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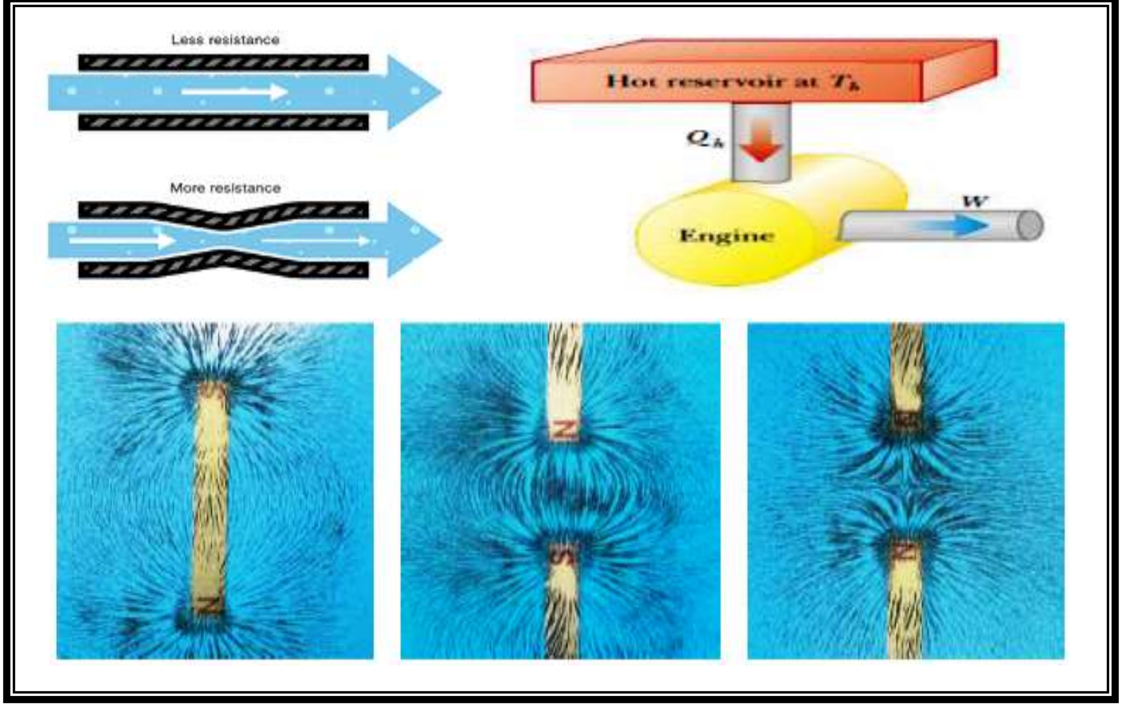
Lecture notes in

Engineering Physics



Assist. Prof. Dr: Eid A. Eid **Dr: Ahmed M. Deghady**

Dr: Mohamed A. Atallah **Dr: Abdel moneim A. Saleh**

CHAPTER (1)
ELECTROSTATICS

1.1 Introduction

Electrostatics is the study of electric charge at rest. The charges at rest are generated due to the friction between two insulating bodies, which are rubbed against each other. When a glass rod is rubbed with a piece of silk, the glass rod acquires the property of attracting small pieces of papers towards its. Then, glass rod is said to be charged with electricity. The Phenomenon of electricity was discovered in the year 600 B.C. by a Greek Philosopher "Thales of Miletus".

1.2 Electrical charges and their kinds

It is well known that, any material was created from atoms which contain equal amount of positive (protons) and negative (electrons) charges, so the atom is named a neutral charge. If the material has lost or gained an electron or more, hence, it can be named an electric charged object. With the help of several experiments, it can be concluded that static electricity has two types of electric charges. When a glass rod is rubbed with silk, glass rod become a *positive charge* $(+Q)$ and the silk carry a *negative charge* $(-Q)$. On the other hand, when ebonite rod is rubbed with wool, the charge on ebonite is found to be $(-Q)$ and wool becomes positively charged $(+Q)$. From figure 1.1a the two glass rods $(+Q)$ repel each other, similarly two ebonite rods $(-Q)$ repel each other as shown in fig 1.1b. But when a glass rod rubbed with silk $(+Q)$ it attracts an ebonite rod rubbed with wool $(-Q)$. figure 1.1c. Thus, we say that bodies having same kind of charge repel each other, while those with an opposite kind of charge attract each other.

Characteristics of electric charge

(i) Electric charge is ***quantized***; the materials cannot loss or gain a fraction of an electron. Therefore, the electric charge either positive or negative is exist only as an integer numbers of electron charge (e) i.e. any positive or negative charge (Q) can be written as: $\boldsymbol{Q = \pm ne}$

where $(n = 1, 2, 3, \ldots)$ and (e) is a charge of an electron $(e = 6 \times 10^{-19}\ Coulomb$ "C")

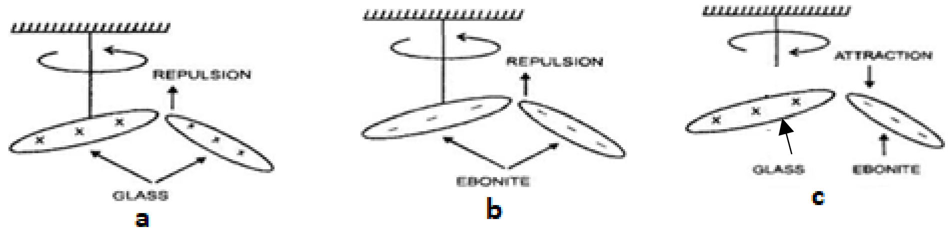


Figure 1.1: Attraction and repulsion between electric charges

(ii) Electric charge is ***additive*** in nature. It implies that the total charge on an extended body is the algebraic sum of the positive and negative charges located at different points of the body. Therefore, the electric charge is a scalar quantity and The S.I. unit of charge is coulomb and it is represented by (C).

(iii) Electric charge is always ***conserved***, means charge can neither be created nor be destroyed in isolation i.e. charge can be created or destroyed only in equal and opposite pairs. Moreover, the charge can be transferred from substance to other as occurring in radioactivity examples.

$$_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4}$$

Example (1.1):

An electrically neutral penny, of mass $\boldsymbol{m = 3.11\ g}$, contains equal amounts of positive and negative charge. Assuming the penny is made entirely of copper, what is the magnitude of $\boldsymbol{q}$ of the total positive (or negative) charge in the penny?

Solution:

Copper $({}^{63.5}_{29}Cu)$ atom has equal positive (Protons) and negative (electrons) charge $(= \pm Ze)$. For copper $(Z = 29)$, which means that copper has (29) protons, so each atom has charge (q).

$$\Longrightarrow q = Ze$$

Then the total positive charge in penny $(q = NZe), N$ is number of Cu atom.

To find (N), we multiply the number of moles of copper in the penny by the number of atoms in a mole (Avogadro's number, $N_A = 6.20 \times 10^{23}\ atoms/mol$). The number of moles of copper in the penny is $(m\ /\ M)$, where (M) is the molar mass of copper, $63.5\ g/mol$. Thus, we have

$$N = N_A \frac{m}{M} = \left[6.02 \times 10^{23} \frac{atoms}{mol}\right]\left[\frac{3.11\ g}{63.5 \frac{g}{mol}}\right]$$

$$N = 2.95 \times 10^{22}\ atoms$$

We then find the magnitude of the total positive or negative charge in the penny to be

$$q = N\ Z\ e = (2.95 \times 10^{22})(29)(1.6 \times 10^{-19}\ C)$$

$$q = 137 \times 10^{3}\ C$$

Example (1.2):

Which is bigger a Coulomb or a charge on an electron? How many electrons in one Coulomb of charge?

Solution:

A Coulomb of charge is bigger than the charge of an electron.

The total number (N) of electrons in one coulomb is

$$N = \frac{q}{e} = \frac{(1\ C)}{(1.6 \times 10^{-19}\ C)} = 0.625 \times 10^{19}\ electrons$$

Example (1.3):

Three small identical balls have charges $-\mathbf{3} \times \mathbf{10^{-12}}\ \boldsymbol{C}, \mathbf{8} \times \mathbf{10^{-12}}\ \boldsymbol{C}$ and $\mathbf{4} \times \mathbf{10^{-12}}\ \mathbf{C}$ respectively. They are brought in contact and then separated. Calculate (a) charge on each ball (b) number of electrons in excess or deficit on each ball after contact.

Solution:

(a) The charge on each ball

$$q = \frac{q_1 + q_2 + q_3}{3} = \frac{[(-3\ ) + (8) + (4)] \times 10^{-12}\ C}{3} = 3 \times 10^{-12} C$$

(b) Since the charge is positive, there is a shortage of electrons on each ball.

$$N = \frac{q}{e} = \frac{(3 \times 10^{-12}\ C)}{(1.6 \times 10^{-19}\ C)} = 1.875 \times 10^7$$

Then, the number of electrons $= 1.875 \times 10^7$ electrons.

Example (1.4):

A polythene (Plastic) piece rubbed with wool is found to have a negative charge of $\mathbf{-3.2 \times 10^{-7}}\ \boldsymbol{C}$. Estimate the number of electrons transferred from which to which? Is there a transfer of mass from wool to polythene?

Solution:

The number (N) of electrons transferred to plastic piece from wool

$$N = \frac{q}{e} = \frac{(-3.2 \times 10^{-7}\ C)}{(1.6 \times 10^{-19}\ C)} = 2 \times 10^{12}\ electrons$$

Yes, there is a transfer of mass, whereas the mass of each electron $(m_e = 9.11 \times 10^{-31}\ kg)$. Then, the total mass (M) that transferred to plastic piece is

$$M = n\, m_e = (2 \times 10^{12})(9.11 \times 10^{-31} kg) = 1.8 \times 10^{-18} kg$$

1.3 Coulomb's law of electrostatic

In 1785, Coulomb measured the force of attraction or repulsion between two electric charges by using a torsion balance. His observation is known as the Coulomb's Law of electrostatics. It states that;

"T***he force of attraction or repulsion between the two stationary electric charges is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between them***"

Consider that two charges (q_1) and (q_2) at a distance (r) apart as shown in figure 1.2, Then force of attraction or repulsion between the two charges is given by

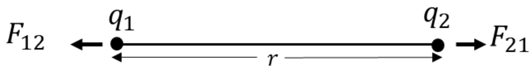


Figure 1.2: The force between two charges

$$F \propto \frac{q_1 q_2}{r^2}$$

$$\Rightarrow F = K\frac{q_1 q_2}{r^2}$$

where, (K) is constant of proportionality, its value depends upon the nature of the medium in which two charges are located and also the system of units adopted to measure (F), (q_1), (q_2) and (r). In S.I., charge is measured in coulomb (C), Force in Newton (N), and distance in meter (m). So that

$$K = \frac{1}{4\pi\varepsilon_o} = 8.997 \times 10^9\ N.m^2/C^2$$

where (ε_o) is called absolute electrical permittivity of free space and its equal $8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$. It is worth to known the physical meaning of (ε_o) , it equal numerically the amount of interaction between the molecules of this medium with electric field that pass through it. Moreover, the modified Coulomb's law for a medium (e.g. plastic or ceramic) could be written as;

$$F = \frac{1}{4\pi\varepsilon_r\varepsilon_o}\frac{q_1 q_2}{r^2}$$

The ratio $(\kappa = \varepsilon/\varepsilon_o)$ is called the relative permittivity or dielectric constant of the medium. The value of κ for air or vacuum is unity.

Example (1.5):

Two equal and similar charges kept **3 *cm*** apart in air repel each other with a force equivalent to weight of mass **4.5 *kg***. Find charges in coulomb.

Solution:

$$\text{Let}, q_1 = q_2 = q$$

Then,

$$F = k\frac{q_1 q_2}{r^2} = k\frac{q^2}{r^2} = \text{mg}$$

$$\Rightarrow q^2 = \frac{mgr^2}{k} = \frac{(4.5\ kg)(9.8\ m/s^2)(0.03\ \text{m})^2}{(9 \times 10^9\ Nm^2/C^2)}$$

$$\Rightarrow q = \pm 2.1 \times 10^{-14} C$$

Example (1.6):

An electron and a proton are at a distance of $\mathbf{10^{-9}\ m}$ from each other in a free space. Compute the force between them.

Solution:

$$F = k\frac{q_1 q_2}{r^2} = k\frac{q^2}{r^2} = (9 \times 10^9\ Nm^2/C^2)\frac{(1.6 \times 10^{-19}\ C)^2}{(10^{-9}\ m)^2} 23.04 \times 10^{-11}\ N$$

Example (1.7):

Two insulated charged spheres of charges $\mathbf{6.5 \times 10^{-7}\ C}$ each are separated by a distance of $\mathbf{0.5\ m}$. Calculate the electrostatic force between them. Also calculate the force (i) when the charges are doubled and the distance of separation is halved. (ii) When the charges are placed in a dielectric medium water $(\boldsymbol{k} = 80)$.

Solution:

$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2} = \frac{(9 \times 10^9\ Nm^2/C^2)(6.5 \times 10^{-7}\ C)^2}{(0.5\ m)^2} = 1.52 \times 10^{-2}\ N$$

(i) If the charge is doubled and separation between them is halved then,

$$F_1 = \frac{1}{4\pi\varepsilon_o}\frac{(2q_1)(2q_2)}{\left(\frac{r}{2}\right)^2} = 16\left[\frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}\right] = 0.24\ N$$

(ii) When placed in water of $(\kappa = 80)$

$$F_2 = \frac{F}{\kappa} = \frac{(1.52 \times 10^{-2}\ N)}{80} = 1.9 \times 10^{-4}\ N$$

Example (1.8):

Compare the magnitude of the electrostatic and gravitational force between an electron and a proton at a distance $(\boldsymbol{r})$ apart in hydrogen atom. ($\boldsymbol{Given}$: $\boldsymbol{me = 9.11 \times 10^{-31}\ kg, m_p = 1.67 \times 10^{-27}\ kg}$, $\boldsymbol{G = 6.67 \times 10^{-11}\ N\mathrm{m^2\ kg^{-2}}}$; $\boldsymbol{e = 1.6 \times 10^{-19}\ C}$).

Solution:

The gravitational force (F_g) between the electron and the proton is found from Newton's law of universal gravitation, as

$$F_g = G\frac{m_P m_e}{r^2}$$

$$F_g = (6.67 \times 10^{-11}\ N\mathrm{m^2\ kg^{-2}})\frac{(1.67 \times 10^{-27}\ kg)(9.11 \times 10^{-31}\ kg)}{(5.29 \times 10^{-11}\ m)^2} = (3.63 \times 10^{-47})\ \mathrm{N}$$

The electric force (F_e) between the electron and the proton is found from Coulomb's law of electrostatics, as

$$F_e = k\frac{q_P q_e}{r^2} = (9 \times 10^9\ Nm^2/C^2)\frac{(1.6 \times 10^{-19}\ C)(1.60 \times 10^{-19}\ C)}{(5.29 \times 10^{-11}\ m)^2}$$

$$F_e = (8.23 \times 10^{-8})\ \text{N}$$

Although both forces seem quite small, let us compare the relative magnitude of these forces by taking the ratio of the electric force to the gravitational force, that is

$$\frac{F_e}{F_g} = \frac{(8.23 \times 10^{-8})\ N}{(3.63 \times 10^{-47})\ N} = (2.27 \times 10^{39})$$

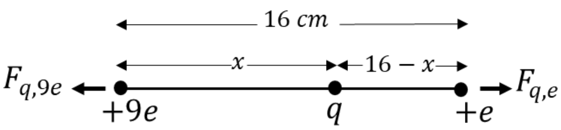
This shows that the electrostatic force is (2.27×10^{39}) times stronger than gravitational force.

