mass*. Or as *the amount of energy needed to raise the temperature of 1gm of sample by* $(1°C)$Therefore, if energy (Q) transfers to a sample of a substance with mass m and the temperature of the sample changes by (ΔT), the specific heat of the substance is

$$c = \frac{Q}{m\Delta T} \qquad (9-\ 3)$$

Specific heat is essentially a measure of how thermally insensitive a substance is to the addition of energy. The greater a material's specific heat, the more energy must be added to a given mass of the material to cause a particular temperature change. **Table** 9-1 lists representative specific heats. From this definition, we can relate the energy (Q) transferred between a sample of mass (m) of a material and its surroundings to a temperature change ΔT as

$$Q = mc\Delta T \qquad (9-\ 4)$$

For example, the energy required to raise the temperature of $(0.5\ kg)$ of water by $(3\ °C)$ is $Q = (0.5\ kg)(4186\ J/kg\ .°C)(3\ °C) = 6.28 \times 103\ J$. Notice that when the temperature increases, Q and ΔT are taken to be positive and energy transfers into the system. When the temperature decreases, Q and ΔT are negative and energy transfers out of the system.

Specific heat varies with temperature. However, if temperature intervals are not too great, the temperature variation can be ignored, and c can be treated as a constant. For example, the specific heat of water varies by only about 1% from $(0\ °C)$ to $100\ °C$ at atmospheric pressure. Unless stated otherwise, we shall neglect such variations.

From **table** 9-1 we note that water has the highest specific heat of common materials. This high specific heat is responsible, in part, for the moderate temperatures found near large bodies of water.

Table 9-1: Specific Heats of Some Substances at $25°C$ and Atmospheric Pressure

Material	**Specific Heat** $(J/kg.°C)$	**Substance**	**Specific Heat** $(J/kg.°C)$
Aluminum	900	Brass	380
Beryllium	1830	Glass	837
Cadmium	230	Ice $(-5$ ℃)	2090
Copper	387	Marble	860
Germanium	322	Wood	1700
Gold	129	Alcohol (ethyl)	2400
Iron	448	Mercury	140
Lead	128	Water (15 ℃)	4186
Silver	234	Steam	2010

Conservation of Energy: Calorimetry

One technique for measuring specific heat involves heating a sample to some known temperature (T_x), placing it in a vessel containing water of known mass and temperature and measuring the temperature of the water after equilibrium has been reached. Because a negligible amount of mechanical work is done in the process, the law of the conservation of energy requires that the amount of energy that leaves the sample (of unknown specific heat) equal the amount of energy that enters the water.

This technique is called calorimetry, and devices in which this energy transfer occurs are called calorimeters. Conservation of energy allows us to write the equation

$$Q_{cold} = -Q_{hot} \qquad (9-5)$$

which simply states that the energy leaving the hot part of the system by heat is equal to that entering the cold part of the system. The negative sign in the equation is necessary to maintain consistency with our sign convention for heat. The heat (Q) hot is negative because energy is leaving the hot sample. The negative sign in the equation ensures that the right-hand side is positive and thus consistent with the left-hand side, which is positive because energy is entering the cold water.

Suppose (m_x) is the mass of a sample of some substance whose specific heat we wish to determine. Let's call its specific heat (c_x) and its initial temperature (T_x) as shown in figure 9.2. Likewise, let (m_w, c_w) and (T_w) represent corresponding values for the water. If (T_f) is the final temperature after the system comes to equilibrium, equation (9-4) shows that the energy transfer for the water is

$\left(m_w c_w(T_f - T_w)\right)$, which is positive because $\left(T_f > T_w\right)$, and that the energy transfer for the sample of unknown specific heat is $\left(m_x c_x(T_f - T_x)\right)$, which is negative. Substituting these expressions into equation (9-5) gives:

$$m_x c_x\left(T_f - T_x\right) = -m_w c_w(T_f - T_w)$$

This equation can be solved for the unknown specific heat (c_x).

Example (9.2):

A $\boldsymbol{0.05\ kg}$ ingot of metal is heated to $\boldsymbol{200\ °C}$ and then dropped into a calorimeter containing $\boldsymbol{0.4\ kg}$ of water initially at $\boldsymbol{20\ °C}$. The final equilibrium temperature of the mixed system is $\boldsymbol{22.4\ °C}$. Find the specific heat of the metal.

Solution:

Use equation (9-4) to evaluate each side of equation (9-5):

$$m_w c_w\left(T_f - T_w\right) = m_x c_x\left(T_f - T_x\right)$$

Solve for (c_x): $c_x = \frac{m_w c_w\left(T_f - T_w\right)}{m_x\left(T_f - T_x\right)} = \frac{(0.4\ kg)(4186\ J/kg.°\text{C})(22.4\ °\text{C} - 20\ °\text{C})}{(0.05\ kg)(200\ °\text{C} - 22.4\ °\text{C})} = 453\,\frac{J}{kg.°\text{C}}$

The temperature of the ingot is initially above the steam point. Therefore, some of the water may vaporize when the ingot is dropped into the water. We assume the system is sealed and this steam cannot escape. Because the final equilibrium temperature is lower than the steam point, any steam that does result re-condenses back into water.

9.5 Latent Heat

A substance can undergo a change in temperature when energy is transferred between it and its surroundings. In some situations, however, the transfer of energy does not result in a change in temperature. That is the case whenever the physical characteristics of the substance change from one form to another; such a change is commonly referred to as a phase change. Two common phase changes are from solid to liquid (melting) and from liquid to gas (boiling); another is a change in the crystalline structure of a solid. All such phase changes involve a change in the system's internal energy but no change in its temperature. The increase in internal energy in boiling, for example, is represented by the breaking of bonds between molecules in the liquid state; this bond breaking allows the molecules to move farther apart in the gaseous state, with a corresponding increase in intermolecular potential energy.

Different substances respond differently to the addition or removal of energy as they change phase because their internal molecular arrangements vary. Also, the amount of energy transferred during a phase change depends on the amount of substance involved. When discussing two phases of a material, we will use the term *higher-phase material* to mean the material existing at the higher temperature. So, for example, if we discuss water and ice, water is the higher-phase material, whereas steam is the higher-phase material in a discussion of steam and water. Consider a system containing a substance in two phases in equilibrium such as water and ice. The initial amount of the higher-phase material, water, in the system is m_i. Now imagine that energy Q enters the system. As a result, the final amount of water is m_f due to the melting of some of the ice. Therefore, the amount of ice that melted, equal to the amount of *new* water, is

$$\Delta m = m_f - m_i$$

We define the latent heat for this phase change as

$$L = \frac{Q}{\Delta m} \qquad (9-6)$$

This parameter is called latent heat (literally, the "hidden" heat) because this added or removed energy does not result in a temperature change. The value of L for a substance depends on the nature of the phase change as well as on the properties of the substance. If the entire amount of the lower-phase material undergoes a phase change, the change in mass Δm of the higher-phase material is equal to the initial mass of the lower-phase material. For example, if an ice cube of mass m on a plate melts completely, the change in mass of the water is $m_f - 0 = m$, which is the mass of new water and is also equal to the initial mass of the ice cube. From the definition of latent heat, and again choosing heat as our energy transfer mechanism, the energy required to change the phase of a pure substance is

$$Q = L\,\Delta m \qquad (9-7)$$

where Δm is the change in mass of the higher-phase material, The latent heats of various substances vary considerably as data in **table** 9-2.

Table 9-2: Latent Heats of Fusion and Vaporization

Material	**Melting Point** (℃)	**Latent Heat of Fusion** (J/kg)	**Boiling Point** (℃)	**Latent Heat of Vaporization** (J/kg)
Helium	-269.665	5.23×10^{3}	-268.93	2.09×10^{4}
Oxygen	-218.79	1.38×10^{4}	-182.97	2.13×10^{5}
Nitrogen	-209.97	2.55×10^{4}	-195.81	2.01×10^{5}
Alcohol(ethyl)	-114	1.04×10^{5}	78	8.54×10^{5}
Water	0.00	3.33×10^{5}	100.00	2.26×10^{6}
Lead	327.3	2.45×10^{5}	1750	8.70×10^{5}
Aluminum	660	3.97×10^{5}	2450	1.14×10^{7}
Silver	960.80	8.82×10^{4}	2193	2.33×10^{6}
Gold	1063.0	6.44×10^{4}	2660	1.58×10^{6}
Copper	1083	1.34×10^{5}	1187	5.06×10^{6}

As illustrated in **table** 9-2 there are latent heat of fusion (L_f) is the term used when the phase change is from solid to liquid, and latent heat of vaporization (L_v) is the term used when the phase change is from liquid to gas (the liquid "vaporizes").

When energy enters a system, causing melting or vaporization, the amount of the higher-phase material increases, so (Δm) is positive and (Q) is positive, consistent with our sign convention. When energy is extracted from a system, causing freezing or condensation, the amount of the higher-phase material decreases, so (Δm) is negative and (Q) is negative, again consistent with our sign convention. Keep in mind that (Δm) in equation (9-7) always refers to the higher-phase material

To understand the role of latent heat in phase changes, consider the energy required to convert a $(1\ g)$ cube of ice at $(230\ °C)$ to steam at $(120\ °C)$. Figure 9.2 indicates the experimental results obtained when energy is gradually added to the ice. The results are presented as a graph of temperature of the system of the ice cube versus energy added to the system. Let's examine each portion of the red-brown curve, which is divided into parts (A) through (E).