Example (1.9):

Two-point charges $(+\mathbf{9}\boldsymbol{e})$ and $(+\mathbf{1}\boldsymbol{e})$ are kept at a distance of $\mathbf{16\ cm}$ from each other. At what point between these charges, should a third charge $(\boldsymbol{q})$ to be placed so that it remains in equilibrium?

Solution:

Let a third charge (q) be kept at a distance (x) from $(+\ 9e)$ and $(r - x)$ from $(+\ e)$



$$F = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2}$$

$$F_{+9e,q} = F_{q,e}$$

$$\frac{1}{4\pi\varepsilon_o}\frac{(9e)(q)}{x^2}$$

$$= \frac{1}{4\pi\varepsilon}\frac{(q)(e)}{(r-x)^2}$$

$$\frac{9}{x^2} = \frac{1}{(r-x)^2}$$

$$\frac{x^2}{(r-x)^2} = 9$$

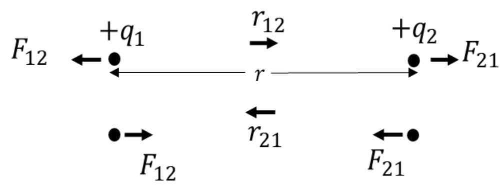
Take the root of both sides

$$\frac{x}{(r-x)} = 3 \quad \Rightarrow \quad \frac{x}{(16-x)} = 3$$

$$x = 12\ cm$$

The third charge should be placed at a distance of $0.12\ m$ from charge $(9e)$.

1.4 Coulomb's law – vector form



(i) If $(\overrightarrow{F_{21}})$ is the force exerted on charge (q_2) by charge (q_1)

$$\overrightarrow{F_{21}} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

where $(\hat{r}_{12})$ is the unit vector from (q_1) to (q_2)

(ii) If $(\overrightarrow{F_{12}})$ is the force exerted on (q_1) due to (q_2);

$$\overrightarrow{F_{12}} = k\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$$

where $(\hat{r}_{21})$ is the unit vector from (q_2) to (q_1), notice, both $(\hat{r}_{21})$ and $(\hat{r}_{12})$ has the same magnitude but oppositely directed.

$$\overrightarrow{F_{12}} = \mathrm{k}\frac{\mathrm{q}_1\mathrm{q}_2}{\mathrm{r}_{12}^2}(-\hat{\mathrm{r}}_{12}) \text{ or } \overrightarrow{F_{12}} = -\mathrm{k}\frac{\mathrm{q}_1\mathrm{q}_2}{\mathrm{r}_{12}^2}\hat{\mathrm{r}}_{12}$$

$$\overrightarrow{F_{12}} = -\overrightarrow{F_{21}}$$

So, the forces exerted by charges on each other are equal in magnitude and opposite in direction.

Principle of superposition

The principle of superposition is to calculate the total electric force experienced by a charge say (q_1) due to other charges q_2, q_3 … …. q_n.

Therefore,

$$\overrightarrow{F_1} = \overrightarrow{F_{12}} + \overrightarrow{F_{13}} + \overrightarrow{F_{14}} + \cdots..\overrightarrow{F_{1n}} =$$

or

$$\overrightarrow{F_1} = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1 q_2}{r_{12}^2}\hat{\mathrm{r}}_{12} + \frac{q_1 q_3}{r_{13}^2}\hat{\mathrm{r}}_{13} + \frac{q_1 q_4}{r_{14}^2}\hat{\mathrm{r}}_{14} + \cdots + \frac{q_1 q_n}{r_{1n}^2}\hat{\mathrm{r}}_{1n}\right]$$

<u>Steps to solve the problems of multi-charges exist at one plane</u>

1. Detriment the charge that calculating the resultant force act on it (from text of problem) نحدد الشحنة المراد حساب محصلة القوي الكهربية عندها من نص المسألة

2. Plot the origin point of $(x - y)$-axis at its position. نرسم محوري س و ص عند هذه الشحنة

3. Apply the charge law (like charge repel and unlike charge attract) between other charge and the desired charge. (i.e find the direction of force component (F_x) and (F_y) .

نطبق قانون التنافر والتجاذب بين الشحنات الكهربية الأخرى وهذه الشحنة لنحدد المركبات الأفقية والرأسية لمحصلة القوة الكهربية.

4. Calculate the horizontal (F_x) and vertical (F_y)component of electric forces values.

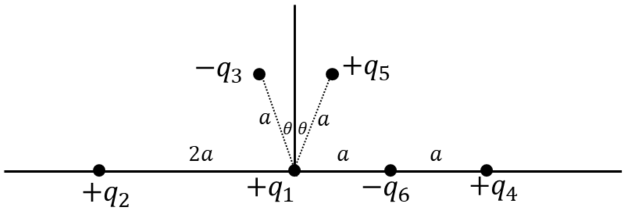
نحسب عدد يا المركبات الأفقية والرأسية لمحصلة القوة الكهربية باستخدام قانون كولوم.

5. Calculate the magnitude of resultant force by using this relation

$$F = \sqrt{F_x^2 + F_y^2}$$

Example (1.10):

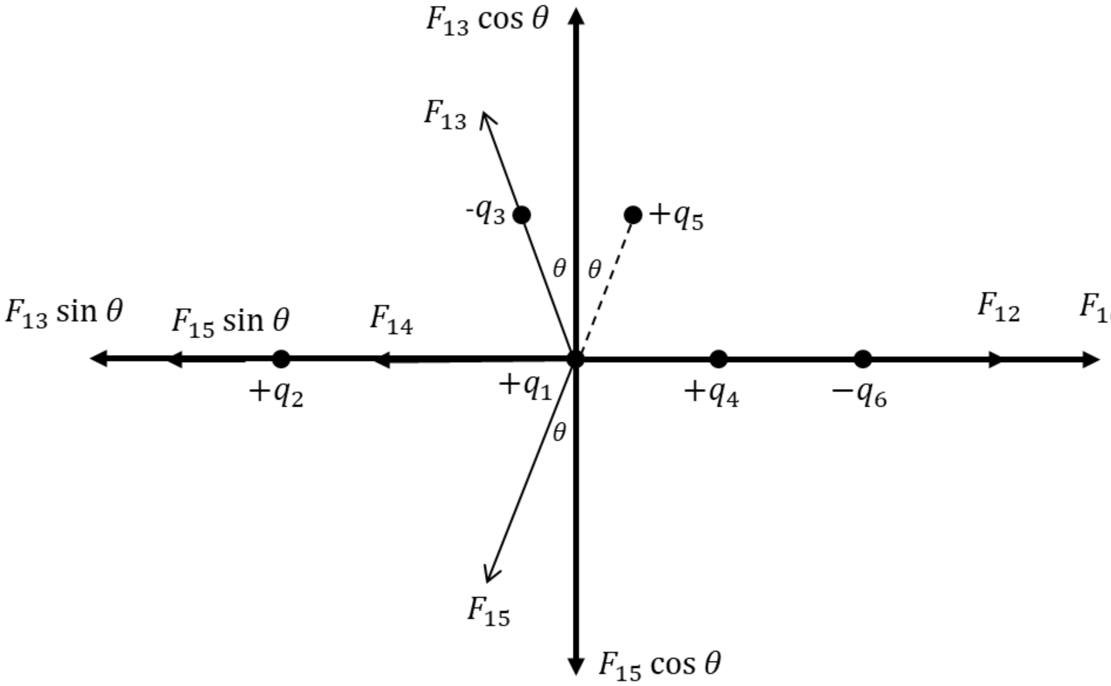
The figure shows an arrangement of six fixed charged particles, where $\boldsymbol{a = 2.0\,cm}$ and $\boldsymbol{\theta = 30}$°. All six particles have the same magnitude of charge, $\boldsymbol{q = 3.0 \times 10^{-6}\,C}$: Their electrical signs are as indicated. What is the net electrostatic force acting on $(\boldsymbol{q_1})$ due to the other charges?



Solution:

$$F_{12} = F_{14} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{(2a)^2}$$

$$F_{13} = F_{15} = F_{16} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_3}{a^2}$$



The above figure is a free body diagram for (q_1). It is cleared that F_{12} and (F_{14}) are equal in magnitude but opposite in direction: thus, those forces cancel.

Inspection of above figure. reveals that the y-components of (F_{13}) and (F_{15}) are equal and opposite direction therefore they cancel each other.

$$F_{13}\cos 30^o - F_{15}\cos 30^o = 0$$

And that their x components are identical in magnitude and both points in the direction of decreasing (x). The figure shows also shows that (F_{16}) points in the direction of increasing (x). thus (F_1) must be parallel to the $x-$axis, its magnitudes are the difference between (F_{16}) and twice the x-component of (F_{13}):

$$F_1 = F_{16} - 2F_{13}\sin\theta$$
$$= \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_6}{a^2} - \frac{2}{4\pi\varepsilon_o}\frac{q_1 q_3}{a^2}\sin\theta$$

Setting $(q_1 = q_6)$ and $(\theta = 30°)$, we find

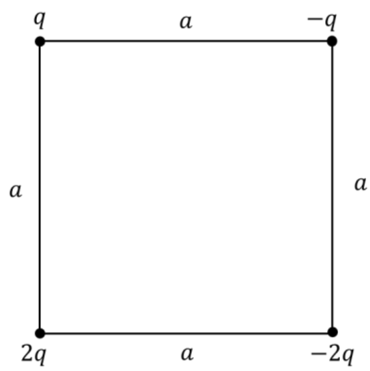
$$F_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_6}{a^2} - \frac{2}{4\pi\varepsilon_o}\frac{q_1 q_2}{a^2}\sin 30^o = 0$$

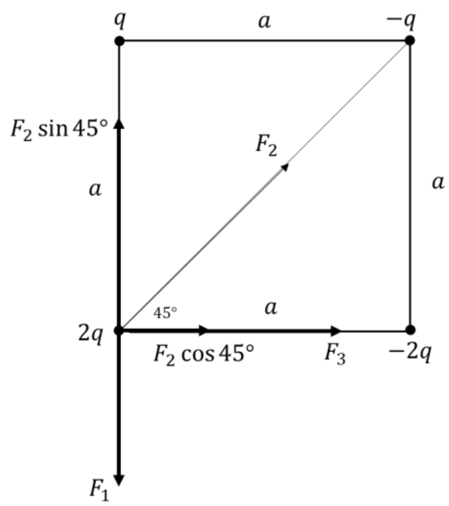
Note that the presence of (q_6) along the line between (q_1) and (q_4) does not in any way alter the electrostatic force exerted by (q_4) on (q_1).

Example (1.11):

What are the horizontal and vertical components of the resultant electrostatic force on the charge in the lower left corner of the square if $\boldsymbol{q = 0.1}$ μ$\boldsymbol{C}$ and $\boldsymbol{a = 5\ cm}$.

Solution:





the repulsion force between charge $(2q)$ and (q) at the corner of square along $y - axis$

$$F_1 = k\frac{(2q)(q)}{a^2}$$

$$F_1 = (9\times10^9 Nm^2/C^2)\left(\frac{2(0.1\times10^{-6}\ C)^2}{(0.05\ m)^2}\right) = 0.072\ N$$

the attraction force between charge $(2q)$ and (q) in the corner on the diagonal of the square.

$$F_2 = k\frac{(2q)(q)}{\left(a\sqrt{2}\right)^2}$$

$$F_2 = (9\times10^9\ Nm^2/C^2)\left(\frac{2(0.1\times10^{-6}\ C)^2}{\left(0.05\sqrt{2}\ m\right)^2}\right) = 0.036\ N$$

the attraction force between charge $(2q)$ and $(-2q)$ in the corner on horizontal axis of the square.

$$F_3 = k\frac{(2q)(2q)}{(a)^2}$$

$$F_3 = (9 \times 10^9 Nm^2/C^2)\left(\frac{4(0.1 \times 10^{-6}\ C)^2}{(0.05\ m)^2}\right) = 0.144\ N$$

The horizontal component of electric force act on the lower left charge. Force component

$$\sum F_x = F_3 + F_2 \cos 45^\circ = (0.144\ N) + (0.036 \cos 45^o\ N) = 0.169\ N$$

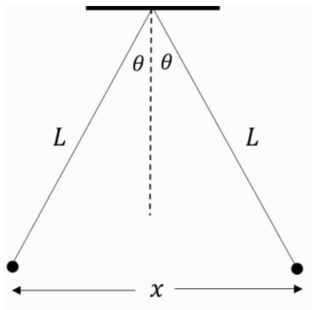
The vertical component of electric force act on the lower left charge.

$$\sum F_y = F_2 \sin 45^\circ - F_1 = (0.036 \sin 45°\ N) - (0.072\ N) = 0.047\ N$$

Example (1.12):

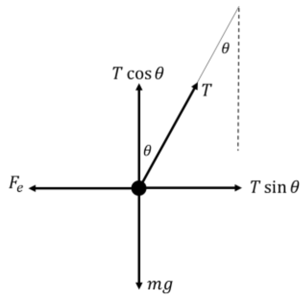
Two tiny conducting balls of identical mass ($\boldsymbol{m}$) and identical charge ($\boldsymbol{q}$) hang from non-conducting threads of length ($\boldsymbol{L}$). Assume that ($\boldsymbol{\theta}$) is so small that ($\boldsymbol{\tan\theta}$) can be replaced by $\boldsymbol{\sin\theta}$; show that, for equilibrium,

$$\boldsymbol{x = (\frac{q^2 L}{2\pi\varepsilon_o mg})^{\frac{1}{3}}}$$



Solution:

The forces effected on each ball (i) tension force (T) in non-conducting thread that make an angle (θ) with vertical plane, (ii) weight of ball and its direction downward and (iii) repulsion electric force between the charged ball. at equilibrium the net forces act on each ball equal zero, therefore the horizontal component of resultant fore is zero



$$\sum F_x = 0 \Rightarrow T\sin\theta = k\frac{q^2}{x^2}$$

Also, the vertical component of resultant force is zero,

$$\sum F_y = 0 \Rightarrow T\cos\theta = mg$$

Divided the both equations, we get

$$\tan\theta = \frac{kq^2}{mgx^2} = \frac{q^2}{4\pi\varepsilon_0 mgx^2}$$

From statement of problem, we get

$$\sin\theta \cong \tan\theta = \frac{x}{2L} = \frac{q^2}{4\pi\varepsilon_0 mgx^2} \quad \Longrightarrow \frac{x^3}{2L} = \frac{q^2}{4\pi\varepsilon_0 mg}$$

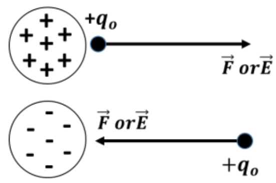
$$x = \left(\frac{q^2 L}{2\pi\varepsilon_o mg}\right)^{\frac{1}{3}}$$

1.5 Electric field (E)

The space around an electrical charge in which its effect can be displayed in this region is known as *electric field region*. Consider an electric charge $(\boldsymbol{Q})$ located in space. If we bring another charge $(\boldsymbol{q_o})$ near the charge $(\boldsymbol{Q})$, it will acquire a force of attraction or repulsion due to presence of charge $(\boldsymbol{q_o})$. The force exerted on charge $(\boldsymbol{q_o})$ is named electric field intensity or strengthen that done on charge $(\boldsymbol{q_o})$.

Electric field intensity

The electric field intensity or strength of electric field at a point may be defined as "***It is electric force act on unit positive test charge*** $+q_o$ ***placed at that point***". If (F) is the force acting on test charge $(+q_o)$ at any point a, then electric field intensity may be given by.



$$\vec{E} = \frac{\vec{F}}{q_o}$$

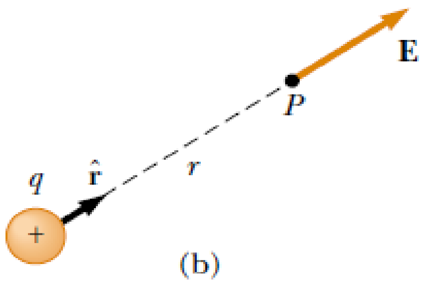
(E) is a vector quantity which has a magnitude and direction, and the S.I unit of (E) is Newton per Coulomb (N/C). Then, the electric force done by an electric field (E) on a charge (q_o) is

$$\vec{F} = q_o\vec{E}$$

<u>Electric field due to a point charge</u>

Let (q) is a point charge and (q_o) is a test charge is placed at point (P) at a distance (r) from (q). According to Coulomb's law, the force acting on (q_o) due to (q) is

$$F = k\frac{qq_o}{r^2}$$



The electric field at a point (P) is, by definition, the force per unit test charge.

$$E = \frac{F}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

<u>Lines of electric field:</u>

Electric field lines are an imaginary straight or curved path along which a unit positive charge tends to move in an electric field. Hence, the electric fields due to simple arrangements of point charges are shown in figure 1.3.

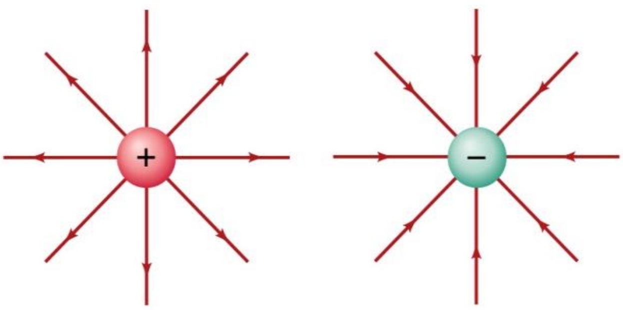


Figure 1.3: The electric field lines for a positive point charge, the lines are directed radially outward and are directed radially inward for a negative point charge

Properties of electric field lines:

1. Electric Lines of forces are imaginary.
2. Electric field lines get out from $(+Q)$ but get into through $(-Q)$
3. Electric Lines represents the strength of the field.
4. he tangent to a line of force at any point gives the direction of the electric field (E) at that point.
5. These lines cannot intersect and crowded near to charges but divergent away of them.

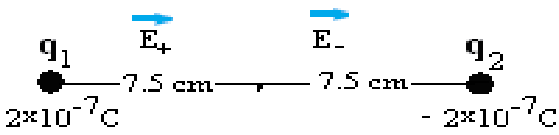
Example (1.13):

Two equal and opposite charges of magnitude $2\,x\,10^{-7}C$ are held 15 cm apart. (a) What are the magnitude and direction of E at the point **midway** between the charges? What are the magnitude and direction of the force that act on an electron placed there?

Solution:

$$\because E_+ = \frac{1}{4\pi\varepsilon_o}\frac{q_2}{r^2}$$

$$E_+ = 9\times10^9 \times \frac{(2\times10^{-7})}{(7.5\times10^{-2})^2} = 0.32\times10^6\,N/C$$



Since $E_+ = E_-$ $\Rightarrow$ The resultant field at the midpoint E equal

$$E = E_+ + E_-$$

$$\therefore E = 2\times E_+ = 2\times0.32\times10^6\,N/C = 0.64\times10^6\,N/C$$

The force act on an electron placed at the midpoint

$$\therefore F = eE = 2\times E_+ = (1.6\times10^{-19}C)\times(0.64\times10^6\,N/C) = 1.024\times10^{-13}N$$

The direction of the force in the opposite direction of the field

Electric dipole

- It is a molecule (figure 1.4) has two charges have same value but one of them is positive $(+q)$ and other is negative $(-q)$ as well as the distance between them is $(d = 2a)$ e.g. salt $(NaCl)$ molecule and water vapor molecule.
- The axis of dipole; it is straight line pass through the two charges of dipole.
- Dipole moment (P); it is product of one of its charge and the distance between them

$$\boldsymbol{P = qd}$$

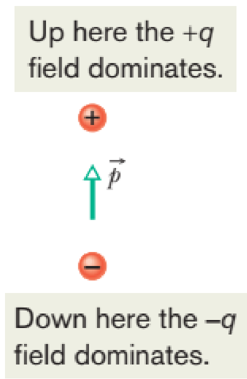


Figure 1.4: An electric dipole

Electric field due to an electric dipole at a point on its axis

As shown in figure 1.5 the total electric field is:

$$E = E_+ - E_- = k\frac{q}{r_+^2} - k\frac{q}{r_-^2}$$

$$= k\frac{q}{(z-d/2)^2} - k\frac{q}{(z+d/2)^2}$$

$$Ex.\ \left(z-\frac{d}{2}\right)^{-2} = z^{-2}(1-\frac{d}{2z})^{-2}$$

$$= k\frac{q}{z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]$$

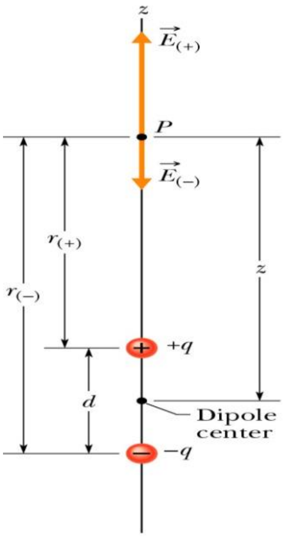


Figure 1.5: Electric field due to an electric dipole at a point on its axis

We can use this approximation and use the binomial equation of negative power $z \gg d \quad \rightarrow \quad 1 \gg d/z$

$$\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2} = \left[\left(1+\frac{d}{z}-\cdots\right) - \left(1-\frac{d}{z}+\cdots\right)\right] \approx \frac{2d}{z}$$

Therefore,

$$E = \left(2k\frac{q}{z^2}\right)\left(\frac{d}{z}\right) = 2k\frac{qd}{z^3} = \boldsymbol{2k\frac{P}{z^3}}$$

Example (1.14):

Calculate the magnitude of the force, due to an electric dipole of dipole moment 3.6 x 10^{-29} C.m, on an electron 25nm away along the dipole axis. Assume that this distance is large relative to the separation of the charges of the dipole.

Solution:

$$\because E_{dipole} = \frac{1}{2\pi\varepsilon_o}\frac{P}{z^3}$$

$$\because F = E_{dipole} \times e = \frac{1}{4\pi\varepsilon_o}\frac{2P}{z^3} \times e$$

$$F = 9 \times 10^9 \times \frac{2\times3.6\times10^{-29}}{(25\times10^{-9})^3} \times 1.6 \times\times 10^{-19} = 6.636 \times 10^{-15} N$$

Electric field due to an electric dipole at a point on the equatorial line.

For the dipole shown in figure 1.6.

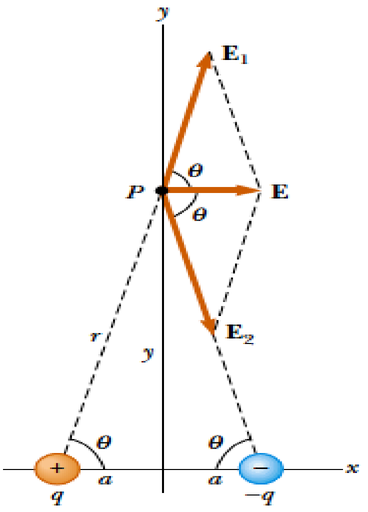


Figure 1.6: Electric field due to an electric dipole at a point on the equatorial line.

- The electric field (E_1) at a point (P) due to the charge $(+q)$ of the dipole,

$$E_1 = \frac{1}{4\pi\varepsilon_o}\frac{q}{(a^2+y^2)}$$

Notice $\left(r = (a^2+y^2)^{1/2}\right)$

- The electric field (E_2) at a point (P) due to the charge $(-q)$ of the dipole

$$E_2 = \frac{1}{4\pi\varepsilon_o}\frac{q}{(a^2+y^2)}$$

- The magnitudes of (E_1) and (E_2) are equal.
- Resolving (E_1) and (E_2) into their horizontal and vertical components as in figure1.6.
- the resultant electric field at the point (P) due to the dipole in vertical axis is zero because $(E_1 = E_2 = E)$

$$\Longrightarrow E = E_1 \sin\theta - E_2 \sin\theta = 0$$

- the resultant electric field at the point (P) due to the dipole in horizontal axis is

$$\Longrightarrow E = E_1 \cos\theta + E_2 \cos\theta$$

$$(E_1 = E_2 = E)$$

$$E = 2E_1 \cos\theta$$

$$E = 2\,\frac{1}{4\pi\varepsilon_o}\frac{q}{(a^2+y^2)}\cos\theta$$

$$But, \cos\theta = \frac{a}{\sqrt{(a^2+y^2)}}$$

$$E = 2\,\frac{1}{4\pi\varepsilon_o}\frac{q}{(a^2+y^2)}\frac{a}{\sqrt{(a^2+y^2)}}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{2qa}{(a^2+y^2)^{\frac{3}{2}}}$$

$$E = \frac{1}{4\pi\varepsilon_o}\frac{P}{(a^2+y^2)^{\frac{3}{2}}}$$

- where (P) is the electric dipole moment $(P = 2qa)$.

- For a dipole, (a^2) is very small when compared to y i.e. $(a \ll y)$ then, you can neglect (a^2) value from above equation

$$E = \frac{1}{4\pi\varepsilon_o}\frac{P}{y^3} \qquad or \qquad E = k\,\frac{P}{y^3}$$

- The direction of (E) is <u>parallel to the axis of the dipole and directed opposite to the direction of dipole moment (P).</u>

<u>Electric charge configuration and distribution</u>

(i) Point charge that represented by (Q)

(ii) Linear charge distribution

When the distribution of charge is uniformly along a line,

$$\lambda = \frac{Q}{L}$$

where (Q) is total charge distributed over a long conductor of length (L) and (λ) is leaner charge density, so the S.I unit of (λ) is C/m. Therefore, the small amount of charge (dq) on this element (dL) is

$$dq = \lambda\, dL$$

(iii) Surface charge distribution

When the distribution of charge is uniformly over a particular area,

$$\sigma = \frac{Q}{A}$$

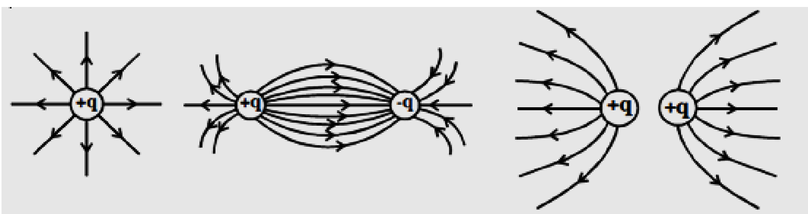
where (Q) is total charge distributed over a particular area (A) and is (σ) surface charge density, so the S.I unit of (σ) is (C/m^2).

<u>(iv) Volume charge distribution</u>

When the distribution of charge is uniformly through a certain volume,

$$\rho = \frac{Q}{V}$$

where (Q) is total charge distributed through a particular volume (V) and (ρ) is volume charge density, so the S.I unit of (ρ) is (C/m^3).



(a) Isolated Charge (b) Unlike Charges (c) Like Charges

The electric field of a uniform charged ring along its central axis

Assume a circular ring of radius (a) carries $(+Q)$ charge that uniformly distributed along its circumference as in figure 1.7.

$$Q = \lambda(2\pi r)$$

The electric field at (P) due to segment of charge $dq = \lambda dl$ is

$$dE = \frac{dq}{4\pi\epsilon_o r^2} = \frac{1}{4\pi\epsilon_o}\frac{\lambda \times dl}{r^2}$$

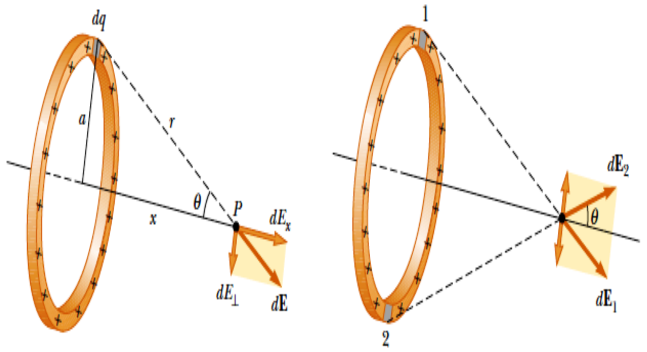


Figure 1.7: The electric field of a uniform charged ring along its central axis.

The electric field component in $x - axis$ is

$$E_x = dE\cos\theta$$

The electric field component in $y\ - axis$ is

$$E_y = dE\ sin\theta$$

Due to symmetry of circular ring, the components of y-axis are Zero.

(The perpendicular components of the field created by any charge element are canceled by the perpendicular component created by an element on the opposite side of the ring as in figure).

$$E_y = dE\ sin\theta - dE\ sin\theta = 0$$

Then, the resultant field at (P) must lie along the $x - axis$.

Notice that, from figure, $r\ =\ (x^2 + a^2)^{\frac{1}{2}}$ and $cos\ \theta\ =\ x/r$, we find that

$$dE_x = dE\ \ cos\,\theta = k\frac{dq}{r^2}.\left(\frac{x}{r}\right) = k\frac{x\,dq}{r^3}$$

All segments of the ring make the same contribution to the field at (P) because they are all equidistant from this point. Thus, we can integrate to obtain the total field at (P):

$$E = \int dE_x = \int k\frac{x\,dq}{(x^2 + a^2)^{\frac{3}{2}}} = k\frac{x}{(x^2 + a^2)^{\frac{3}{2}}}\int dq$$

$$= \boldsymbol{k}\frac{\boldsymbol{Qx}}{(\boldsymbol{x}^2 + \boldsymbol{a}^2)^{\frac{3}{2}}}$$

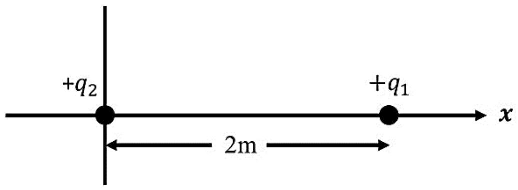
PROBLEMS

1. The sum of two-point charges is $6\ \mu C$. They attract each other with a force of $0.9\ N$, when kept $40\ cm$ apart in vacuum. Calculate the charges.

2. Two small, charged spheres repel each other with a force of $2 \times 10^{-3}\ N$. The charge on one sphere is twice that on the other. When one of the charges is moved $10\ cm$ away from the other, the force is $5 \times 10^{-4}\ N$. Calculate the charges and the initial distance between them.