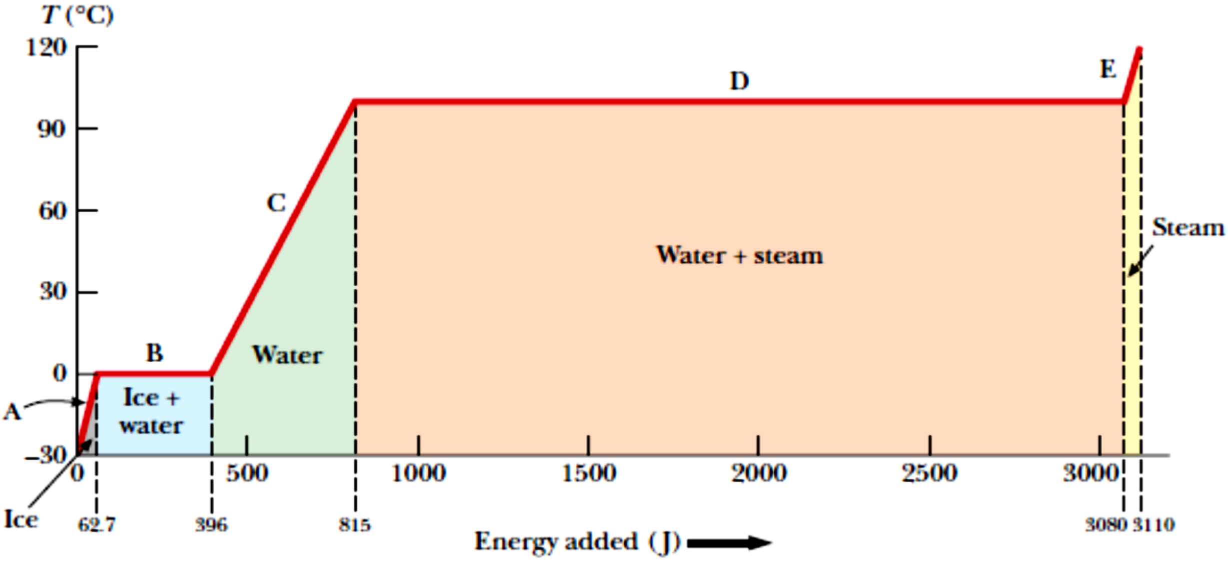


Figure 9.2: A plot of temperature versus energy added when $(1g)$ of ice initially at $(230\ °C)$ is converted to steam at $(120\ °C)$.

Part A. On this portion of the curve, the temperature of the ice changes from $(230\ °C)$ to $(0\ °C)$. Equation (9-4) indicates that the temperature varies linearly with the energy added, so the experimental result is a straight line on the graph. Because the specific heat of ice is $(2090\ J/kg.°C)$, we can calculate the amount of energy added by using equation (9-4):

$$Q = m_i c_i \Delta T = (1.00 \times 10^{-3}\ kg)(2090\ J/kg\ .°C)(30\ °C) = 62.7\ J$$

Part B. When the temperature of the ice reaches $(0\ °C)$, the ice–water mixture remains at this temperature even though energy is being added until all the ice melts. The energy required to melt $(1\ g)$ of ice at $(0\ °C)$ is, from equation (9-7),

$$Q = L_f\, \Delta m_w = L_f m_i = (3.33 \times 10^5\ J/kg)(1.00 \times 10^{-3}\ kg) = 333\ J$$

At this point, we have moved to the $(62.7\ J + 333\ J) = 396\ J$ mark on the energy axis in figure 9.2.

Part C. Between $(0\ °C)$ and $(100\ °C)$, nothing surprising happens. No phase change occurs, and so all energy added to the water is used to increase its temperature. The amount of energy necessary to increase the temperature from $(0\ °C)$ to $(100\ °C)$ is

$$Q = m_w c_w\, \Delta T = (1.00 \times 10^{-3}\ kg)(4.19 \times 10^3\ J/kg\ .°C)(100\ °C) = 419\ J$$

Part D. At $(100\ °C)$, another phase change occurs as the water changes from water at $(100\ °C)$ to steam at $(100\ °C)$. Similar to the ice–water mixture in part B, the water– steam mixture remains at $(100\ °C)$ even though energy is being added until all the liquid has been converted to steam. The energy required to convert $(1g)$ of water to steam at $(100\ °C)$ is

$$Q = L_v\, \Delta m_s = L_v m_w = (2.26 \times 10^6\ J/kg)(1.00 \times 10^{-3}\ kg) = 2.26\ x\ 103\ J$$

Part E. On this portion of the curve, as in parts (A) and (C), no phase change occurs; therefore, all energy added is used to increase the temperature of the steam. The energy that must be added to raise the temperature of the steam from $(100\ °C)$ to $(120\ °C)$ is

$$Q = m_s c_s\, \Delta T = (1.00 \times 10^{-3}\ kg)(2.01 \times 10^3\ J/kg\ .°C)(20.0°C) = 40.2\ J$$

The total amount of energy that must be added to change $(1\ g)$ of ice at $(230\ °C)$ to steam at $(120\ °C)$ is the sum of the results from all five parts of the curve, which is $(3.11 \times 10^3\ J)$. Conversely, to cool $(1g)$ of steam at $(120\ °C)$ to ice at $(230\ °C)$, we must remove $(3.11 \times 10^3\ J)$ of energy.

Notice in figure 9.2 the relatively large amount of energy that is transferred into the water to vaporize it to steam. Imagine reversing this process, with a large amount of energy transferred out of steam to condense it into water. That is why a burn to your skin from steam at $(100\ °C)$ is much more damaging than exposure of your skin to water at $(100\ °C)$. A very large amount of energy enters your skin from the steam, and the steam remains at $(100\ °C)$ for a long time while it condenses. Conversely, when your skin makes contact with water at $(100\ °C)$, the water immediately begins to drop in temperature as energy transfers from the water to your skin.

If liquid water is held perfectly still in a very clean container, it is possible for the water to drop below $(0\,°C)$ without freezing into ice. This phenomenon, called supercooling, arises because the water requires a disturbance of some sort for the molecules to move apart and start forming the large, open ice structure that makes the density of ice lower than that of water as discussed in Section 1.4. If supercooled water is disturbed, it suddenly freezes. The system drops into the lower-energy con-figuration of bound molecules of the ice structure, and the energy released raises the temperature back to $(0\,°C)$.

It is also possible to create superheating. For example, clean water in a very clean cup placed in a microwave oven can sometimes rise in temperature beyond $(100\,°C)$ without boiling because the formation of a bubble of steam in the water requires scratches in the cup or some type of impurity in the water to serve as a nucleation site.

Example (9.3):

What mass of steam initially at $\mathbf{130\,°C}$ is needed to warm $\mathbf{200\ g}$ of water in a $\mathbf{100\ g}$ glass container from $\mathbf{20\,°C}$ to $\mathbf{50\,°C}$?

Solution:

Write equation (9-5) to describe the calorimetry process:

$$Q_{cold} = -Q_{hot} \quad (1)$$

The steam undergoes three processes: first a decrease in temperature to $(100\,°C)$, then condensation into liquid water, and finally a decrease in temperature of the water to $(50\,°C)$.

$$Q_1 = m_s c_s \Delta T_s$$

Find the energy transfer in the first process using the unknown mass (m_s) of the steam:

$$Q_2 = L_v \Delta m_s = L_v(0 - m_s) = -\,msL_v$$

Find the energy transfer in the second process:

$$Q_3 = m_s c_w \Delta T_{hot\,water}$$

Find the energy transfer in the third process:

$$Q_{hot} = Q_1 + Q_2 + Q_3 = m_s(c_s \Delta T_s - L_v + c_w \Delta T_{hot\,water}) \quad (2)$$

Add the energy transfers in these three stages:

$$Qcold = m_w c_w \Delta T_{cold\,water} + m_g c_g \Delta T_{glass} \quad (3)$$

The $(20\,°C)$ water and the glass undergo only one process, an increase in temperature to $(50\,°C)$. Find the energy transfer in this process:

Substitute equations (2) and (3) into equation (1):

$$m_w c_w \Delta T_{cold\,water} + m_g c_g \Delta T_{glass} = -m_s(c_s \Delta T_s - L_v + c_w \Delta T_{hot\,water})$$

Solve for (m_s):

$$m_s = -\frac{m_w c_w \Delta T_{\text{cold water}} + m_g c_g \Delta T_{\text{glass}}}{c_s \Delta T_s - L_v + c_w \Delta T_{\text{hot water}}}$$

Substitute numerical values:

$$m_s = -\frac{(0.2kg)(4.186 J/kg.°\text{C})(50°\text{C} - 20°\text{C}) + (0.1kg)(837 J/kg.°\text{C})(50°\text{C} - 20°\text{C})}{(2010 J/kg.°\text{C})(100°\text{C} - 130°\text{C}) - (2.26 \times 10^6 J/kg) + (4.186 J/kg.°\text{C})(50°\text{C} - 100°\text{C})}$$

$$= 1.09 \times 10^{-2}\ kg = 10.9\ g$$

9.6 Work and Heat in Thermodynamic Processes

In the macroscopic approach to thermodynamics we describe the state of a system with such variables as pressure, volume, and internal energy. The number of macroscopic variables needed to characterize a system depends on the nature of the system. For a homogeneous system, such as a gas containing only one type of molecule, usually only two variables are needed. However, it is important to note that a macroscopic state of an isolated system can be specified only if the system is in thermal equilibrium internally. In the case of a gas in a container, internal thermal equilibrium requires that every part of the gas be at the same pressure and temperature.

Consider a gas contained in a cylinder fitted with a movable piston (figure 9.3). At equilibrium, the gas occupies a volume (V) and exerts a uniform pressure (P) on the cylinder's walls and on the piston. If the piston has a cross-sectional area A, the force exerted by the gas on the piston is $(F = PA)$. Now let us assume that the gas expands quasi-statically, that is, slowly enough to allow the system to remain essentially in thermal equilibrium at all times. As the piston moves up a distance (dy), the work done by the gas on the piston is

$$dw = Fdy = PAdy$$

Because $(A\ dy)$ is the increase in volume of the gas (dV), we can express the work done by the gas as

$$dW = PdV \qquad (9-8)$$

Since the gas expands, (dV) is positive, and so the work done by the gas is positive. If the gas were compressed, $d(V)$ would be negative, indicating that the work done by the gas (which can be interpreted as work done on the gas) was negative. Clearly, the work done by the gas is zero when the volume remains constant. The total work done by the gas as its volume changes from (V_i) to (V_f) is given by the integral of equation (9-8):

$$W = \int_{V_i}^{V_f} PdV \qquad (9-9)$$

To evaluate this integral, it is not enough that we know only the initial and final values of the pressure. We must also know the pressure at every instant during the expansion; we would know this if we had a functional dependence of (P) with respect to (V). This important point is true for any processesthe expansion we are discussing here, or any other. To fully specify a process, we must know the values of the thermodynamic variables at every state through which the system passes between the initial and final states. In the expansion we are considering here, we can plot the pressure and volume at each instant to create a PV diagram like the one shown in figure 9.4. The value of the integral in equation (9-9) is the area bounded by such a curve. Thus, we can say that

The work done by a gas in the expansion from an initial state to a final state is *the area under the curve connecting the states in a* PV *diagram*.