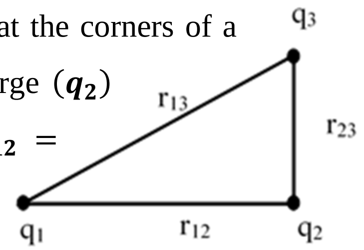
3. Two identical metal spheres are placed $0.2\ m$ apart. A charge (q_1) of $9\ \mu C$ is placed on one sphere while a charge (q_2) of $-3\ \mu C$ is placed upon the other. (a) What is the force on each of the spheres? (b) If the two spheres are brought together and touched and then returned to their original positions, what will be the force on each sphere?

4. Two charges lie along the $x - axis$ as in figure. The positive charge $q_1 = 15\ \mu C$ is at $x = 2\ m$, and the positive charge $q_2 = 6\ \mu C$ is at the origin. Where a negative charge (q_3) must be placed on the $x - axis$ so that the resultant electric force on it is $zero$?



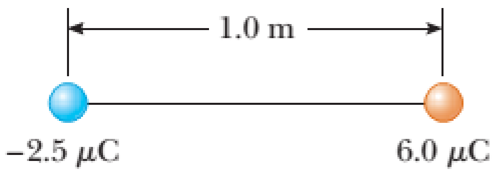
5. Charges (q_1) and (q_2) lie on the $x - axis$ at points $(x = -a)$, and $(x = a)$, respectively, (a) How must (q_1) and (q_2) be related for the net electrostatic force on charge (Q) placed at $(x = a/2)$, to be zero?

6. Three-point charges, $q_1 = 3\ \mu C, q_2 = 5\ \mu C, q_3 = 4\ \mu C$, are fixed at the corners of a right triangle, as shown in figure. (a) Find the resultant force on charge (q_2) and (b) Find the resultant force on charge (q_3). $(r_{12} = 0.4\ m\ and\ r_{23} = 0.3\ m)$.

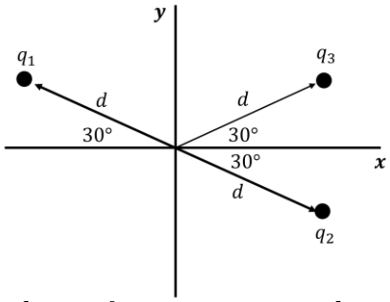


7. Four equal point charges each of $3\ mC$ are placed at the corners of a square of side length $1\ m$. Calculate the electric field at the intersection of the diagonals of the square.

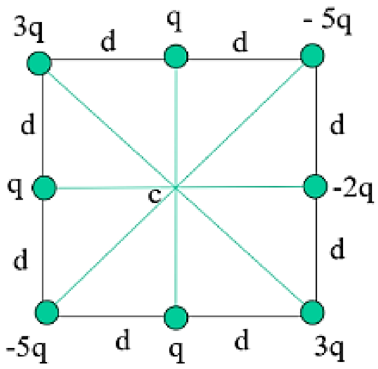
8. In the opposite figure determine the point (other than infinity) at which total electric field is zero.



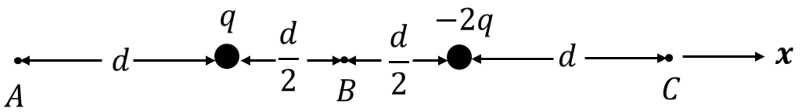
9. Figure shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance (d) from the origin. What net electric field is produced at the origin?



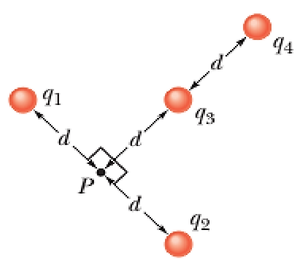
10. In the figure, four charges form the corners of a square and four more charges lie at the midpoints of the sides of the square. The distance between adjacent charges on the perimeter of the square is (d). What are the magnitude and direction of the electric field at the center of the square?



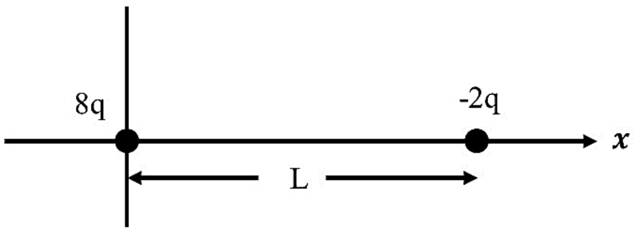
11. In the figure, charges $Q_1 = q$ and $Q_2 = -2q$ are fixed a distance "d" apart (i) find (E) at points (A), (B), and (C) (b) sketch the electric field lines.



11. In figure, the four particles are fixed in place and have charges $q_1 = q_2 = +5e$, $q_3 = +3e$, and $q_4 = -12e$. Distance $d = 5\ mm$. What is the magnitude of the net electric field at point P due to the particles?



13. At which points is the net electric field due to these two charges is zero?

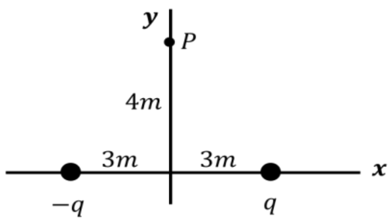


14. An electron is accelerated eastward at $\mathbf{1.8 \times 10^{9}}\ \boldsymbol{m/s^{2}}$ by an electric field. Determine the magnitude and direction of the electric field.

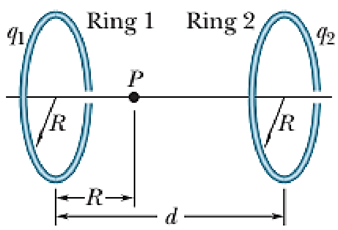
15. An electric dipole of charges $\mathbf{2 \times 10^{-6}}\ \boldsymbol{C}, \mathbf{-2 \times 10^{-6}}\ \boldsymbol{C}$ are separated by a distance $\mathbf{1}\ \boldsymbol{cm}$. Calculate the electric field due to dipole at a point on its. (i) Axial line $\mathbf{1}\ \boldsymbol{m}$ from its center (ii) equatorial line $\mathbf{1}\ \boldsymbol{m}$ from its center.

16. A neutral water molecule $(\boldsymbol{H_2O})$ in its vapor state has an electric dipole moment of magnitude $\mathbf{6.2 \times 10^{-30}}\ \boldsymbol{C/m}$.(i) How far apart are the molecule's centers of positive and negative charge? (ii) If the molecule is placed in an electric field of $\mathbf{1.5 \times 10^{4}}\ \boldsymbol{N/C}$, what maximum force act on each charge?

17. In figure shows charged particles on an $x - axis$: $\boldsymbol{q} = \mathbf{-3.2 \times 10^{-19}}\ \boldsymbol{C\ at\ x} = \mathbf{-3}\ \boldsymbol{m\ and\ q} = \mathbf{3.2 \times 10^{-19}}\ \boldsymbol{C\ at\ x} = \mathbf{+3}\ \boldsymbol{m}$. What are the (a) magnitude and direction of the net electric field produced at point $\boldsymbol{P}$ at $\boldsymbol{y} = \mathbf{4.00}\ \boldsymbol{m}$?



18. The Figure shows two parallel non conducting rings with their central axes along a common line. Ring 1 has uniform charge $(\boldsymbol{Q_1})$ and radius $(\boldsymbol{R})$; ring **2** has uniform charge $(\boldsymbol{Q_2})$ and the same radius $(\boldsymbol{R})$. The rings are separated by distance $\boldsymbol{d} = \mathbf{3}\boldsymbol{R}$.The net electric field at point P on the common line, at distance $(\boldsymbol{R})$ from ring1, is zero. What is the ratio $(\boldsymbol{Q_1}/\ \boldsymbol{Q_2})$.



CHAPTER (2)
GAUSS' LAW

2.1 Introduction

In physics, Gauss's law, also known as Gauss's flux theorem, is a law relating the distribution of electric charge to the resulting electric field. The surface under consideration may be a closed one enclosing a volume such as a spherical surface.

The law was first formulated by Joseph-Louis Lagrange in 1773, followed by Carl Friedrich Gauss in 1813, both in the context of the attraction of ellipsoids. It is one of Maxwell's four equations, which form the basis of classical electrodynamics. Gauss's law can be used to derive Coulomb's law and vice versa. and is used to calculate the electric field results for many shapes of electric charge distribution.

2.2 Electric flux (Φ)

"***Electric flux may be defined as the total number of electric filed lines passing through the normal surface area placed at this point***"

Consider a small area element (dA) of a closed surface area (A) (figure 2.1). (E) is the electric field at the area element (dA). Let (θ) is the angle between area vector $(\overrightarrow{dA})$ and electric field vector $(\vec{E})$, then electric flux $(\emptyset)$ through the area element (dA) is given by

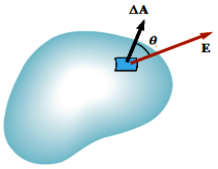


Figure 2.1: A small element of surface area (ΔA)

$$\Phi = \oint \vec{E}.\overrightarrow{dA} = \oint EdA\ Cos\theta$$

The above equation represents the electric flux for a small area element (dA), and the normal component of the electric field.

The electric flux through the whole surface (A), may be calculate by applying the closed integration in the right side. Over the surface (A),

$$\Phi = E.A_{normal}$$

It is the number of electric filed lines pass through a normal unit area placed at certain point (figure 2.2).

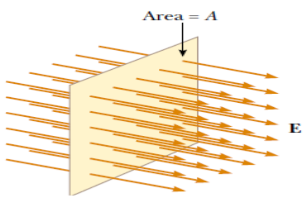


Figure 2.2: Field lines representing a uniform electric field penetrating a plane of area (A) perpendicular to the field.

The number of electric field lines and/or electric flux is directly proportional to the magnitude of the charge.

$$\Phi \propto Q \implies \Phi = \frac{Q}{\varepsilon_o}$$

2.3 Some physical concepts

1. Gaussian surface (G.S.):

- It is an imaginary closed surface, with high symmetry and surrounded the charge from all direction.
- It is suggested according the shape of electric charge distribution.

2. Closed integration($\oint dA$):

- It is one kind of integration and applying on closed surface (i.e G. S)
- Its value equals the area of Gaussian surface.

3. Vector area:

Consider a small area element (dA) (figure 2.3) of a closed surface as shown in figure. The arrow representing the area vector is drawn perpendicular to the area of the element. If $(\hat{n})$ represents unit vector along the outdrawn normal to the area element, then,

$$\overrightarrow{dA} = \hat{n} dA$$

Therefore, it is a vector its value is one and its direction outward and normal on Gaussian surface.

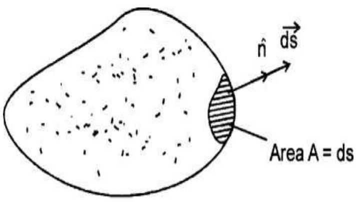


Figure 2.3: A small element of surface area (ΔA) of a closed

4. Scalar product

It is the kind of vectors product which obtains a numerical value positive or negative. The scalar product depends on the angle between two vectors (figure 2.4) and general formula can be written as

$$\vec{A} \odot \vec{B} = |\vec{A}| \times |\vec{B}| \cos\theta$$

Therefore;

$$\theta = 0 \implies \vec{A} \odot \vec{B} = |\vec{A}|.|\vec{B}| \cos 0° = AB$$

$$\theta = \pi \implies \vec{A} \odot \vec{B} = |\vec{A}|.|\vec{B}| \cos \pi^o = -AB$$

$$\theta = 90^o \implies \vec{A} \odot \vec{B} = |\vec{A}|.|\vec{B}| \cos 90^o = 0$$

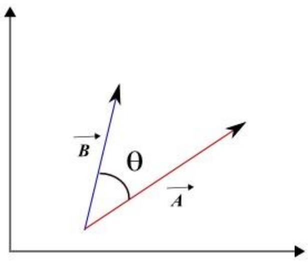


Figure 2.4: A and B vectors

2.4 Formula of Gauss' law

$$\Phi = \frac{Q_{in\,G.S}}{\varepsilon_0} = \oint \vec{E}.\overrightarrow{dA}$$

where, (Φ) is electric flux, (Q_{in}) is sum of electric charge within Gaussian surface, $\varepsilon_o = 8.85 \times 10^{-12}\ C^2/Nm^2$ is electric permittivity of air, $(\vec{E})$is electric field vector, $(\overrightarrow{dA})$ is vector area and $\oint is$ closed integration that apply on Gaussian closed surface.

Example (2.1):

A point charge of $\mathbf{1.8}\ \mu c$ is at the center of a cubical Gaussian surface $\mathbf{55}\ \boldsymbol{cm}$ on edge. What is the net electric flux through the surface?

Solution:

$$\Phi = \frac{q}{\varepsilon_0} = \frac{(1.8 \times 10^{-6}\ C)}{(8.854 \times 10^{-12}\ C^2/Nm^2)} = 2 \times 10^5\ N.m^2/C$$

2.5 Derivation of Coulomb' s law by applying Gauss' law:

To calculate the electric field at a distance (r) from a point charge (q) using Gauss' law consider a spherical Gaussian surface has a radius (r), and the angle (θ) between $(\vec{E})$ and $(\overrightarrow{dA})$ is Zero

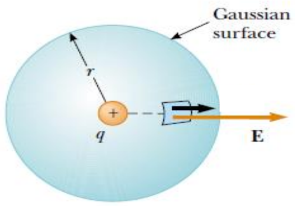


Figure: 2.5: Gauss' law for sphere

$$\frac{q}{\varepsilon_0} = \oint \vec{E}.\overrightarrow{dA}$$

$$\frac{q}{\varepsilon_0} = \oint E\ dA \cos\theta = E \oint dA = EA$$

$$\frac{q}{\varepsilon_o} = E(4\pi r^2)$$

$$\Rightarrow \boldsymbol{E} = \frac{\mathbf{1}}{\mathbf{4}\boldsymbol{\pi\varepsilon_o}}\frac{\boldsymbol{q}}{\boldsymbol{r}^2} \Rightarrow \boldsymbol{E} = \boldsymbol{k}\frac{\boldsymbol{q}}{\boldsymbol{r}^2}$$

⇒ Procedure for deriving or solving the Gauss' law problem;

لحل مسائل الشحنات الموزعة باستخدام قانون جاوس يتم الاتى

1. Detriment the shape of electric charge (تحديد شكل الشحنة)
   - Point charge or charged sphere $(\boldsymbol{Q})$
   - Charged line or charged cylinder $\boldsymbol{Q = \lambda l}$
   - Charged surface (plate, plane, sheet) $\boldsymbol{Q = \sigma A}$
   - Charged surface (sphere, cube) $\boldsymbol{Q = \rho V}$
2. Assume Gaussian surface according to the shape of charge

   إختيار سطح مغلق(جاوسى) لاحتواء الشحنة الموزعة وللحصول على مجال كهربي منتظم
3. Apply Gauss law تطبيق قانون جاوس

$$\Phi = \frac{\boldsymbol{Q_{in\ G.S}}}{\boldsymbol{\varepsilon_o}} = \oint \vec{E}.\overrightarrow{dA}\ \boldsymbol{\Phi}$$

$$\frac{Q_{in\,G.S}}{\varepsilon_o} = \oint E\, dA \cos\theta = E\oint dA$$

$$\frac{Q_{in\,G.S}}{\varepsilon_o} = EA_{G.S.}$$

4. Find the formula of electric field (E).

2.6 Electric field due to charged line or cylinder (Cylindrical symmetry)

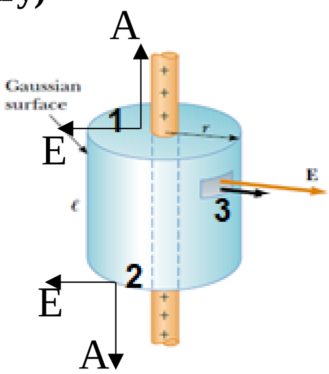


Figure 2.6: Cylindrical symmetry

- The electric field at a distance (r) from an infinite line of charges of constant charge per unit length (λ) $\Rightarrow q = \lambda l$
- Gaussian surface is a cylinder of radius (r) and its length is (L).
- **The cylinder has 3 faces the total flux from both circular bases is zero** because the angle (θ) between $(\vec{E})$ and $(\overrightarrow{dA})$ is $(90°)$

$$\Phi = \oint \vec{E}.\overrightarrow{dA} = \oint E\, dA \cos 90^o = 0$$

- Then, all electric flux gets from the side area of cylinder because the angle (θ) between $(\vec{E})$ and $(\overrightarrow{dA})$ is Zero in all position.

$$\frac{q}{\varepsilon_o} = \oint \vec{E}.\overrightarrow{dA}$$

$$\frac{\lambda l}{\varepsilon_o} = \oint E\, dA \cos\theta = E\oint dA$$

$$\frac{\lambda l}{\varepsilon_0} = E(4\pi r l)$$

$$\Rightarrow \boldsymbol{E = \frac{1}{2\pi\varepsilon_o}\frac{\lambda}{r}} \qquad \boldsymbol{or} \qquad \boldsymbol{E = 2k\frac{\lambda}{r}}$$

Example (2.2):

An infinite line of charge produces a field of $\boldsymbol{4.5\ x\ 10^4\ N/C}$ at a distance of $\boldsymbol{2\ m}$. Calculate the linear charge density.

Solution:

$$E = \frac{\lambda}{2\pi\varepsilon_o r}$$

$$\lambda = 2\pi\varepsilon_o rE = (2\pi)(8.854 \times 10^{-12}\ C^2/Nm^2)(2\ m)(4.5 \times 10^4\ N/C) = 5\mu C/m$$

2.7 Electric field due to charged planer symmetry

(i) For conducting sheet (conducting material)

- This material has a free charge so it concentrated on one face of sheet.
- To find the electric field due to an infinite-conducting sheet that has a uniform surface charge density (σ).

$$q = \sigma A$$

- Gaussian surface is a cylinder one of its base on the charged sheet as in figure, and the angle (θ) between $(\vec{E})$ and $(\overrightarrow{dA})$ is Zero

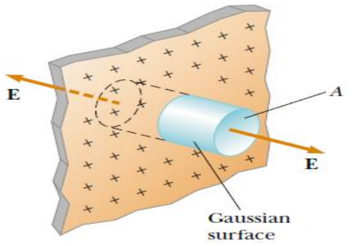


Figure 2.7: Electric field for conducting sheet

$$\frac{q}{\varepsilon_0} = \oint \vec{E}.\overrightarrow{dA}$$

$$\frac{\sigma A}{\varepsilon_0} = \oint E\, dA\, cos\,\theta = E \oint dA$$

$$\frac{\sigma A}{\varepsilon_0} = E(A)$$

$$\Longrightarrow E = \frac{\sigma}{\varepsilon_o}$$

(ii) For non-conducting Sheet (Insulating material)

- This material hasn't free charge so it can be placed on both faces of sheet.
- To find the electric field due to an infinite non-conducting sheet that has a uniform surface charge density (σ) on each face of sheet

$$q = \sigma A$$

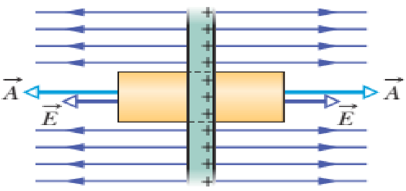


Figure 2.8: Electric field for non conducting sheet

- Gaussian surface is two cylinder their one of base on both sides of the sheet as in figure, and the angle (θ) between $(\overrightarrow{E})$ and $(\overrightarrow{dA})$ is Zero

$$\frac{q}{\varepsilon_0} = \oint \vec{E}.\overrightarrow{dA}$$

$$\frac{\sigma A}{\varepsilon_0} = \oint E\, dA\, cos\,\theta + \oint E\, dA\, cos\,\theta = 2E \oint dA$$

$$\frac{\sigma A}{\varepsilon_0} = 2E(A)$$

$$\Longrightarrow \boldsymbol{E} = \frac{\boldsymbol{\sigma}}{\boldsymbol{2\varepsilon_o}}$$

(iii) Electric field due to two parallel conducting charged sheets

- Consider two plane parallel infinite non-conducting sheets with equal and opposite charge densities $(+\sigma)$ and $(+\sigma)$ as shown in figure.
- The electric field on either side of a plane is $(E = \sigma/2\varepsilon_0)$, directed outward (if the charge is positive) or inward (if the charge is negative).

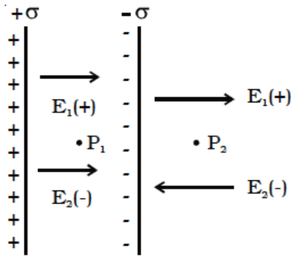


Figure 2.9: Two parallel infinite non-conducting sheets

➢ At a point (P_1) between the two sheets,

$$E = E_1 + E_2 = \frac{\sigma}{2\varepsilon_o} + \frac{\sigma}{2\varepsilon_o} = \frac{\sigma}{\varepsilon_o}\ (twords\ the\ right)$$

➢ At a point (P_2) outside the two sheets,

$$E = E_1 - E_2 = \frac{\sigma}{2\varepsilon_o} - \frac{\sigma}{2\varepsilon_o} = 0$$

Notice; If the two plate are conducting sheets

➢ At a point (P_1) between the two sheets,

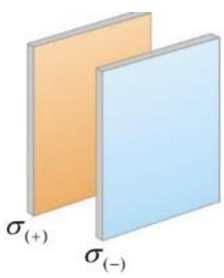
$$E = E_1 - E_2 = \frac{\sigma}{\varepsilon_o} - \frac{\sigma}{\varepsilon_o} = 0$$

➢ At a point (P_2)outside the two sheets,

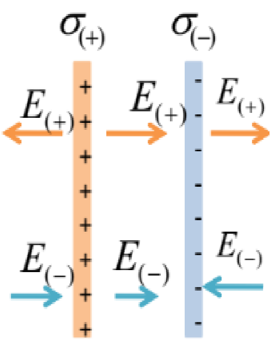
$$E = E_1 + E_2 = \frac{\sigma}{\varepsilon_o} + \frac{\sigma}{\varepsilon_o} = \frac{2\sigma}{\varepsilon_o}\ (twords\ the\ left\ ward)$$

Example (2.3):

The figure shows portions of two large non-conducting sheets, each with a fixed uniform charge on one side. The magnitude of the surface charge densities is $\boldsymbol{\sigma_+ = 6.8\ \mu C/m^2}$ and $\boldsymbol{\sigma_- = -4.3\ \mu C/m^2}$. Find the electric field, (i) between the Sheets, (ii) to the right of the sheets and (iii) to the left of the sheets.



Solution:



(i) Between two sheets (Towards $+ve\ x - axis$)

$$E_{bet} = E_{(-)} + E_{(+)} = \frac{\sigma_{(-)}}{2\varepsilon_o} + \frac{\sigma_{(+)}}{2\varepsilon_o} \frac{(4.3 + 6.8) \times 10^{-6}\ C/m^2}{2(8.854 \times 10^{-12}\ C^2/Nm^2)}$$

$$= 6.2 \times 10^5\ N/C$$

(ii) At the right from two sheet (Towards $+ve\ x - axis$)

$$E_{right} = E_{(+)} - E_{(-)} = \frac{\sigma_{(+)}}{2\varepsilon_o} - \frac{\sigma_{(-)}}{2\varepsilon_o} = \frac{(6.8 - 4.3) \times 10^{-6}\ C/m^2}{2(8.854 \times 10^{-12}\ C^2/Nm^2)}$$

$$= 1.4 \times 10^5\ N/C$$

(iii) At the left from two sheet (Towards $-ve\ x-axis$)

$$E_{right} = E_{(-)} - E_{(+)} = \frac{\sigma_{(-)}}{2\varepsilon_o} - \frac{\sigma_{(+)}}{2\varepsilon_o}\frac{(4.3-6.8)\times 10^{-6}\ C/m^2}{2(8.854\times 10^{-12}\ C^2/Nm^2)} = -1.4\times 10^5\ N/C$$

Example (2.4):

The magnitude of the average electric field normally present in the earth's atmosphere just above the surface of the earth is about $\mathbf{150\ N/C}$, directed downward. What is the total net surface charge of the earth? Assume the earth to be a conductor with a uniform surface charge density

Solution:

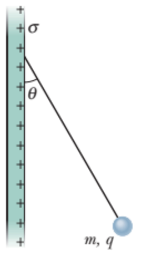
$$E = \sigma/\varepsilon_0$$

$$\Longrightarrow \sigma = \varepsilon_o E = \left(8.854\times 10^{-12}\frac{C^2}{Nm^2}\right)\left(150\frac{N}{C}\right) = 1.33\times 10^{-9}\ C/m^2$$

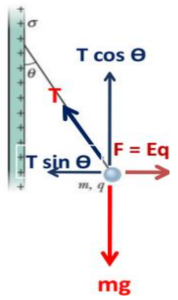
$$q = \sigma A = \sigma(4\pi r^2) = (1.33\times 10^{-9}\ C/m^2)(4\pi)(6.37\times 10^6\ m)^2 = -6.8\times 10^5\ C$$

Example (2.5):

In the figure, a small, non-conducting ball of mass $\boldsymbol{m = 1\,mg}$ and uniformly distributed charge $\boldsymbol{q = 2\times 10^{-8}\ \mathrm{C}}$ hangs from an insulating thread that makes an angle $\boldsymbol{\theta = 30^o}$ with a vertical, uniformly charged non-conducting sheet. Considering the ball' s weight and assuming that the sheet extends far in all directions, calculate the surface charge density σ of the sheet.



Solution:



The sheet is non-conducting, so

$$F = qE = \frac{q\sigma}{2\varepsilon_o}$$

$$\sum F_x = 0 \Rightarrow T\sin\theta = \frac{q\sigma}{2\varepsilon_o}$$

$$\sum F_y = 0 \Rightarrow T\cos\theta = mg$$

$$\tan\theta = \frac{q\sigma}{2mg\varepsilon_o}$$

$$\Rightarrow \sigma = \frac{2mg\varepsilon_o\tan\theta}{q} = \frac{2(10^{-6}\ kg)(9.8\ m/s^2)(8.854\times 10^{-12}\ C^2/Nm^2)\tan 30°}{(2\times 10^{-8}\ C)} = 5\ nC/m^2$$

2.8 The Electric field due to charged sphere (Spherical symmetry)

To find the electric field inside, on the surface and outside an insulating solid sphere of radius (a) has a uniform volume charge density ρ and carries a total charge (Q).

- **First inside the sphere**

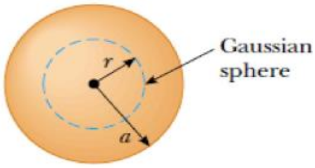


Figure 2.10: Spherical symmetry

(i) The Gaussian surface is a sphere of radius $r < a$

(ii) The charge within the sphere is direct proportional with its volume

$$Q \propto \frac{4}{3}\pi a^3 \quad and \quad q \propto \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{q}{Q} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi a^3} = \frac{r^3}{a^3}$$

$$\Rightarrow q = \frac{r^3}{a^3}Q$$

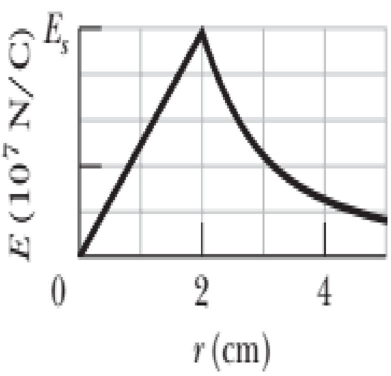


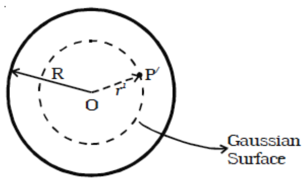
Figure 2.11: Relation between E&r

Using above equation in electric field formula for point charge

$$\Rightarrow E = k\frac{q}{r^2} = k\frac{r^3 Q}{r^2\, a^3} = k\frac{Q}{a^3}r$$

$$\Rightarrow E = k\frac{Q}{a^3}r$$

Notice: At a point inside the *conducting spherical shell* (metal sphere):

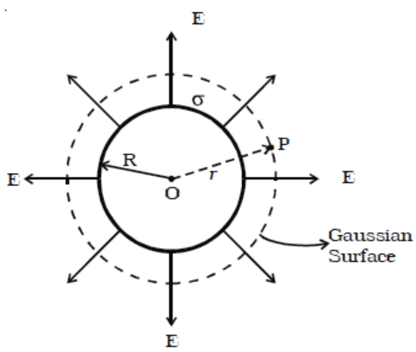


$$E\,A = \frac{q_{in}}{\varepsilon_o}$$

$$q_{in} = 0$$

$$\Rightarrow E = 0$$

(i.e) the field inside the conducting sphere is zero because the charge inside conductor is zero.



- At a point outside the charged sphere of radius $(r > R)$

  For conducting (metal) or nonconducting (plastic) sphere.

$$\Rightarrow E = k\frac{q}{r^2}$$

- At a point on the surface $(r = R)$

  For conducting (metal) or nonconducting (plastic) sphere.

$$\Rightarrow E = k\frac{q}{R^2}$$

2.9 The two-shell theorem of spherical conductor.

- The charges on the surface of metallic spherical shell have attracted or repelled until they uniformly distributed on its surface. Moreover, these charges are accumulated outside the surface of metallic shell and can be imagined them as if were concentrated at its center.
- A charged metallic spherical shell does not generate an electrostatic force on any charged particle that is located inside this metallic shell. So, the electric field (E) inside the metallic shell is equal zero this because the charge is equal zero ($Q = 0$) inside it. Furthermore, if excess charge is placed on a spherical shell that is made of conducting material, this excess charge spreads uniformly outside the (external) surface. For example, if we place excess electrons on a spherical metal shell, those electrons repel one another and tend to move apart, spreading over the available surface until they are uniformly distributed.

  We can summarize the two-shell theorem as:

  1. The electric field inside a uniform shell of charge is zero.
  2. The electric field outside the uniform shell of charge is the same as that of a point charge with the same total charge as the shell.