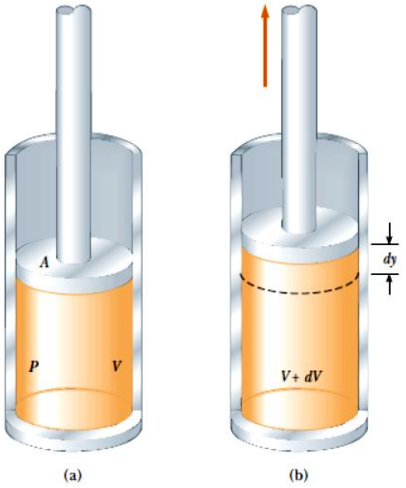


Figure 9.3: Work is done on a gas contained in a cylinder at a pressure (P) as the piston is pushed downward so that the gas is compressed.

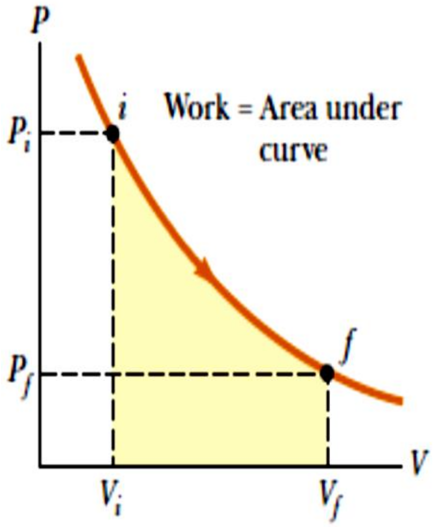


Figure 9.4: A gas is compressed quasi-statically (slowly) from state (i) to state (f). An outside agent must do positive work on the gas to compress it.

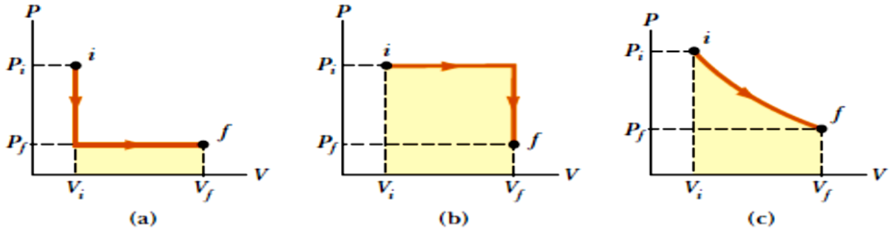


Figure 9.5: The work done by a gas as it is taken from an initial state to a final state depends on the path between these states.

As figure 9.4 shows, the work done in the expansion from the initial state (i) to the final state (f) depends on the path taken between these two states. To illustrate this important point, consider several paths connecting (i) and (f) (figure 9.5). In the process depicted in figure 9.5a, the pressure of the gas is first reduced from (P_i) to (P_f) by cooling at constant volume (V_i). The gas then expands from (V_i) to (V_f)

at constant pressure (P_f). The work done along this path is equal to the area of the shaded rectangle, which is equal to $(P_f\ (V_f - V_i))$.

In figure 9.5b, the gas first expands from (V_i) to (V_f) at constant pressure (P_i). then, its pressure is reduced to (P_f) at constant volume (V_f). The value of the work done along this path is $(P_i\ (V_f\ -\ V_i))$, which is greater than that for the process described in figure 9.5a. Finally, for the process described in figure 9.5c, where both (P) and (V) change continuously, the work done has some value intermediate between the values obtained in the first two processes. Therefore, the work done by a system depends on the initial and final states and on the path followed by the system between these states. The energy transfers by heat (Q) into or out of a system also depends on the process.

Consider the situations depicted in figure 9.6. In figure 9.6a, the gas is thermally insulated from its surroundings except at the bottom of the gas-filled region, where it is in thermal contact with an energy reservoir. An energy reservoir is a source of energy that is considered to be so great that a finite transfer of energy from the reservoir does not change its temperature. During this expansion to the final volume (V_f), just enough energy is transferred by heat from the reservoir to the gas to maintain a constant temperature (T_i).

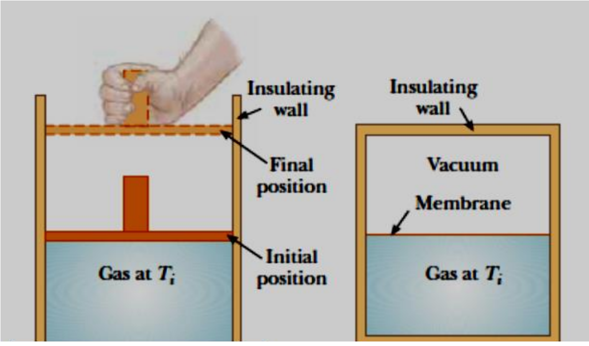


Figure 9.6: (a) A gas at temperature T_i expands slowly while absorbing energy from a reservoir in order to maintain a constant temperature. **(b)** A gas expands rapidly into an evacuated region after a membrane is broken.

Now consider the completely thermally insulated system shown in figure 9.6b. When the membrane is broken, the gas expands rapidly into the vacuum until it occupies a volume $V(\quad)_f$ and is at a pressure (P_f). In this case, the gas does no work because there is no movable piston on which the gas applies a force. Furthermore, no energy is transferred by heat through the insulating wall.

The initial and final states of the ideal gas in figure 9.6a are identical to the initial and final states in figure 9.6b, but the paths are different. In the first case, the gas does work on the piston, and energy is transferred slowly to the gas. In the second case, no energy is transferred, and the value of the work done is zero. Therefore, we conclude that energy transfer by heat, like work done, depends on the initial, final, and intermediate states of the system. In other words, because heat and work depend on the path, neither quantity is determined solely by the end points of a thermodynamic process.

9.7 The First Law of Thermodynamics

The law of conservation of energy stated that the change in the energy of a system is equal to the sum of all transfers of energy across the system's boundary. The **first law of thermodynamics** is a special case of the law of conservation of energy that describes processes in which only the internal energy (It is an unfortunate accident of history that the traditional symbol for internal energy is (U), which is also the traditional symbol for potential energy. To avoid confusion between potential energy and internal energy, we use the symbol (E_{int}) for internal energy in this book. If you take an advanced course in thermodynamics, however, be prepared to see (U) used as the symbol for internal energy in the first law.) changes and the only energy transfers are by heat and work:

$$\Delta E_{int} = Q + W \qquad (9-10)$$

An important consequence of the first law of thermodynamics is that there exists a quantity known as internal energy whose value is determined by the state of the system. The internal energy is therefore a state variable like pressure, volume, and temperature.

Let us investigate some special cases in which the first law can be applied. First, consider an *isolated system,* that is, one that does not interact with its surroundings. In this case, no energy transfer by heat takes place and the work done on the system is zero; hence, the internal energy remains constant. That is, because $(Q = W = 0)$, it follows that $(\Delta E_{int} = 0)$; therefore, $\left(E_{int,i} = E_{int,f}\right)$. We conclude that the internal energy (E_{int}) of an isolated system remains constant.

Next, consider the case of a system that can exchange energy with its surroundings and is taken through a cyclic process, that is, *a process that starts and ends at the same state.* In this case, the change in the internal energy must again be zero because (E_{int}) is a state variable; therefore, the energy (Q) added to the system must equal the negative of the work (W) done on the system during the cycle. That is, in a cyclic process,

$$\Delta E_{int} = 0 \text{ and } Q = -W \text{ (cyclic process)}$$

On a PV diagram, a cyclic process appears as a closed curve. (The processes described in figure 9.6 are represented by open curves because the initial and final states differ.) It can be shown that in a cyclic process, the net work done on the system per cycle equals the area enclosed by the path representing the process on a PV diagram.

9.8 Some Applications of the First Law of Thermodynamics

1- Adiabatic process: an adiabatic process is one during which no energy enters or leaves the system by heat; that is, $(Q = 0)$. An adiabatic process can be achieved either by thermally insulating the walls of the system or by performing the process rapidly so that there is negligible time for energy to transfer by heat. Applying the first law of thermodynamics to an adiabatic process gives

$$\Delta E_{int} = W \; (adiabatic\ process) \qquad (9-11)$$

This result shows that if a gas is compressed adiabatically such that (W) is positive, then ΔE_{int} is positive and the temperature of the gas increases. Conversely, the temperature of a gas decreases when the gas expands adiabatically.

Adiabatic processes are very important in engineering practice. Some common examples are the expansion of hot gases in an internal combustion engine, the liquefaction of gases in a cooling system, and the compression stroke in a diesel engine.

2- Isobaric process: a process that occurs at constant pressure**.** In such a process, the values of the heat and the work are both usually nonzero. The work done on the gas in an isobaric process is simply

$$W = -P(V_f - V_i) \; (isobaric\ process) \qquad (9-12)$$

where, (P) is the constant pressure of the gas during the process.

3- Isovolumetric process: a process that takes place at constant volume**.** Because the volume of the gas does not change in such a process, the work given by equation (9-9) is zero. Hence, from the first law we see that in an isovolumetric process, because $(W = 0)$,

$$\Delta E_{int} = Q \; (isovolumetric\ process) \qquad (9-13)$$

This expression specifies that if energy is added by heat to a system kept at constant volume, all the transferred energy remains in the system as an increase in its internal energy. For example, when a can of spray paint is thrown into a fire, energy enters the system (the gas in the can) by heat through the metal walls of the can. Consequently, the temperature, and therefore the pressure, in the can increases until the can possibly explodes.

4- Isothermal process: a process that occurs at constant temperature. A plot of (P) versus (V) at constant temperature for an ideal gas yields a hyperbolic curve called an isotherm. The internal energy of an ideal gas is a function of temperature only. Hence, in an isothermal process involving an ideal gas, $(\Delta E_{int} = 0)$. For an isothermal process, we conclude from the first law that the energy transfer Q must be equal to the negative of the work done on the gas; that is, $(Q = -W)$. Any energy that enters the system by heat is transferred out of the system by work; as a result, no change in the internal energy of the system occurs in an isothermal process.

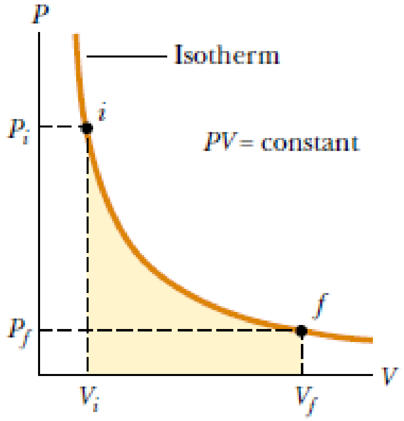


Figure 9.7: The *PV* diagram for an isothermal expansion of an ideal gas from an initial state to a final state. The curve is a hyperbola.

9.9 Energy Transfer Mechanisms in Thermal Processes

It is important to understand the rate at which energy is transferred between a system and its surroundings and the mechanisms responsible for the transfer. let us look at three common energy transfer mechanisms that can result in a change in internal energy of a system.

Thermal Conduction

The energy transfer process that is most clearly associated with a temperature difference is thermal conduction. In this process, the transfer can be represented on an atomic scale as an exchange of kinetic energy between microscopic particles molecules, atoms, and electrons in which less energetic particles gain energy in collisions with more energetic particles. For example, if you hold one end of a long metal bar and insert the other end into a flame, you will find that the temperature of the metal in your hand soon increases. The energy reaches your hand by means of conduction. We can understand the process of conduction by examining what is happening to the microscopic particles in the metal. Initially, before the rod is inserted into the flame, the microscopic particles are vibrating about their equilibrium positions. As the flame heats the rod, those particles near the flame begin to vibrate with greater and greater amplitudes. These particles, in turn, collide with their neighbors and transfer some of their energy in the collisions. This increased vibration represents an increase in the temperature of the metal and of your potentially burned hand.

The rate of thermal conduction depends on the properties of the substance being heated. In general, metals are good thermal conductors because they contain large numbers of electrons that are relatively free to move through the metal and so can transport energy over large distances. Therefore, in a good conductor such as copper, conduction takes place by means of both the vibration of atoms and the motion of free electrons. Materials such as wool, cork, paper, and fiberglass are poor conductors. Gases also are poor conductors because the separation distance between the particles is so great.

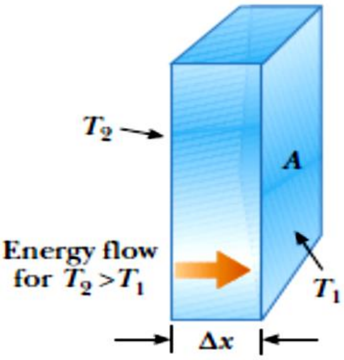


Figure 9.8: Energy transfer through a conducting slab with a cross-sectional area (A) and a thickness (Δx).

Conduction occurs only if there is a difference in temperature between two parts of the conducting medium. Consider a slab of material of thickness (Δx) and cross-sectional area (A). One face of the slab is at a temperature (T_1), and the other face is at a temperature $(T_2 > T_1)$ (figure 9.8). Experimentally, it is found that energy (Q) transfers in a time interval (Δt) from the hotter face to the colder one. The rate $(P = Q/\Delta t)$ at which this energy transfer occurs is found to be proportional to the cross-sectional area and the temperature difference $\Delta(T = T_2 - T_1)$ and inversely proportional to the thickness:

$$P = \frac{Q}{\Delta T} \; \alpha \; A \frac{\Delta T}{\Delta x}$$

Notice that (P) has units of watts when (Q) is in joules and (Δt) is in seconds. For a slab of infinitesimal thickness (Δx) and temperature difference (ΔT), we can write the law of thermal conduction as

$$P = KA\left|\frac{\Delta T}{\Delta x}\right| \qquad (9-14)$$

where the proportionality constant (K) is the thermal conductivity of the material and $|\Delta T/\Delta x|$ is the temperature gradient (the rate at which temperature varies with position).

Substances that are good thermal conductors have large thermal conductivity values, whereas good thermal insulators have low thermal conductivity values. **Table** 9-3 lists thermal conductivities for various substances. Notice that metals are generally better thermal conductors than nonmetals.

Suppose a long, uniform rod of length (L) is thermally insulated so that energy cannot escape by heat from its surface except at the ends as shown in figure 9.9. One end is in thermal contact with an energy reservoir at temperature (T_1), and the other end is in thermal contact with a reservoir at temperature $(T_2 > T_1)$. When a steady state has been reached, the temperature at each point along the rod is constant in time. In this case, if we assume (K) is not a function of temperature, the temperature gradient is the same everywhere along the rod and is

$$\left|\frac{\Delta T}{\Delta x}\right| = \frac{T_2 - T_1}{L}$$

Table 9-3: Thermal Conductivities of some substances.

Substance	**Thermal Conductivity** $(W/m\,.\,℃)$
Matals $(at\ 25\ ℃)$	
Aluminum	238
Copper	397
Gold	314
Iron	79.5
Lead	34.7
Silver	427
Nonmetals (approximate values)	
Asbestos	0.08
Concrete	0.8
Diamond	2300
Glass	0.8
Ice	2
Rubber	0.2
Water	0.6
Wood	0.08
Gases (at 20 °C)	
Air	0.0234
Helium	0.138
Hydrogen	0.172
Oxygen	0.0238
Nitrogen	0.0234

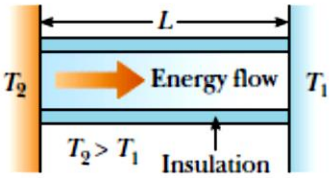


Figure 9.9: Conduction of energy through a uniform, insulated rod of length (L). The opposite ends are in thermal contact with energy reservoirs at different temperatures.

Therefore, the rate of energy transfer by conduction through the rod is

$$P = KA\left(\frac{T_2 - T_1}{L}\right) \qquad (9-15)$$

For a compound slab containing several materials of thicknesses $(L_1, L_2, \dots)$and thermal conductivities $(K_1, K_2, \dots)$ the rate of energy transfer through the slab at steady state is

$$P = \frac{A(T_2 - T_1)}{\sum_i (L_i/K_i)} \qquad (9-16)$$

where (T_2) and (T_1) are the temperatures of the outer surfaces (which are held constant) and the summation is over all slabs. Example (9.4) shows how equation (9-16) results from a consideration of two thicknesses of materials.

Example (9.4):

Two slabs of thickness $\boldsymbol{L_1}$ and $\boldsymbol{L_2}$ and thermal conductivities $\boldsymbol{K_1}$ and $\boldsymbol{K_2}$ are in thermal contact with each other as shown in figure 9.10. The temperatures of their outer surfaces are $\boldsymbol{T_1}$ and $\boldsymbol{T_2}$, respectively, and $\boldsymbol{T_2} > \boldsymbol{T_1}$. Determine the temperature at the interface and the rate of energy transfer by conduction through an area *A* of the slabs in the steady-state condition.

Solution:

Use equation (9-15) to express the rate at which energy is transferred through an area (A) of slab (1):

$$P_1 = K_1 A\left(\frac{T - T_1}{L_1}\right) \quad (1)$$

Express the rate at which energy is transferred through the same area of slab (2):

$$P_2 = K_2 A\left(\frac{T_2 - T}{L_2}\right) \quad (2)$$

Set these two rates dual to represent the steady-state situation:

$$K_1 A\left(\frac{T - T_1}{L_1}\right) = K_2 A\left(\frac{T_2 - T}{L_2}\right)$$

Solve for (T):

$$T = \frac{K_1 L_2 T_1 + K_2 L_1 T_2}{K_1 L_2 + K_2 L_1} \quad (3)$$

Substitute equation (3) into either equation (1) or equation (2):

$$P = \frac{A(T_2 - T_1)}{(L_1/K_1) + (L_2/K_2)} \quad (4)$$

Convection

At one time or another, you probably have warmed your hands by holding them over an open flame. In this situation, the air directly above the flame is heated and expands. As a result, the density of this air decreases and the air rises. This warmed mass of air heats your hands as it flows by. *Energy transferred by the movement of a heated substance is said to have been transferred by* convection. When the movement results from differences in density, as with air around a fire, it is referred to as natural convection. Air flow at a beach is an example of natural convection, as is the mixing that occurs as surface water in a lake cools and sinks. When the heated substance is forced to move by a fan or pump, as in some hot-air and hot-water heating systems, the process is called forced convection.

Figure 9.10: Convection currents are set up in a room heated by a radiator.

If it were not for convection currents, it would be very difficult to boil water. As water is heated in a teakettle, the lower layers are warmed first. The heated water expands and rises to the top because its density is lowered. At the same time, the denser, cool water at the surface sinks to the bottom of the kettle and is heated. The same process occurs when a room is heated by a radiator. The hot radiator warms the air in the lower regions of the room. The warm air expands and rises to the ceiling because of its lower density. The denser, cooler air from above sinks, and the continuous air current pattern shown in figure 9.10 is established.

Radiation

The third means of energy transfer we shall discuss is **thermal radiation.** All objects radiate energy continuously in the form of electromagnetic waves produced by thermal vibrations of the molecules. You are likely familiar with electromagnetic radiation in the form of the orange glow from an electric stove burner, an electric space heater, or the coils of a toaster. The rate at which an object radiates energy is proportional to the fourth power of its absolute temperature. Known as Stefan's law, this behavior is expressed in equation form as

$$P = \sigma A e T^4 \qquad (9-18)$$

where (P) is the power in watts of electromagnetic waves radiated from the surface of the object, (σ) is a constant equal to $5.6696 \times 10^{-8}\ W/m^2\ K^4$, (A) is the surface area of the object in square meters, (e) is the emissivity, and (T) is the surface temperature in kelvins. The value of (e) can vary between zero and unity depending on the properties of the surface of the object. The emissivity is equal to the absorptivity, which is the fraction of the incoming radiation that the surface absorbs. A mirror has very low absorptivity because it reflects almost all incident light. Therefore, a mirror surface also has a very low emissivity. At the other extreme, a black surface has high absorptivity and high emissivity. An **ideal absorber** is defined as an object that absorbs all the energy incident on it, and for such an object, $e = 1$. An object for which $(e = 1)$ is often referred to as a **black body.** We shall investigate experimental and theoretical approaches to radiation from a black body.