Example (2.6):

The nucleus of an atom of gold has a radius $\mathbf{6.2\ x10^{-15}\ m}$ and a positive charge $\boldsymbol{q = Ze}$, where the atomic number $\boldsymbol{Z}$ of gold is $\mathbf{79}$. Plot the magnitude of the electric field from the center of the gold nucleus outward to a distance of about twice its radius. (Assume that the nucleus is spherical and that the charge is distributed uniformly throughout its volume)

Solution:

(i) First inside the nucleus

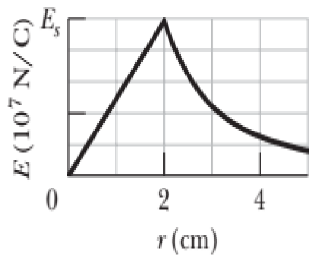
$$q = Ze = (79)(1.6 \times 10^{-19}\ C) = 1.264 \times 10^{-17} C$$

$$E = k\frac{qr}{a^3} = (9 \times 10^9\ Nm^2/C^2)\frac{(1.264 \times 10^{-17}\ C)r}{(6.2 \times 10^{-15}\ m)^3}$$

$$E_{inside} = 4.8 \times 10^{35}\ r\ N/C$$

(ii) Second on the surface of the nucleus

$$E = k\frac{q}{a^2} = (9 \times 10^9\ Nm^2/C^2)\frac{(1.264 \times 10^{-17}\ C)}{(6.2 \times 10^{-15}\ m)^2} = 3 \times 10^{21}\ N/C$$



(iii) Third outside the sphere

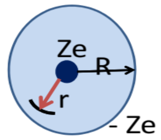
$$E = k\frac{q}{r^2} = \left(9 \times 10^9\frac{Nm^2}{C^2}\right)\frac{(1.264 \times 10^{-17}\ C)}{r^2} = \frac{1.1 \times 10^7}{r^2} N/C$$

Example (2.7):

Consider an atom containing a point positive charge $\boldsymbol{Ze}$ at its center and surrounded by a distribution of negative electricity, $-\boldsymbol{Ze}$ uniformly distributed within a sphere of radius $\boldsymbol{R}$. Prove that the electric field $\boldsymbol{E}$ at a distance $\boldsymbol{r}$ from the center for a point inside the atom is:

$$E = \frac{Ze}{4\pi\varepsilon_o}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

Solution:



There are two electric fields at this point inside the sphere

$$E = E_{out} - E_{in}$$

1. E get out due to the $+ve$ point charge **(Ze)** at the center

$$E_{out} = \frac{kq}{r^2} = \frac{Ze}{4\pi\varepsilon_o r^2}$$

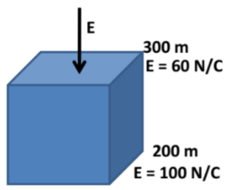
2. E get into due to the $-ve$ charge is uniform distributed in volume of atom

$$E_{in} = \frac{kqr}{R^3} = \frac{Zer}{4\pi\varepsilon_o R^3}$$

$$E = \frac{Ze}{4\pi\varepsilon_o r^2} - \frac{Zer}{4\pi\varepsilon_o R^3}$$

$$\therefore E = \frac{Ze}{4\pi\varepsilon_o}\left(\frac{1}{r^2} - \frac{r}{R^3}\right)$$

Example (2.8):



It is found experimentally that the electric field in a certain region of the earth's atmosphere is directed vertically down. At an altitude of $\boldsymbol{300\,m}$ the field has magnitude $\boldsymbol{60\ N/C}$, and at an altitude of $\boldsymbol{200\ m}$, $\boldsymbol{100\ N/C}$. Find the net amount of charge contained in a cube $\boldsymbol{100\,m}$ on edge, with horizontal faces at altitudes of $\boldsymbol{200}$ and $\boldsymbol{300\ m}$. Neglect the curvature of the earth.

Solution:

Apply the Gauss law on upper and lower surface only of cube because the angel between E and A of the other surfaces is 90, i.e $\varphi = 0$.

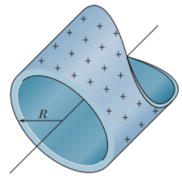
$$\because \Phi_{upper} = \oint E\ dA\ cos\ \theta = E\oint dA\ cos\pi\ = -EA = -(60\ N/C)(100\ m)^2$$

$$\Phi_{lower} = \oint E\ dA\ cos\ \theta = E\oint dA\cos 0\ = EA = (100\ N/C)(100\ m)^2$$

$$\Delta\Phi = \frac{Q_{in}}{\varepsilon_o}$$

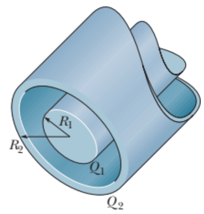
$$\Longrightarrow Q_{in} = \varepsilon_o\left(\Phi_{upper} - \Phi_{lower}\right) = (8.854 \times 10^{-12}\ C^2/Nm^2)(100\ m)^2(100 - 60)N/C\ = -3.54\ \mu C$$

ROBLEMS



1. The figure shows a section of a long, thin-walled metal tube of radius $\boldsymbol{R = 3\ cm}$, with a charge per unit length of $\boldsymbol{\lambda = 2 \times 10^{-8}\ C/m}$. What is the magnitude $(\boldsymbol{E})$ of the electric field at radial distance (a) $\boldsymbol{r = R/2}$ and (b) $\boldsymbol{r = 2\ R}$?

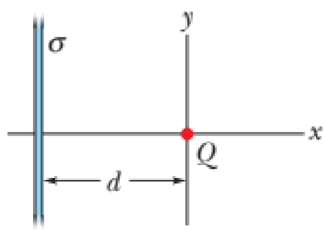
2. An electron is released $\boldsymbol{9\ cm}$ from a very long non-conducting rod with a uniform $\boldsymbol{6\ mC/m}$. What is the magnitude of the electron's initial acceleration?



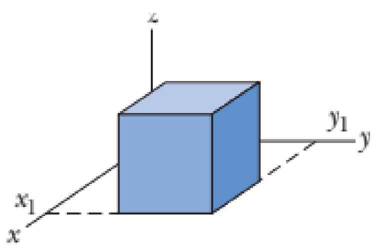
3. In figure is a section of a conducting rod of radius $\boldsymbol{R_1 = 0.3}$ mm and length $\boldsymbol{L = 11\ m}$ inside a thin-walled coaxial conducting cylindrical shell of radius $\boldsymbol{R_2 = 10\ R_1}$ and the same length $(\boldsymbol{L})$. The net charge on the rod is $\boldsymbol{Q_1 = +3.4 \times 10^{-12}\ C}$; that on the shell is $\boldsymbol{Q_2 = -2\ Q_1}$. What are the magnitude and direction at radial distance $\boldsymbol{r = 2\ R_1}$ and $\boldsymbol{r = 5\ R_1}$?

4. Two charged concentric spherical shells have radii $\boldsymbol{10\ cm}$ and $\boldsymbol{15\ cm}$. The charge on the inner shell is $\boldsymbol{10^{-8}\ C}$, and that on the outer shell is $\boldsymbol{2 \times 10^{-8}\ C}$. Find the electric field (i) at $\boldsymbol{r = 12\ cm}$ and (ii) at $\boldsymbol{r = 20\ cm}$.

5. The figure shows a very large non-conducting sheet that has a uniform surface charge density of $\boldsymbol{\sigma = 2\ mC/m^2}$; it also shows a particle of charge $\boldsymbol{Q = 6\ \mu C}$, at distance $(\boldsymbol{d})$ from the sheet. If $\boldsymbol{d = 0.2\ m}$, at what (i) positive and (ii) negative coordinate on the $\boldsymbol{x - axis}$ (other than infinity) is the net electric field $(\boldsymbol{E_{net}})$ of the sheet and charged particle is zero?



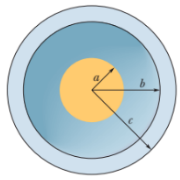
6. The figure shows a closed Gaussian surface in the shape of a cube of edge length $\boldsymbol{2\ m}$, with one corner at $\boldsymbol{x_1 = 5\ m}$, $\boldsymbol{y = 4\ m}$. The cube lies in a region where the electric field vector is given by $\boldsymbol{\vec{E} = -3\hat{\imath} - 4y^2\,\hat{\jmath} + 3\hat{k}}$, with y in meters. What is the net charge contained by the cube?



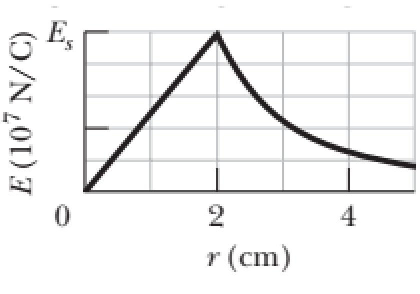
7. Charge is distributed uniformly throughout the volume of an infinitely long solid cylinder of radius $(\boldsymbol{R})$. (a) Show that, at a distance $(\boldsymbol{r < R})$ from the cylinder axis, $\left(\boldsymbol{E = \frac{\rho r}{2\varepsilon_o}}\right)$ where $(\boldsymbol{\rho})$ is the volume charge density. (b) Write an expression for $(\boldsymbol{E})$ when $(\mathbf{r > R})$.

8. The drum of a photocopying machine has a length of $\mathbf{42\ cm}$ and a diameter of $\mathbf{12\ cm}$. The electric field just above the drum's surface is $\mathbf{2.3 \times 10^5\ N/C}$. What is the total charge on the drum?

9. In figure, a plastic solid sphere of radius $\boldsymbol{a = 2\ cm}$ is concentric with a spherical conducting shell of inner radius $\boldsymbol{b = 2a}$ and outer radius $\boldsymbol{c = 2.4a}$. The sphere has a net uniform charge $\boldsymbol{q_1 = 5\ f\ C}$; the shell has a net charge $\boldsymbol{q_2 = -q_1}$.What is the magnitude of the electric field at radial distances (a) $\boldsymbol{r = 0}$, (b) $\boldsymbol{r = a/2}$, (c) $\boldsymbol{r = a}$, (d) $\boldsymbol{r = 1.5a}$, (e) $\boldsymbol{r = 2.3\ a}$, and (f) $\boldsymbol{r = 3.5\ a}$?



10. The figure gives the magnitude of the electric field inside and outside a sphere with a positive charge distributed uniformly throughout its volume. The scale of the vertical axis is set by $\boldsymbol{E_s = 5 \times 10^7\ N/C}$. What is the charge on the sphere?



CHAPTER (3)
ELECTROSTATIC POTENTIAL ENERGY

3.1 Introduction

The concept of potential energy has a great value in the study of electricity. Moreover, most of electrostatic phenomena can be described in terms of an electric potential energy. This idea enables us to define a scalar quantity named as "***electric potential***".

The electric potential at any point in an electric field region is a scalar quantity. Therefore, it helps us to describe the electrostatic phenomena more simply than the vector quantities (e.g. the electric field and electrostatic forces). Furthermore, the concept of electric potential has effective value in the operation of electric circuits and devices that we will study in later chapters.

When an external force (F) does work to rise up a body from a point to another against gravitational force, this work is stored as potential energy for body in the field of gravity. When the external force is removed, the body moves down toward the earth surface and its gravitational potential energy change into a kinetic based on the conservation energy law. The sum of kinetic and potential energies is thus conserved. So, forces of this kind are called ***conservative forces.***

It is known that, Coulomb force $\left(F = k\frac{q_1 q_2}{r^2}\right)$ between two (stationary) charges, like the gravitational force $\left(F = G\frac{m_1 m_2}{r^2}\right)$, is also a conservative force. This is not surprising, since both have inverse-square dependence on distance and differ mainly in the proportionality constants. The masses in the gravitational law are replaced by charges in Coulomb's law. Thus, like the potential energy of a mass in a gravitational field, we can define electrostatic potential energy of a charge in an electrostatic field.

Consider an electrostatic field (E) due to a charge $(+Q)$ placed at the origin. Now, imagine that, we transport a test charge $(+q)$ from a point (R) to a point (P) against the repulsive force that produce between two positive charges.

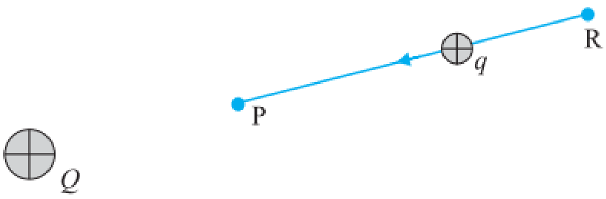


Figure 3.1: Electric field at test charge $(+q)$ due to a charge $(+Q)$

In figure 3.1, to bring the charge $(+q)$ from (i) to (f), we must apply an external force (F_{ext}). to overcome the repulsive electric force $(F_e = qE)$ between them, i.e. $(F_{ext}. = -F_e)$. In this situation, **work done by the external force (F_{ext}) is the negative but the work done by the electric force (F_e) is the positive**. This work is fully stored in the form of electric potential energy for the charge $(+q)$. If the external force is removed after reaching to point (P), the electric force will take the charge away from $(+Q)$ and the stored energy (i.e. electric potential energy) at point (f) is converted into a kinetic energy to the charge $(+q)$, So the sum of the kinetic and potential energies is conserved.

3.2 The electric potential energy

Thus, work done by external forces (F_{ext}). to move a charge $(+q)$ from initial point $(\boldsymbol{i})$ to final point (f) is

$$W_{if} = \int_i^f F_{ext} \cdot\ dr \qquad (3-1)$$

Also, we can write that,

$$W_{if} = -\int_i^f F_e \cdot\ dr = -q \int_i^f E \cdot\ dr \qquad (3-2)$$

This work done is against electrostatic repulsive force (F_e) and gets stored as electric potential energy which is negative value of the work done by external force. (Note, the displacement direction of charge $(+q)$ is opposite to the electric force (F_e) direction and hence work done by electric field is negative).

$$U_{if} = -W_{if} = -q \int_i^f \vec{E} \cdot \overrightarrow{dr} \qquad (3-3)$$

Furthermore, at every point in electric field (E), a particle with charge $(+q)$ has a certain electrostatic potential energy (U), this work done increases its potential energy by an amount equal to potential energy difference between points (i) and (f). Thus, potential energy difference is; $\left(\Delta U_{if} = U_f - U_i\right)$ Therefore, we can define electric potential energy difference between two points as ***"it is work done by an external force to move charge $(+\boldsymbol{q})$ between two points in an electric field"***

Two important comments may be made at this stage.

(i) The right side of above equation depends only on the initial and final positions of the charge. It means that the work done by an electric field to move a charge from one point to another is not dependent on its path of motion.

(ii) We can select an infinity point (∞) at which the electric potential energy is zero $(U_\infty = 0)$. Then, we take the initial point (i) at infinity, so $(U_i = 0)$ and have obtained the value of electric potential energy at point (f).

$$U_{\infty f} = U_f - U_\infty \qquad (3-4)$$

Since the (f) is an arbitrary point, So the equation (3-4) provides us with a definition of potential energy (U) of a charge $(+q)$ at any point. "***It is the work done by the external force (equal and opposite to the electric force) to transport the charge $(+\boldsymbol{q})$ from infinity to that point.***

Example (3.1)

Helium balloon has a charge $\boldsymbol{q = -5.5 \times 10^{-8}\, C}$ rises vertically into the air by a distance $\boldsymbol{520\, m}$ from an initial position (i) to a final position (f). If the magnitude of electric field exists in the atmosphere near to the earth surface $\boldsymbol{150\, N/C}$ and is directed downward. What is the difference in electric potential energy of the balloon between positions (i) and (f)?

Solution:

The work W done by the electric force ($F_e = qE$)on the balloon is

$$W = \int_i^f F_e \cdot \, dr$$

$$\therefore W = q \int_0^D E \cdot \, dr \quad = q \int_0^D Edr \, cos180^o$$

$$W = qE(-1) \int_0^D dr \; = (-1)qED$$

$$W = (-1)(-5.5 \times 10^{-8}\; C)(150 \;\; N/C)(520\, m) = 4.3 \times 10^{-3} J$$

- The work done by the electrostatic force on the balloon is positive.
- Thus, the difference in the electric potential energy of the balloon is

$$\Delta U_{if} = \; U_f - U_i = -W_{if}$$

$$\Delta U_{if} = \; -4.3 \times 10^{-3} J = \; -4.3 \;\; mJ$$

- The minus sign indicates that, the electric potential (U) decreases as the negatively charged balloon rises. Because its motion opposite the direction of electric field (downward directed).