Every second, approximately $(1370\,J)$ of electromagnetic radiation from the Sun passes perpendicularly through each $(1\,m^2)$ at the top of the Earth's atmosphere. This radiation is primarily visible and infrared light accompanied by a significant amount of ultraviolet radiation. Enough energy arrives at the surface of the Earth each day to supply all our energy needs on this planet hundreds of times over, if only it could be captured and used efficiently. The growth in the number of solar energy–powered houses built in the United States reflects the increasing efforts being made to use this abundant energy.

What happens to the atmospheric temperature at night is another example of the effects of energy transfer by radiation. If there is a cloud cover above the Earth, the water vapor in the clouds absorbs part of the infrared radiation emitted by the Earth and re-emits it back to the surface. Consequently, temperature levels at the surface remain moderate. In the absence of this cloud cover, there is less in the way to prevent this radiation from escaping into space; therefore, the temperature decreases more on a clear night than on a cloudy one.

As an object radiates energy at a rate given by equation (9-18), it also absorbs electromagnetic radiation from the surroundings, which consist of other objects that radiate energy. If the latter process did not occur, an object would eventually radiate all its energy and its temperature would reach absolute zero. If an object is at a temperature (T) and its surroundings are at an average temperature (T_o), the net rate of energy gained or lost by the object as a result of radiation is

$$P_{net} = \sigma A e (T^4 - T_o^4) \qquad (9-19)$$

When an object is in equilibrium with its surroundings, it radiates and absorbs energy at the same rate and its temperature remains constant. When an object is hotter than its surroundings, it radiates more energy than it absorbs and its temperature decreases.

The Dewar Flask

The *Dewar flask* (Invented by Sir James Dewar (1842–1923).) is a container designed to minimize energy transfers by conduction, convection, and radiation. Such a container is used to store cold or hot liquids for long periods of time. (An insulated bottle, such as a Thermos, is a common household equivalent of a Dewar flask.) The standard construction (figure 9.11) consists of a double-walled Pyrex glass vessel with silvered walls. The space between the walls is evacuated to minimize energy transfer by conduction and convection.

The silvered surfaces minimize energy transfer by radiation because silver is a very good reflector and has very low emissivity. A further reduction in energy loss is obtained by reducing the size of the neck. Dewar flasks are commonly used to store liquid nitrogen (boiling point $77\ K$) and liquid oxygen (boiling point $90\ K$). To confine liquid helium (boiling point $4.2\ K$), which has a very low heat of vaporization, it is often necessary to use a double Dewar system in which the Dewar flask containing the liquid is surrounded by a second Dewar flask. The space between the two flasks is filled with liquid nitrogen.

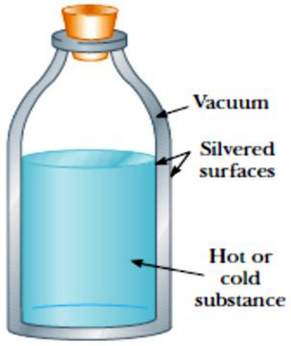


Figure 9.11: A cross-sectional view of a Dewar flask, which is used to store hot or cold substances.

Newer designs of storage containers use “superinsulation” that consists of many layers of reflecting material separated by fiberglass. All this material is in a vacuum, and no liquid nitrogen is needed with this design.

PROBLEMS

1. Consider Joule's apparatus described in figure 9.1. Each of the two masses is $\mathbf{1.50\ kg}$, and the tank is filled with $\mathbf{200\ g}$ of water. What is the increase in the temperature of the water after the masses fall through a distance of 3 m?

2. The temperature of a silver bar rises by $\mathbf{10\ °C}$ when it absorbs $\mathbf{1.23\ kJ}$ of energy by heat. The mass of the bar is $\mathbf{525\ g}$. Determine the specific heat of silver.

3. A $\mathbf{50\ g}$ sample of copper is at $\mathbf{25\ °C}$. If $\mathbf{1200\ J}$ of energy is added to it by heat, what is its final temperature?

4. A $\mathbf{1.5\ kg}$ iron horseshoe initially at $\mathbf{600\ °C}$ is dropped into a bucket containing $\mathbf{20\ kg}$ of water at $\mathbf{25\ °C}$. What is the final temperature? (Neglect the heat capacity of the container and assume that a negligible amount of water boils away.)

5. An aluminum cup with a mass of $\mathbf{200\ g}$ contains $\mathbf{800\ g}$ of water in thermal equilibrium at $\mathbf{80\ °C}$. The combination of cup and water is cooled uniformly so that the temperature decreases at a rate of $\mathbf{1.5\ °C/min}$. At what rate is energy being removed by heat? Express your answer in watts.

6. A $\mathbf{3\ g}$ copper penny at $\mathbf{25\ °C}$ drops from a height of $\mathbf{50\ m}$ to the ground. (a) If $\mathbf{60}$ % of the change in potential energy goes into increasing the internal energy, what is its final temperature? (b) Does the result you obtained in (a) depend on the mass of the penny?

7. How much energy is required to change a $\mathbf{40\ g}$ ice cube from ice at $\mathbf{-10\ °C}$ to steam at $\mathbf{110\ °C}$?

8. A $\mathbf{3\ g}$ lead bullet at $\mathbf{30\ °C}$ is fired at a speed of $\mathbf{240\ m/s}$ into a large block of ice at $\mathbf{0\ °C}$, in which it becomes embedded. What quantity of ice melts?

9. A $\mathbf{1\ kg}$ block of copper at $\mathbf{20\ °C}$ is dropped into a large vessel of liquid nitrogen at $\mathbf{77.3\ K}$. How many kilograms of nitrogen boil away by the time the copper reaches $\mathbf{77.3\ K}$? (The specific heat of copper is $\mathbf{0.092\ cal/g.°C}$. The latent heat of vaporization of nitrogen is $\mathbf{48\ cal/g}$.)

10. A $\mathbf{50\ g}$ copper calorimeter contains $\mathbf{250\ g}$ of water at $\mathbf{20\ °C}$. How much steam must be condensed into the water if the final temperature of the system is to reach $\mathbf{50\ °C}$?

11. A glass window pane has an area of $\mathbf{3\ m^2}$ and a thickness of $\mathbf{0.6\ cm}$. If the temperature difference between its faces is $\mathbf{25\ °C}$, what is the rate of energy transfer by conduction through the window?

12. Gas in a container is at a pressure of $\mathbf{1.50\ atm}$ and a volume of $\mathbf{4.00\ m^3}$. What is the work done by the gas (a) if it expands at constant pressure to twice its initial volume? (b) If it is compressed at constant pressure to one quarter of its initial volume?

13. An ideal gas is enclosed in a cylinder with a movable piston on top. The piston has a mass of $\mathbf{8000\ g}$ and an area of $\mathbf{5\ cm^2}$ and is free to slide up and down, keeping the pressure of the gas constant. How much work is done as the temperature of $\mathbf{0.2\ mol}$ of the gas is raised from $\mathbf{20\ °C}$ to $\mathbf{300\ °C}$?

CHAPTER (10)
HEAT ENGINES, ENTROPY AND THE SECOND LAW OF THERMODYNAMICS

10.1 Introduction

Although the first law of thermodynamics is very important, it makes no distinction between processes that occur spontaneously and those that do not. Only certain types of energy conversion and energy transfer processes actually take place in nature, however. The *second law of thermodynamic* establishes which processes do and do not occur. The following are examples of processes that do not violate the first law of thermodynamics if they proceed in either direction, but are observed in reality to proceed in only one direction:

- When two objects at different temperatures are placed in thermal contact with each other, the net transfer of energy by heat is always from the warmer object to the cooler object, never from the cooler to the warmer.
- A rubber ball dropped to the ground bounces several times and eventually comes to rest, but a ball lying on the ground never gathers internal energy from the ground and begins bouncing on its own.
- An oscillating pendulum eventually comes to rest because of collisions with air molecules and friction at the point of suspension. The mechanical energy of the system is converted to internal energy in the air, the pendulum, and the suspension; the reverse conversion of energy never occurs.

All these processes are *irreversible;* that is, they are processes that occur naturally in one direction only. No irreversible process has ever been observed to run backward. If it were to do so, it would violate the second law of thermodynamics

10.2 Heat Engines and the Second Law of Thermodynamics

A heat engine is a *device that takes in energy by heat and operating in a cyclic process*, expels a fraction of that energy by means of work. For instance, in a typical process by which a power plant produces electricity, a fuel such as coal is burned and the high-temperature gases produced are used to convert liquid water to steam. This steam is directed at the blades of a turbine, setting it into rotation. The mechanical energy associated with this rotation is used to drive an electric generator.

A heat engine carries some working substance through a cyclic process during which (1) the working substance absorbs energy by heat from a high-temperature energy reservoir, (2) work is done by the engine, and (3) energy is expelled by heat to a lower-temperature reservoir. As an example, consider the operation of a steam engine, which uses water as the working substance. The water in a boiler absorbs energy from burning fuel and evaporates to steam, which then does work by expanding against a piston. After the steam cools and condenses, the liquid water produced returns to the boiler and the cycle repeats.

It is useful to represent a heat engine schematically as in figure 10.1. The engine absorbs a quantity of energy $|Q_h|$ from the hot reservoir. For the mathematical discussion of heat engines, we use absolute values to make all energy transfers by heat positive, and the direction of transfer is indicated with an explicit positive or negative sign. The engine does work (W_{eng}) (so that *negative* work $(W = 2W_{eng})$ is done *on* the engine) and then gives up a quantity of energy $|Q_c|$ to the cold reservoir. Because the working substance goes through a cycle, its initial and final internal energies are equal: $(\Delta E_{int} = 0)$. Hence, from the first law of thermodynamics, $(\Delta E_{int} = Q + W = Q - W_{eng} = 0)$, and the net work (W_{eng}) done by a heat engine is equal to the net energy (Q_{net}) transferred to it. As you can see from figure 10.2, $Q_{net} = |Q_h| - |Q_c|$; therefore,

$$W_{eng} = |Q_h| - |Q_c| \qquad (10-1)$$

The **thermal efficiency** (e) of a heat engine is defined as *the ratio of the net work done by the engine during one cycle to the energy input at the higher temperature during the cycle*:

$$e = \frac{W_{eng}}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|} \qquad (10-2)$$

You can think of the efficiency as the ratio of what you gain (work) to what you give (energy transfer at the higher temperature). In practice, all heat engines expel only a fraction of the input energy (Q_h) by mechanical work; consequently, their efficiency is always less than 100%. For example, a good automobile engine has an efficiency of about 20%, and diesel engines have efficiencies ranging from 35% to 40%. Equation (10-2) shows that a heat engine has 100% efficiency ($e = 1$) only if $|Q_c| = 0$, that is, if no energy is expelled to the cold reservoir. In other words, a heat engine with perfect efficiency would have to expel all the input energy by work. Because efficiencies of real engines are well below 100 %, the **Kelvin–Planck form of the second law of thermodynamics** states the following:

It is impossible to construct a heat engine that, operating in a cycle, produces no effect other than the input of energy by heat from a reservoir and the performance of an equal amount of work.

This statement of the second law means that during the operation of a heat engine, (W_{eng}) can never be equal to $|Q_h|$ or, alternatively, that some energy $|Q_c|$ must be rejected to the environment. Every heat engine *must* have some energy exhaust. Figure 10.2 is a schematic diagram of the impossible “perfect” heat engine.

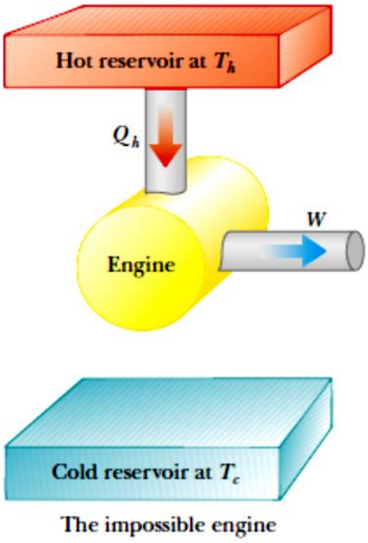


Figure 10.1: Schematic representation of a heat engine.

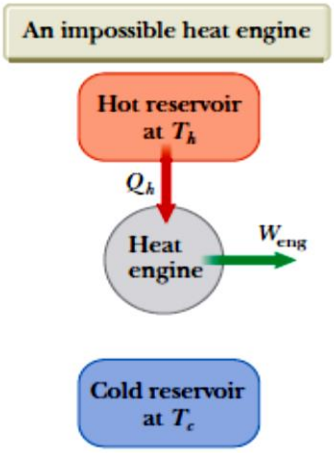


Figure 10.2: Schematic diagram of a heat engine that takes in energy from a hot reservoir and does an equivalent amount of work. It is impossible to construct such a perfect engine.

Example (10.1):

An engine transfers $\mathbf{2 \times 10^3}$ $\boldsymbol{J}$ of energy from a hot reservoir during a cycle and transfers $\mathbf{1.5 \times 10^3}$ $\boldsymbol{J}$ as exhaust to a cold reservoir. **(a)** Find the efficiency of the engine.

Solution:

From equation (10-2):

$$e = 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{1.5 \times 10^3 j}{2 \times 10^3 j} = 0.25 \; or \; 25\%$$

Find the work done by the engine by taking the difference between the input and output energies:

$$W_{eng} = |Q_h| - |Q_c| = 2.00 \times \mathbf{10^3} \, J - 1.50 \times \mathbf{10^3} \, J = 5.0 \times \mathbf{10^2} J$$

(b) How much work does this engine do in one cycle?

Solution:

If you were told that the engine operates at $\mathbf{2000 \; rpm}$ (revolutions per minute), however, you could relate this rate to the period of rotation $\boldsymbol{T}$ of the mechanism of the engine. Assuming there is one thermodynamic cycle per revolution, the power is

$$P = \frac{W_{eng}}{T} = \frac{5.0 \times 10^2 J}{\left(\frac{1}{2000 \; min}\right)} \left(\frac{1 \; min}{60 \; s}\right) = 1.7 \times 10^4 \text{ w}$$

10.3 Reversible and Irreversible Processes

To discuss a theoretical heat engine, we must first examine the meaning of reversible and irreversible processes. In a **reversible** process, the system undergoing the process can be returned to its initial conditions along the same path on a PV diagram, and every point along this path is an equilibrium state. A process that does not satisfy these requirements is **irreversible.**

All natural processes are known to be irreversible. Consider a gas in a thermally insulated container as shown in figure 10.3. A membrane separates the gas from a vacuum. When the membrane is punctured, the gas expands freely into the vacuum. As a result of the puncture, the system has changed because it occupies a greater volume after the expansion. Because the gas does not exert a force through a displacement, it does no work on the surroundings as it expands. In addition, no energy is transferred to or from the gas by heat because the container is insulated from its surroundings. Therefore, in this adiabatic process, the system has changed but the surroundings have not. For this process to be reversible, we must return the gas to its original volume and temperature without changing the surroundings. Imagine trying to reverse the process by compressing the gas to its original volume. To do so, we fit the container with a piston and use an engine to force the piston inward. During this process, the surroundings change because work is being done by an outside agent on the system. In addition, the system changes because the compression increases the temperature of the gas. The temperature of the gas can be lowered by allowing it to come into contact with an external energy reservoir. Although this step returns the gas to its original conditions, the surroundings are again affected because energy is being added to the surroundings from the gas. If this energy could be used to drive the engine that compressed the gas, the net energy transfer to the surroundings would be zero. In this way, the system and its surroundings could be returned to their initial conditions and we could identify the process as reversible. The Kelvin–Planck statement of the second law, however, specifies that the energy removed from the gas to return the temperature to its original value cannot be completely converted to mechanical energy in the form of the work done by the engine in compressing the gas. Therefore, we must conclude that the process is irreversible.

We could also argue that the adiabatic free expansion is irreversible by relying on the portion of the definition of a reversible process that refers to equilibrium states. For example, during the sudden expansion, significant variations in pressure occur throughout the gas. The PV diagram for an adiabatic free expansion would show the initial and final conditions as points, but these points would not be connected by a path. Therefore, because the intermediate conditions between the initial and final states are not equilibrium states, the process is irreversible.

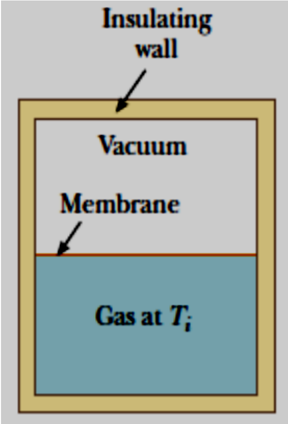


Figure 10.3: Adiabatic free expansion of a gas.

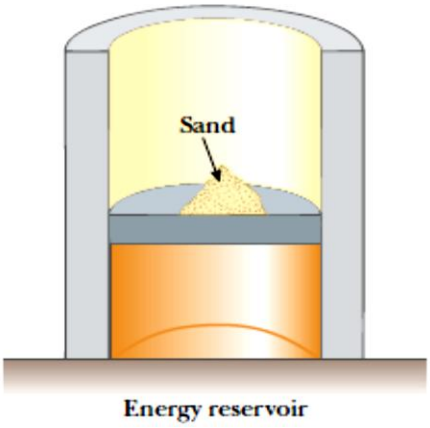


Figure 10.4: A method for compressing a gas in a reversible isothermal process

Although all real processes are irreversible, some are almost reversible. If a real process occurs very slowly such that the system is always very nearly in an equilibrium state, the process can be approximated as being reversible. Suppose a gas is compressed isothermally in a piston–cylinder arrangement in which the gas is in thermal contact with an energy reservoir and we continuously transfer just enough energy from the gas to the reservoir to keep the temperature constant. For example, imagine that the gas is compressed very slowly by dropping grains of sand onto a frictionless piston as shown in figure 10.4. As each grain lands on the piston and compresses the gas a small amount, the system deviates from an equilibrium state, but it is so close to one that it achieves a new equilibrium state in a relatively short time interval. Each grain added represents a change to a new equilibrium state, but the differences between states are so small that the entire process can be approximated as occurring through continuous equilibrium states. The process can be reversed by slowly removing grains from the piston.

A general characteristic of a reversible process is that no dissipative effects (such as turbulence or friction) that convert mechanical energy to internal energy can be present. Such effects can be impossible to eliminate completely. Hence, it is not surprising that real processes in nature are irreversible.

10.4 The Carnot Engine

In 1824, a French engineer named Sadi Carnot described a theoretical engine, now called a Carnot engine, that is of great importance from both practical and theoretical viewpoints. He showed that a heat engine operating in an ideal, reversible cycle called a Carnot cycle between two energy reservoirs is the most efficient engine possible. Such an ideal engine establishes an upper limit on the efficiencies of all other engines. That is, the net work done by a working substance taken through the Carnot cycle is the greatest amount of work possible for a given amount of energy supplied to the substance at the higher temperature. Carnot's theorem can be stated as follows:

No real heat engine operating between two energy reservoirs can be more efficient than a Carnot engine operating between the same two reservoirs.

To prove the validity of this theorem, imagine two heat engines operating between the *same* energy reservoirs. One is a Carnot engine with efficiency (e_C), and the other is an engine with efficiency e, where we assume $(e > e_C)$. Because the cycle in the Carnot engine is reversible, the engine can operate in reverse as a refrigerator. The more efficient engine is used to drive the Carnot engine as a Carnot refrigerator. The output by work of the more efficient engine is matched to the input by work of the Carnot refrigerator. For the *combination* of the engine and refrigerator, no exchange by work with the surroundings occurs. Because we have assumed the engine is more efficient than the refrigerator, the net result of the combination is a transfer of energy from the cold to the hot reservoir without work being done on the combination. According to the Clausius statement of the second law, this process is impossible. Hence, the assumption that $(e > e_C)$ must be false.