3.3 Electrostatic potential (V)

In the previous section, we define the electric potential energy (U) of a test charge $(+q)$ in terms of the work done on it. This work is clearly proportional to magnitude of $(+q)$, since the electric force at any point is $(F_e = \; qE)$, where (E) is the electric field at that point due to the given charge configuration. It is suitable to divide the work by the amount of charge $(+q)$, so that the resulting quantity is independent of $(+q)$. In other words, work done per unit test charge is characteristic of the electric field associated with the charge configuration. This leads to the idea of electrostatic potential (V) due to a given charge configuration. From equation (3-2), we get: "***Work done by external force to move a unit positive charge between two points***"

$$V_{if} = \frac{U_{if}}{q} = \frac{-W_{if}}{q} = \frac{U_f}{q} - \frac{U_i}{q} = V_f - V_i \qquad (3-5)$$

where, (V_f) and (V_i) are the electrostatic potentials at points (f) and (i), respectively. Note, as before, that it is not the actual value of potential but the potential difference that is physically significant.

If, we take the electric potential at initial point (i) which was placed at infinity is equal zero $(V_\infty = 0)$ in equation (3-5). Then, the work done by an external force to transport a unit of positive charge from infinity to a point is equal the electrostatic potential (V) at this point.

In other words, ***the electrostatic potential (V) at any point in a region with electrostatic field is the work done to transport a unit positive charge from infinity to this point.***

$$V_f = \frac{U_f}{q} \qquad (3-6)$$

Moreover, the **S.I.** unit of both electric potential and potential difference is joules per coulomb, which is defined as a volt (V):

$$1\text{V} \equiv 1\ J/C$$

Electron volt:

The energy usually measures in the S.I. unit of joules but in atomic, solid state and nuclear physics often needs a very small unit of energy. A more suitable unit for that is the **electron-volt** (eV), which is defined as; ***"It is energy required to transport an elementary charge*** $(\boldsymbol{e = 1.6 \times 10^{-19} C})$ **(*e.g. an electron or proton*) *through an electric potential difference of one volt".*** Furthermore, the electron volt (eV) is related to the joule as follows:

$$1\ eV = 1.6 \times 10^{-19}\ J \qquad (3-7)$$

3.4 Calculating the potential (V) from the field (E)

Consider an arbitrary *nonuniform electric field* (E), represented by the field lines in figure, and a positive test charge q_o that moves along the path shown from point (i) to point (f). At any point on the path, an electric force $\left(\vec{F_e} = q_o\vec{E}\right)$ acts on the charge as it moves through a differential displacement $(\overrightarrow{ds})$. We know that the differential work (dW) done on a particle by a force during a displacement is given by the dot product of the force and the displacement:

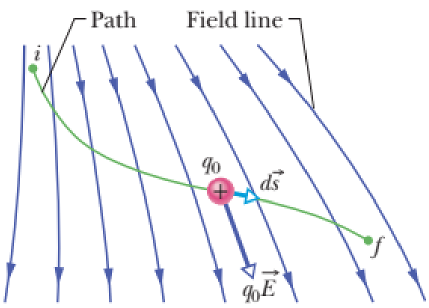


Figure 3.2: Moveing of test charge (q_0) in an **e**lectric field.

$$dW = \vec{F_e} \cdot \overrightarrow{ds} = q_o\vec{E} \cdot \overrightarrow{ds}$$

To find the total work (W) done on the particle by the field as the particle moves from point (i) to point (f), we integration the differential works done on the charge as it moves through all the displacements $(\overrightarrow{ds})$ along the path:

$$W_{if} = q_o \int_i^f \vec{E} \cdot \overrightarrow{ds}$$

$$\frac{W_{if}}{q_o} = \int_i^f \vec{E} \cdot \overrightarrow{ds}$$

but,

$$V_{if} = \frac{U_{if}}{q_o} = \frac{-W_{if}}{q_o}$$

therefore,

$$V_{if} = -\frac{W_{if}}{q_o} = -\int_i^f \vec{E} \cdot \overrightarrow{ds}$$

or,

$$V_f - V_i = -\int_i^f \vec{E} \cdot \overrightarrow{ds} = -\int_i^f E\, ds \cos\theta \qquad (3-8)$$

Thus, the potential difference $\left(V_f - V_i\right)$ between any two points (i) and (f) in an electric field (E) is equal to the negative of the *line integral* (meaning the integral along a particular path) of from (i) to (f). However, because the electric force is conservative, all paths (whether easy or difficult to use) produce the same result. Equation (3-8) helps us to calculate $(\Delta V_{if} = V_f - V_i)$ between any two points in the field. If we set potential $(V_i = 0)$ at infinity (∞) point, then equation (3-8) becomes

$$V_f - V_i = -\int_{\mathrm{i}}^f \vec{E} \cdot \overrightarrow{ds} \qquad (3-9)$$

In above equation we have dropped the subscript (f) on (V_f) term to obtain the potential (V) at any point (f) in the electric field relative to the zero potential at point (i) was existed at infinity.

• Relation between uniform electric field and potential difference.

Let’s apply equation (3-8) for a uniform field as shown in figure 3.2. We start at point (i) on an equipotential line with potential (V_i) and move to point (f) on an equipotential line with a lower potential (V_f).The separation between the two equipotential lines is (D). Let’s also move along a path that is parallel to the electric field (and thus perpendicular to the equipotential lines). The angle between them is zero $(\theta = 0)$ and the dot product gives us,

$$V_f - V_i = -\int_i^f \vec{E} \cdot \overrightarrow{ds} = -\int_i^f E\, ds \cos 0$$

$$\Delta V = V_f - V_i = -E \int_i^f ds$$

The integral is simply an instruction for us to add all the displacement elements ds from (i) to (f), but we already know that the sum is length (D). Thus, we can write the change in potential (ΔV) in this uniform field as

$$\Delta V = -ED \qquad (3-10)$$

This is the change in voltage $(\boldsymbol{\Delta V})$ between two equipotential lines in a uniform field of magnitude (E), separated by distance (D). If we move in the direction of the field by distance (D), the potential decreases. In the opposite direction, it increases. Equation (3-10) shows that potential difference also has units of electric field times distance. From this, it follows that the SI unit of electric field (N/C) can also be expressed in volts per meter (V/m).

Equipotential surface

Adjacent points on a certain surface that have the same electric potential is named an ***equipotential surface,*** which can be either an imaginary or a real surface. There is no work (W) is done on a charged particle by an electric field when the particle moves between two points (i) and (f) on the same equipotential surface.

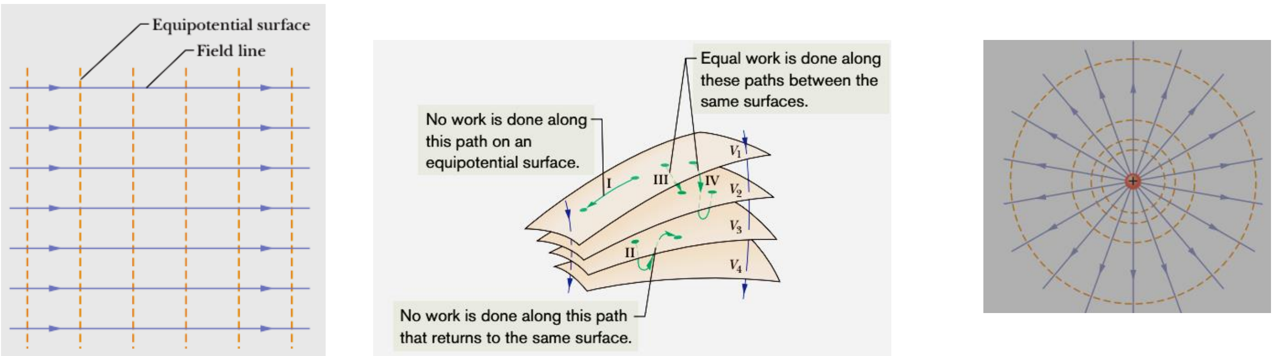


Figure 3.3: Equipotential Surfaces.

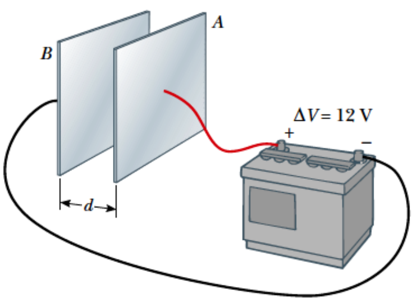
$$W = -\Delta U = -q\Delta V = -q(V_f - V_i)$$

The (W) must be zero if $(V_f = V_i)$. This because the work is independent on the path of charge motion. Thus, $(W = 0)$ for *any* path connecting between points (i) and (f) on a given equipotential surface regardless the path that lies entirely on the surface. The above figure shows a family of equipotential surfaces associated with the electric field due to some distribution of charges. The work done by the electric field on a charged particle when the particle moves from one end to the other of paths $\boldsymbol{I}$ and $\boldsymbol{II}$ is zero because each of these paths begins and ends on the same equipotential surface and thus there is no net change in potential. The work done as the charged particle moves from one end to the other of paths $\boldsymbol{III}$ and $\boldsymbol{IV}$ is not zero but has the same value for both these paths because the initial and final potentials are identical for the two paths; that is, paths $\boldsymbol{III}$ and $\boldsymbol{IV}$ connect the same pair of equipotential surfaces.

Furthermore, from symmetry, the equipotential surfaces produced by a charged particle (q) or a spherically symmetrical charge distribution are a family of concentric spheres. (i.e. points of each surface have same radius, so they have same potential). Moreover, for a uniform electric field (E) is constant at all point), the surfaces are a family of planes perpendicular to the field lines. In fact, equipotential surfaces are always perpendicular to electric field lines.

Example (3.2)

A battery produces a potential difference $\boldsymbol{\Delta V = 12\ V}$ between two metal plates. The plates are separated by $\boldsymbol{0.3\ cm}$. a uniform electric field is formed between the plates. (i) Find the electric field between the plates. (ii) what is the surface charge density on each plate.



Solution:

(i) The electric field is directed from the positive plate (A) to the negative plate (B), and the positive plate is at a higher electric potential than the negative plate is.

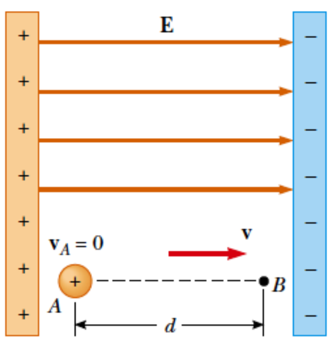
$$\Delta V = ED \Longrightarrow E = \frac{\Delta V}{D} = \frac{(12\ V)}{(0.30 \times 10^{-2}\ m)} = 4 \times 10^{3}\ V/m$$

(ii) The electric field (E) of charged metal plate related with surface charge density (σ) by this relation

$$E = \frac{\sigma}{\varepsilon_o} \Longrightarrow \sigma = \varepsilon_o E = (8.85 \times 10^{-12}\ C^2/Nm^2)(4 \times 10^{3}\ N/C) = 3.54\ \times 10^{9}\ \ C/m^2$$

Example (3.3)

In front figure, a proton is released from rest in uniform field of $\boldsymbol{8 \times 10^{4}\ V/m}$. The proton is displaced by $\boldsymbol{0.5\ m}$ in the direction of (E). (i) Find the change in electric potential (ΔV) between points $(\boldsymbol{A})$ and $(\boldsymbol{B})$. (ii) evaluate the change in potential energy (ΔU) that acquired by the proton in the field. (iii) calculate the speed of the proton. $(\boldsymbol{m_{proton}\ = 1.76 \times 10^{-27}\ kg})$



Solution:

(i) $\Delta V_{AB} = -Ed = -(8.0 \times 10^4\ V/m)(0.50\ m) = -4 \times 10^4\ V$

The negatively sign indicates that, the proton move with same direction of field (E), (i.e. high potential position to low potential position.

(ii) The change in electric potential energy

$$\Delta U = q\Delta V = (+e)\Delta V = (+1.60 \times 10^{-19}\ C)(-4.0 \times 10^4\ V) = -\ 6.40 \times 10^{-15}\ J$$

The negative sign means the potential energy of proton decreases and changed into kinetic energy.

(iii) The charge field system is isolated, so the mechanical energy of the system is conserved:

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}m_p v^2 - 0\right) + e\Delta V = 0$$

$$v = \sqrt{\frac{(2e(-\Delta V))}{m_p}}$$

$$v = \sqrt{\frac{(2 \times (1.60 \times 10^{-19}\ C)(4.0 \times 10^4\ V)}{1.67 \times 10^{-27}\ kg}} = 2.8 \times 10^6\ m/s$$

3.5 Potential due to a charged particle

Now we derive the equation of electric potential (V) due to a point charged particle (q) at the space around it. Consider a point (P) at distance (R) from a fixed particle of positive charge $(+q)$ (figure 3.4). if you imagine that, we move a positive test charge (q_o) from point (P) to infinity (∞) in the direction of electric field of positive charge $(+q)$. let us select a line that extends radially from the fixed particle through (P) to infinity (∞). To use equation (3-9), we must evaluate the dot product of

$$\vec{E} \cdot \overrightarrow{ds} = E\ ds\ (\cos\theta)$$

The electric field $(\vec{E})$ is directed radially outward from the positive charged particle. Thus, the differential displacement $(\overrightarrow{ds})$ of the test particle along its path has the same direction of $(\vec{E})$, then $(\vec{E}\ //\ \overrightarrow{ds})$.

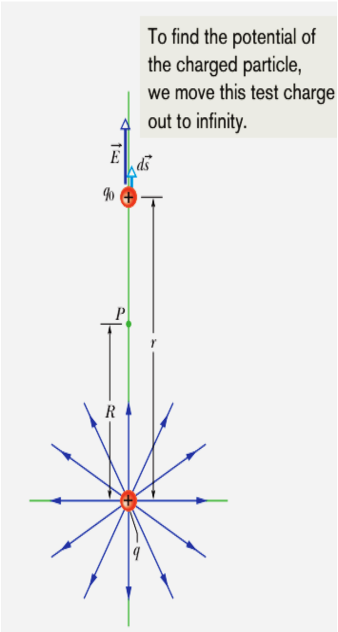


Figure 3.4: Potential due to point charge

So, in the above equation the angle is zero $(\theta = 0°)$ and then, $(cos\ 0° = 1)$. And also, the path is radial, then we can write ds as

dr. Then, substituting the limits of integration as initial point (i) is (P) which located at (R) distance from the origin point as well as the final point (f) at infinity (∞). So, we can write this equation

$$\Delta V = -\int_{i}^{f} \vec{E} \cdot \overrightarrow{ds}$$

$$Notice\ that, \quad ds \equiv dr\,, \;\; \theta = 0^o \;\; and \;\; \cos 0^o = 1$$

$$\Delta V = -\int_{R}^{\infty} \vec{E} \cdot \overrightarrow{dr} = -\int_{R}^{\infty} E\ cos0\ dr = -\int_{R}^{\infty} E\ dr$$

The magnitude of electric filed (E) due to point charge (q) is

$$\because E = k\frac{q}{r^2}$$

we substitute the electric field formula in integral equation we obtain

$$\Delta V = -\int_{R}^{\infty} k\frac{q}{r^2}\ dr = -kq\int_{R}^{\infty} \frac{dr}{r^2} = -kq\left[\frac{-1}{r}\right]_R^\infty$$

$$V_\infty - V_R = kq\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

Notice, the electric potential (V_∞) at infinity is equal zero.

$$0 - V_R = kq\left[\frac{1}{\infty} - \frac{1}{R}\right] = -k\frac{q}{R}$$

$$\Rightarrow V_R = k\frac{q}{R}$$

If replacing (R) to (r)**,** we have obtained the electric potential (V) formula due to a particle of charge $(\pm q)$ at any radial distance (r) from the charged particle.

$$\Rightarrow V = k\frac{\pm q}{r} \qquad (3-11)$$

Moreover, equation (3-11) gives the electric potential either ***outside or on the external surface*** *of* a spherically symmetric charge distribution. We can prove this by using one of the shell theorems that illustrate in the chapter (2) by replacing the actual spherical charge distribution with an equal charge concentrated at its center.