All real engines are less efficient than the Carnot engine because they do not operate through a reversible cycle. The efficiency of a real engine is further reduced by such practical difficulties as friction and energy losses by conduction. To describe the Carnot cycle taking place between temperatures (T_c) and (T_h), let's assume the working substance is an ideal gas contained in a cylinder fitted with a movable piston at one end. The cylinder's walls and the piston are thermally nonconducting. Four stages of the Carnot cycle are shown in figure 10.5, and the *PV* diagram for the cycle is shown in figure 10.5. The Carnot cycle consists of two adiabatic processes and two isothermal processes, all reversible:

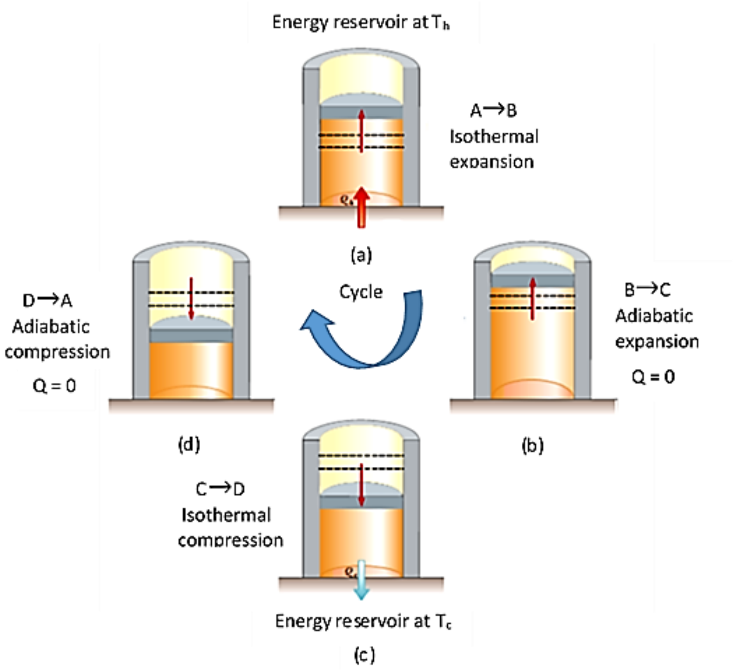


Figure 10.5: The Carnot cycle. The letters *A*, *B*, *C*, and *D* refer to the states of the gas shown in Active.

1. Process $A \rightarrow B$ (figure 10.5a) is an isothermal expansion at temperature (T_h). The gas is placed in thermal contact with an energy reservoir at temperature (T_h). During the expansion, the gas absorbs energy $|Q_h|$ from the reservoir through the base of the cylinder and does work (W_{AB}) in raising the piston.

2. In process $(B \to C)$ (figure 10.5b), the base of the cylinder is replaced by a thermally nonconducting wall and the gas expands adiabatically; that is, no energy enters or leaves the system by heat. During the expansion, the temperature of the gas decreases from (T_h) to (T_c) and the gas does work W_{BC} in raising the piston
3. In process $(C \to D)$ (figure 10.5c), the gas is placed in thermal contact with an energy reservoir at temperature T_c and is compressed isothermally at temperature (T_c). During this time, the gas expels energy $|Q_c|$ to the reservoir and the work done by the piston on the gas is W_{CD}.
4. In the final process $(D \to A)$ (figure 10.5d), the base of the cylinder is replaced by a nonconducting wall and the gas is compressed adiabatically. The temperature of the gas increases to (T_h), and the work done by the piston on the gas is (W_{DA}).

The thermal efficiency of the engine is given by equation (10-2):

$$e = 1 - \frac{|Q_c|}{|Q_h|}$$

For a Carnot engine, the following relationship between the thermal energy transfers and the absolute temperatures can be derived:

$$\frac{|Q_c|}{|Q_h|} = \frac{T_c}{T_h} \qquad (10-5)$$

Hence, the thermal efficiency of a Carnot engine is

$$e_C = 1 - \frac{T_c}{T_h} \qquad (10-6)$$

Efficiency of a Carnot engine

This result indicates that all Carnot engines operating between the same two temperatures have the same efficiency (For the processes in the Carnot cycle to be reversible, they must be carried out infinitesimally slowly. Therefore, although the Carnot engine is the most efficient engine possible, it has zero power output because it takes an infinite time interval to complete one cycle! For a real engine, the short time interval for each cycle results in the working substance reaching a high temperature lower than that of the hot reservoir and a low temperature higher than that of the cold reservoir).

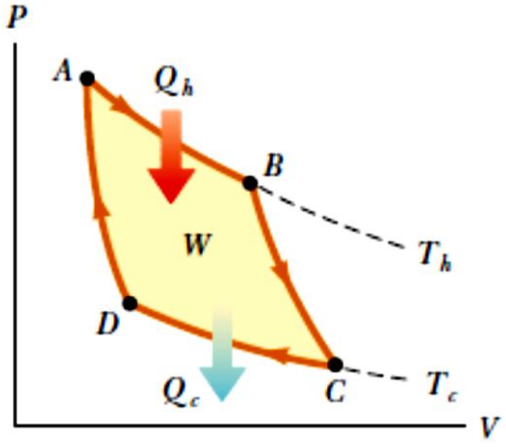


Figure 10.6: PV diagram for the Carnot cycle. The net work done (W_{eng}) equals the net energy transferred into the Carnot engine in one cycle, $|Q_h| - |Q_c|$.

Equation (10-6) can be applied to any working substance operating in a Carnot cycle between two energy reservoirs. According to this equation, the efficiency is zero if $(T_c = T_h)$, as one would expect. The efficiency increases as (T_c) is lowered and (T_h) is raised. The efficiency can be unity (100 %), however, only if $(T_c = 0\,K)$. Such reservoirs are not available; therefore, the maximum efficiency is always less than 100%. In most practical cases, (T_c) is near room temperature, which is about $300\,K$.

Therefore, one usually strives to increase the efficiency by raising (T_h). Theoretically, a Carnot-cycle heat engine run in reverse constitutes the most effective heat pump possible, and it determines the maximum (COP) for a given combination of hot and cold reservoir temperatures. Using equations (10-1) and (10-4), we see that the maximum (COP) for a heat pump in its heating mode is

$$COP_c(heating\ mode) = \frac{|Q_h|}{W}$$

$$= \frac{|Q_h|}{|Q_h| - |Q_c|} = \frac{1}{1 - \frac{|Q_c|}{|Q_h|}} = \frac{1}{1 - \frac{T_c}{T_h}} = \frac{T_h}{T_h - T_c}$$

The Carnot (COP) for a heat pump in the cooling mode is

$$COP_c(cooling\ mode) = \frac{T_c}{T_h - T_c}$$

As the difference between the temperatures of the two reservoirs approaches zero in this expression, the theoretical (COP) approaches infinity. In practice, the low temperature of the cooling coils and the high temperature at the compressor limit the (COP) to values below 10.

10.5 Entropy

The zeroth law of thermodynamics involves the concept of temperature, and the first law involves the concept of internal energy. Temperature and internal energy are both state variables; that is, the value of each depends only on the thermodynamic state of a system, not on the process that brought it to that state. Another state variable this one related to the second law of thermodynamics is *entropy* (S).

Entropy was originally formulated as a useful concept in thermodynamics. Its importance grew, however, as the field of statistical mechanics developed because the analytical techniques of statistical mechanics provide an alternative means of interpreting entropy and a more global significance to the concept. In statistical mechanics, the behavior of a substance is described in terms of the statistical behavior of its atoms and molecules. An important finding in these studies is that isolated systems tend toward disorder, and entropy is a measure of this disorder. For example, consider the molecules of a gas in the air in your room. If half the gas molecules had velocity vectors of equal magnitude directed toward the left and the other half had velocity vectors of the same magnitude directed toward the right, the situation would be very ordered. Such a situation is extremely unlikely, however. If you could view the molecules, you would see that they move haphazardly in all directions, bumping into one another, changing speed upon collision, some going fast and others going slowly. This situation is highly disordered.

The cause of the tendency of an isolated system toward disorder is easily explained. To do so, we distinguish between microstates and macrostates of a system. A microstate is *a particular description of the properties of the individual molecules of the system*. For example, the description we just gave of the velocity vectors of the air molecules in your room being very ordered refers to a particular microstate, and the more likely likely haphazard motion is another microstate—one that represents disorder. A macrostate is *a description of the conditions of the system from a macroscopic point of view and makes use of macroscopic variables* such as pressure, density, and temperature. For example, in both of the microstates described for the air molecules in your room, the air molecules are distributed uniformly throughout the volume of the room; this uniform density distribution is a macrostate. We could not distinguish between our two microstates by making a macroscopic measurement—both microstates would appear to be the same macroscopically, and the two macrostates corresponding to these microstates are equivalent.

For any given macrostate of the system, a number of microstates are possible, or accessible. Among these microstates, it is assumed that all are equally probable. However, when all possible microstates are examined, it is found that far more of them are disordered than are ordered. Because all of the microstates are equally probable, it is highly likely that the actual macrostate is one resulting from one of the highly disordered microstates, simply because there are many more of them. Similarly, the probability of a macrostate's forming from disordered microstates is greater than the probability of a macrostate's forming from ordered microstates.

All physical processes that take place in a system tend to cause the system and its surroundings to move toward more probable macrostates. The more probable macrostate is always one of greater disorder. If we consider a system and its surroundings to include the entire Universe, then the Universe is always moving toward a macrostate corresponding to greater disorder. Because entropy is a measure of disorder, an alternative way of stating this is the entropy of the Universe increases in all real processes. This is yet another statement of the second law of thermodynamics that can be shown to be equivalent to the Kelvin–Planck and Clausius statements.

The original formulation of entropy in thermodynamics involves the transfer of energy by heat during a reversible process. Consider any infinitesimal process in which a system changes from one equilibrium state to another. If (dQ_r) is the amount of energy transferred by heat when the system follows a reversible path between the states, the change in entropy (dS) is equal to this amount of energy for the reversible process divided by the absolute temperature of the system:

$$dS = \frac{dQ_r}{T} \tag{10-8}$$

We have assumed the temperature is constant because the process is infinitesimal. Because entropy is a state variable, the change in entropy during a process depends only on the endpoints and therefore is independent of the actual path followed. Consequently, the entropy change for an irreversible process can be determined by calculating the entropy change for a *reversible* process that connects the same initial and final states.

The subscript (r) on the quantity (dQ_r) is a reminder that the transferred energy is to be measured along a reversible path even though the system may actually have followed some irreversible path. When energy is absorbed by the system, (dQ_r) is positive and the entropy of the system increases. When energy is expelled by the system, (dQ_r) is negative and the entropy of the system decreases. Notice that equation. (10-8) does not define entropy but rather the *change* in entropy. Hence, the meaningful quantity in describing a process is the *change* in entropy.

To calculate the change in entropy for a *finite* process, first recognize that (T) is generally not constant during the process. Therefore, we must integrate equation (10-8):

$$\Delta S = \int_i^f dS = \int_i^f \frac{dQ_r}{T} \tag{10-9}$$

As with an infinitesimal process, the change in entropy (ΔS) of a system going from one state to another has the same value for *all* paths connecting the two states. That is, the finite change in entropy (ΔS) of a system depends only on the properties of the initial and final equilibrium states. Therefore, we are free to choose a particular reversible path over which to evaluate the entropy in place of the actual path as long as the initial and final states are the same for both paths.

Let's consider the changes in entropy that occur in a Carnot heat engine that operates between the temperatures (T_c) and (T_h). In one cycle, the engine takes in energy $|Q_h|$ from the hot reservoir and expels energy $|Q_c|$ to the cold reservoir. These energy transfers occur only during the isothermal portions of the Carnot cycle; therefore, the constant temperature can be brought out in front of the integral sign in equation (10-9). The integral then simply has the value of the total amount of energy transferred by heat. Therefore, the total change in entropy for one cycle is

$$\Delta S = \frac{|Q_h|}{T_h} - \frac{|Q_c|}{T_c}$$

where the minus sign represents that energy is leaving the engine. In example (10), we showed that for a Carnot engine,

$$\frac{|Q_c|}{|Q_h|} - \frac{T_h}{T_c}$$

Using this result in the previous expression for $(\varDelta S)$, we find that the total change in entropy for a Carnot engine operating in a cycle is *zero:*

$$\varDelta S = 0$$

Now consider a system taken through an arbitrary (non-Carnot) reversible cycle. Because entropy is a state variable and hence depends only on the properties of a given equilibrium state we conclude that $(\varDelta S = 0)$ for *any* reversible cycle. In general, we can write this condition as

$$\oint \frac{dQ_r}{T} = 0 \text{ (reversible cycle)} \qquad (10-10)$$

where the symbol (r) indicates that the integration is over a closed path.

10.6 Entropy and the Second Law

By definition, a calculation of the change in entropy for a system requires information about a reversible path connecting the initial and final equilibrium states. To calculate changes in entropy for real (irreversible) processes, remember that entropy (like internal energy) depends only on the *state* of the system. That is, entropy is a state variable, and the change in entropy depends only on the initial and final states.

You can calculate the entropy change in some irreversible process between two equilibrium states by devising a reversible process (or series of reversible processes) between the same two states and computing $\left(\Delta S = \oint \frac{dQ_r}{T}\right)$ for the reversible process. In irreversible processes, it is important to distinguish between (Q), the actual energy transfer in the process, and (Q_r), the energy that would have been transferred by heat along a reversible path. Only (Q_r) is the correct value to be used in calculating the entropy change.

If we consider a system and its surroundings to include the entire Universe, the Universe is always moving toward a higher-probability macrostate, corresponding to greater disorder. Because entropy is a measure of disorder, an alternative way of stating this behavior is as follows:

The entropy of the Universe increases in all real processes.

This statement is yet another wording of the second law of thermodynamics that can be shown to be equivalent to the Kelvin-Planck and Clausius statements.

When dealing with a system that is not isolated from its surroundings, remember that the increase in entropy described in the second law is that of the system *and* its surroundings. When a system and its surroundings interact in an irreversible process, the increase in entropy of one is greater than the decrease in entropy of the other. Hence, the change in entropy of the Universe must be greater than zero for an irreversible process and equal to zero for a reversible process. Ultimately, because real processes are irreversible, the entropy of the Universe should increase steadily and eventually reach a maximum value. At this value, the Universe will be in a state of uniform temperature and density. All physical, chemical, and biological processes will have ceased at this time because a state of perfect disorder implies that no energy is available for doing work. This gloomy state of affairs is sometimes referred to as the *heat death* of the Universe.

Entropy Change in Thermal Conduction

Let's now consider a system consisting of a hot reservoir and a cold reservoir that are in thermal contact with each other and isolated from the rest of the Universe. A process occurs during which energy (Q) is transferred by heat from the hot reservoir at temperature (T_h) to the cold reservoir at temperature (T_c). The process as described is irreversible (energy would not spontaneously flow from cold to hot), so we must find an equivalent reversible process. Because the temperature of a reservoir does not change during the process, we can replace the real process for each reservoir with a reversible, isothermal process in which the same amount of energy is transferred by heat. Consequently, for a reservoir, the entropy change does not depend on whether the process is reversible or irreversible.

Because the cold reservoir absorbs energy (Q), its entropy increases by (Q/T_c). At the same time, the hot reservoir loses energy (Q), so its entropy change is $(-Q/T_h)$. Because $(T_h > T_c)$, the increase in entropy of the cold reservoir is greater than the decrease in entropy of the hot reservoir. Therefore, the change in entropy of the system (and of the Universe) is greater than zero:

$$\Delta S_U = \frac{Q}{T_c} + \frac{-Q}{T_h} > 0$$

Suppose energy were to transfer spontaneously from a cold object to a hot object, in violation of the second law. This impossible energy transfer can be described in terms of disorder. Before the transfer, a certain degree of order is associated with the different temperatures of the objects. The hot object's molecules have a higher average energy than the cold object's molecules. If energy spontaneously transfers from the cold object to the hot object, the cold object becomes colder over a time interval and the hot object becomes hotter. The difference in average molecular energy becomes even greater, which would represent an increase in order for the system and a violation of the second law.

In comparison, the process that does occur naturally is the transfer of energy from the hot object to the cold object. In this process, the difference in average molecular energy decreases, which represents a more random distribution of energy and an increase in disorder.

PROBLEMS

1. An engine absorbs $\mathbf{1.7\ kJ}$ from a hot reservoir at $\mathbf{277\ °C}$ and expels $\mathbf{1.2\ kJ}$ to a cold reservoir at $\mathbf{27\ °C}$ in each cycle. (a) What is the engine's efficiency? (b) How much work is done by the engine in each cycle? (c) What is the power output of the engine if each cycle lasts $\mathbf{0.3\ s}$?

2. The work done by an engine equals one-fourth the energy it absorbs from a reservoir. (a) What is its thermal efficiency? (b) What fraction of the energy absorbed is expelled to the cold reservoir?

3. A heat engine takes in $\mathbf{360\,J}$ of energy from a hot reservoir and performs $\mathbf{25\,J}$ of work in each cycle. Find (a) the efficiency of the engine and (b) the energy expelled to the cold reservoir in each cycle.

4. A particular heat engine has a mechanical power output of $\mathbf{5\ kW}$ and an efficiency of $\mathbf{25}\%$. The engine expels $\mathbf{8 \times 10^3\ J}$ of exhaust energy in each cycle. Find (a) the energy taken in during each cycle and (b) the time interval for each cycle.

5. Suppose a heat engine is connected to two energy reservoirs, one a pool of molten aluminum ($\mathbf{660\ °C}$) and the other a block of solid mercury ($\mathbf{238.9\ °C}$). The engine runs by freezing $\mathbf{1\ g}$ of aluminum and melting $\mathbf{15\ g}$ of mercury during each cycle. The heat of fusion of aluminum is $\mathbf{3.97 \times 10^5\ J/kg}$; the heat of fusion of mercury is $\mathbf{1.18 \times 10^4\ J/kg}$. What is the efficiency of this engine?

6. During each cycle, a refrigerator ejects $\mathbf{625\ kJ}$ of energy to a high-temperature reservoir and takes in $\mathbf{550\,kJ}$ of energy from a low-temperature reservoir. Determine (a) the work done on the refrigerant in each cycle and (b) the coefficient of performance of the refrigerator.

7. One of the most efficient heat engines ever built is a coal fired steam turbine in the Ohio River valley, operating between $\mathbf{1870\ °C}$ and $\mathbf{430\ °C}$. (a) What is its maximum theoretical efficiency? (b) The actual efficiency of the engine is $\mathbf{42}\%$. How much mechanical power does the engine deliver if it absorbs $\mathbf{1.4 \times 10^5\ J}$ of energy each second from its hot reservoir?

8. A heat engine operates between a reservoir at $\mathbf{25\ °C}$ and one at $\mathbf{375\ °C}$. What is the maximum efficiency possible for this engine?

9. A Carnot engine has a power output of $\boldsymbol{150\ kW}$. The engine operates between two reservoirs at $\boldsymbol{20\ °C}$ and $\boldsymbol{500\ °C}$. (a) How much energy enters the engine by heat per hour? (b) How much energy is exhausted by heat per hour?

10. A Carnot engine has a power output $\boldsymbol{P}$. The engine operates between two reservoirs at temperature $\boldsymbol{T_c}$ and $\boldsymbol{T_h}$. (a) How much energy enters the engine by heat in a time interval $\boldsymbol{\Delta t}$? (b) How much energy is exhausted by heat in the time interval $\boldsymbol{\Delta t}$? (c) What is the coefficient of performance of a refrigerator that operates with Carnot efficiency between temperatures $\boldsymbol{23\ °C}$ and $\boldsymbol{127\ °C}$?

11. A heat engine is being designed to have a Carnot efficiency of $\boldsymbol{65}$ % when operating between two energy reservoirs. (a) If the temperature of the cold reservoir is $\boldsymbol{20\ °C}$, what must be the temperature of the hot reservoir? (b) Can the actual efficiency of the engine be equal to $\boldsymbol{65}$ %? Explain.

12. What is the maximum possible coefficient of performance of a heat pump that brings energy from outdoors at $\boldsymbol{23\ °C}$ into a $\boldsymbol{22\ °C}$ house? *Note:* The work done to run the heat pump is also available to warm the house.

13. How much work does an ideal Carnot refrigerator require to remove $\boldsymbol{1\ J}$ of energy from liquid helium at $\boldsymbol{4\ K}$ and expel this energy to a room-temperature $(\boldsymbol{293\ K})$ environment?

14. If a $\boldsymbol{35}$ %-efficient Carnot heat engine is run in reverse so as to form a refrigerator (figure 10.3), what would be this refrigerator's coefficient of performance?

15. A Carnot heat engine operates between temperatures $\boldsymbol{T_h}$ and $\boldsymbol{T_c}$. (a) If $\boldsymbol{T_h = 500}$ K and $\boldsymbol{T_c = 350\ K}$, what is the efficiency of the engine? (b) What is the change in its efficiency for each degree of increase in $\boldsymbol{T_h}$ above $\boldsymbol{500\ K}$? (c) What is the change in its efficiency for each degree of change in $\boldsymbol{T_c}$? (d) Does the answer to part (c) depend on $\boldsymbol{T_c}$? Explain.