Furthermore, we can obtain the electric potential resulting from two or more-point charges by applying the superposition principle. That is, the total electric potential at point (P) due to several point charges is the sum of the potentials due to the individual charges. For a group of point charges, we can write the total electric potential at (P) in the form

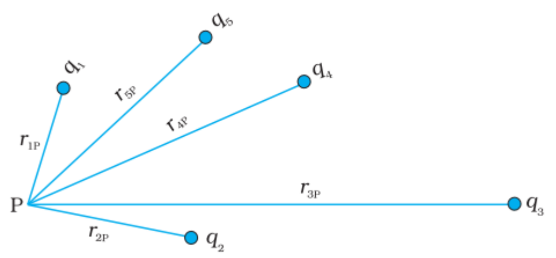


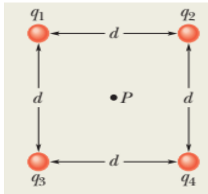
Figure 3.5: Potential due to group of charges.

$$V = \sum_{i}^{n} k\frac{q_i}{r_i} \qquad (3-12)$$

where the potential is again taken to be zero at infinity and (r_i) is the distance from the point (P) to the charge (q_i). Note that the sum in equation (3-12) is an algebraic sum of scalars rather than a vector sum (which we use to calculate the electric field of a group of charges). Thus, it is often much easier to evaluate (V) than to evaluate (E).

Example (3.4)

What is the electric potential at point $(\boldsymbol{P})$, located at the center of the square of charged particles shown in adjacent Figure? The distance $(\boldsymbol{d})$ is $\mathbf{1.3}\ \boldsymbol{m}$, and the charges are $\boldsymbol{q_1 = +12\ nC, q_2 = +31\ nC, q_3 = \ 24\ nC\ and\ q_4 = +17\ nC}$



Solution

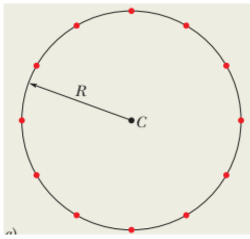
$$r = d/\sqrt{2} = \frac{1.3}{\sqrt{2}} = 0.019\ m$$

$$V = \sum_{i=1}^{4} k\frac{q_i}{r_i} = k\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right)$$

$$V_p = k\left(\frac{q_1 + q_2 + q_3 + q_4}{r}\right) = (9 \times 10^9\ N/m^2C^2)\left(\frac{(12 + 31 - 24 + 17) \times 10^{-9}\ C}{0.019\ m}\right) \approx 350\ V$$

Example (3.4)

In This figure electrons are equally spaced and fixed around a circle of radius $(\boldsymbol{R})$. considered $(\boldsymbol{V\ =\ 0})$ at infinity, what are the electric potential and electric field at the center $(\boldsymbol{C})$ of the circle due to these electrons?



Solution

(i) Electric potential is a scalar, So the orientations of the electrons do not matter. Because the electrons all have the same negative charge e and all have the same distance (R) from (C), then

$$V = k\sum_{i=1}^{12} \frac{q_i}{r_i} = -12k\frac{e}{R}$$

(ii) the electric field $(\vec{E})$ at (C) is a vector quantity and thus the orientation of the electrons is important but due to of the symmetry of the arrangement in figure, the electric field vector at (C) due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, $(\vec{E} = 0)$ at (C).

3.6 Potential due to an electric dipole

The electric potential (V) of electric dipole is created due to its two charges. To evaluate the (V) at any arbitrary point (P) placed at the dipole plane as shown in adjacent figure. At point (P), the positively charged particle $(+q)$ creates potential (V_+) [placed at distance $(r_{(+)})$] and the negatively charged particle $(-q)$ [at distance $(r_{(-)})$] generates potential (V_-) . (figure 3.6a) Then the net potential at point (P) is given by

$$V_p = \mathrm{k}\sum_{i=1}^{2}\frac{\mathrm{q_i}}{\mathrm{r_i}} = k\left(\frac{+q}{r_+} + \frac{-q}{r_-}\right) = kq\left(\frac{r_- - r_+}{r_+ r_-}\right)$$

The most of electric dipoles are very diminutive $(\approx 10^{-10}\ m)$, so we are interested only the points that are relatively far from the dipole $(r \ggg d)$. If (d) is the distance between the charges and (r) is the distance from the dipole's midpoint to point (P).

In this case as in adjacent figure, we can approximate the two lines to the point (P) as being parallel lines and their length difference as being the adjacent side of a right triangle with hypotenuse (d).

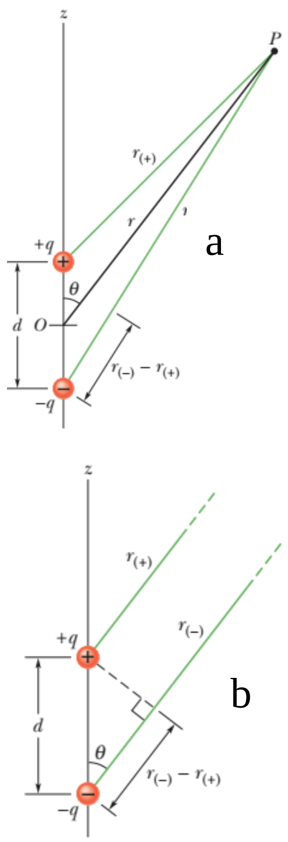


Figure 3.6: Potential due to dipole.

Also, this difference is so small and the product of the lengths is approximately r^2. Thus from figure 3.6b

$$\Rightarrow r_{(-)} - r_{(+)} \approx d\cos\theta \quad and \quad \therefore\ r_{(-)} r_{(+)} \approx r^2$$

If we substitute these quantities into the above equation of potential, we can calculate the approximate value of (V) in polar coordinate to be

$$V_p = kq\left(\frac{r_- - r_+}{r_+ r_-}\right) = k\left(\frac{qd}{r^2}\right)\cos\theta$$

where (θ) is measured from the dipole axis as shown in figure and $(\vec{P} = qd)$ is ***dipole moment*** vector. The $(\vec{P})$ is directed along the dipole axis (i.e. from the negative to the positive charge)

$$V_p = kq\left(\frac{r_- - r_+}{r_+ r_-}\right) = k\left(\frac{P}{r^2}\right)\cos\theta$$

It is worth to note that, (P) is magnitude of dipole moment the (θ) is measured from the direction of $(\overrightarrow{P})$. To calculate the electric potential along dipole axis we can substitute by the value of angle (θ) by $(zero\ or\ 180°)$. To obtain the potential on the central axis of dipole toward its positive charge $(+q)$. Then substitute $(\theta = 0)$ in the potential equation of dipole

$$V_p = k\left(\frac{P}{r^2}\right)\cos 0 \qquad \Rightarrow \qquad V_p = k\left(\frac{P}{r^2}\right)$$

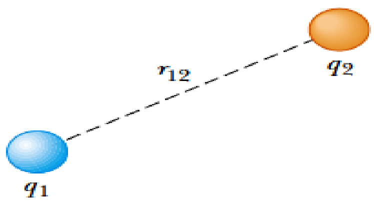
3.7 Electric potential energy of a system of charged particles

In this section we are going to calculate the potential energy (U) of a system of two charged particles and then briefly discuss how to expand the result to a system of more than two particles.

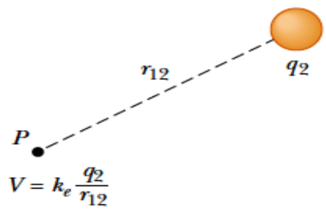
- ***The potential energy of two charged particle system***

Our starting point is to examine the work we must do (as an external agent) to transport the two charged particles that are initially infinitely far apart and that end up near each other and stationary.

(i) If the two particles have the same sign of charge, we must fight against their mutual repulsion of electric force. Our work is then positive and results in a positive potential energy for the final two-particle system.



(ii) If the two particles have opposite signs of charge, our job is easy due to the mutual attraction of the particles. Our work is then negative and results in a negative potential energy for the system.



If (V_2) is the electric potential at a point (P) due to charge (q_2), then the work done by an external force to bring a second charge (q_1) from infinity point (∞) (i.e at ∞ the potential is zero) to point (P) is

$$W_{12} = q_1 V_2.$$

$$V_2 = k\frac{q_2}{r}$$

Notice, an initial potential energy (U_i) is zero for the two-charged system. After bring (q_1) to its final position (r_{12}), the system's potential energy is (U_{12}). This work represents a transfer of energy into the system and the energy appears in the system as potential energy (U_{12}) when the particles are separated by a distance(r_{12}). Therefore, we can express the potential energy of the system as

$$U_{12} = k\frac{q_1 q_2}{r_{12}}$$

- ***The potential energy of multi-charged particle***

If the system consists of more than two charged particles, we can obtain the total potential energy by calculating (U) for every pair of charges and summing the terms algebraically. As an example, the total potential energy of the system of ***three charges*** shown in figure 3.7 is

$$U = U_{12} + U_{13} + U_{23}$$

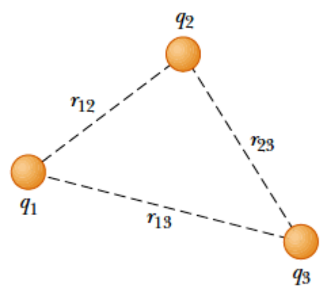


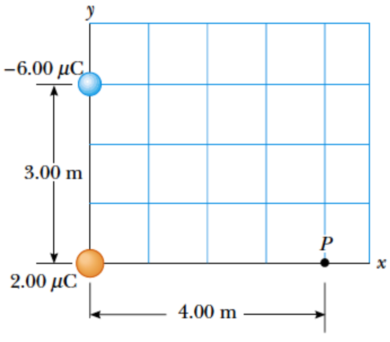
Figure 3.7: Potential energy due to system of charges.

$$U = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

Physically, we can explain this as follows: imagine that (q_1) is fixed at the position shown in the Figure but that (q_2) and (q_3) are at infinity. The work done by an external agent to bring (q_2) from infinity to its position near (q_1) is ($k\frac{q_1 q_2}{r_{12}}$), which is the first term in above equation. The last two terms represent the work required to bring (q_3) from infinity to its position near (q_1) and (q_2).

Example (3.6)

A charge $\boldsymbol{q_1 = 2}$ μ$\boldsymbol{C}$ is located at the origin, and a charge $\boldsymbol{q_2 = -6}$ μ$\boldsymbol{C}$ is located at $(\mathbf{0,3})\ \boldsymbol{m}$, as shown in figure (a). **(i)** Find the total electric potential due to these charges at the point ***(P)***, whose coordinates are $(\mathbf{4,0})\ \boldsymbol{m}$. **(ii)** Find the change in potential energy of the system of two charges plus a charge $\boldsymbol{q_3 = 3.0}$ μ$\boldsymbol{C}$ as the latter charge moves from infinity to point ***(P)*** in figure (b). (iii) calculate the electric potential energy that needed to collect the three charge in their position as shown in figure b?



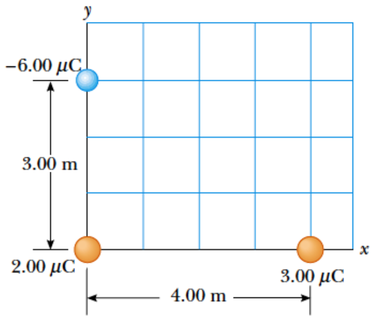
(a)

Solution:

(i) For two charges, the sum in equation (3-12) gives

$$V = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = \left(9 \times 10^9\ \frac{Nm^2}{C^2}\right)\left(\frac{2.0 \times 10^{-6}\ C}{4\ m} - \frac{6.0 \times 10^{-6}\ C}{5\ m}\right) = -6.29 \times 10^3\ V$$



(b)

(ii) When the charge (q_3) is at infinity, let us define ($U_i = 0$) for the system, and when the charge is at (P), ($Uf = q_3 V_P$); therefore,

$$\Delta U = q_3 V_p - 0 = (3 \times 10^{-6}\ C)(-6.29 \times 10^3\ V) = -1.89 \times 10^{-2}\ J$$

Therefore, because the potential energy of the system has decreased, positive work would have to be done by an external agent to remove the charge from point P back to infinity.

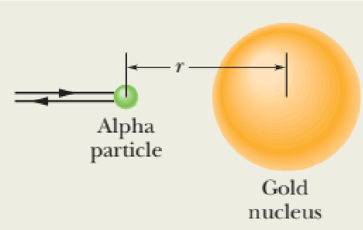
(iii) the total potential energy of the system of ***three charges*** shown in adjacent Figure is

$$U = U_{12} + U_{13} + U_{23}$$

$$U = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$U_t = \left(9 \times 10^9\ \frac{Nm^2}{C^2}\right)\left(\frac{(2)(-6)}{(3\ m)} + \frac{(2)(3)}{(4\ m)} + \frac{(-6)(3)}{(5\ m)}\right) \times 10^{-12}\ C = 0.082\ J$$

Example (3.7)



An alpha particle ($^4_2\boldsymbol{He}$) moves into a stationary gold atom ($^{118}_{79}\boldsymbol{Au}$) and headed directly toward the nucleus. The alpha particle slows until it momentarily stops when its center is at radial distance $\boldsymbol{r = 9.23\ fm}$ from the nuclear center. Then it moves back along its incoming path. What was the kinetic energy $\boldsymbol{K_i}$ of the $^4_2\boldsymbol{He}$ when it was initially far away? (*Assume that the only force acting between the* $^4_2\boldsymbol{He}$ *and the* $^{118}_{79}\boldsymbol{Au}$ *is the Coulomb force and treat each as a single charged particle.)*

Solution

From the principle of conservation of mechanical energy;

$$K_i + U_i = K_f + U_f$$

We know two values: $(Ui = 0)$ and $(K_f = 0)$. We also know that the potential energy (U_f) at the stopping point is given by

$$U_f = k\frac{q_1 q_2}{r} = k\frac{2 \times 79 \times e^2}{r}$$

where $(q_1 = 2e, q_2 = 79e)$ (in which (e) is the elementary charge, $1.6 \times 10^{-19}\ C$), and $(r = 9.23\ fm)$.

$$\therefore U_f = (9 \times 10^9\ \frac{Nm^2}{C^2})\left(\frac{22 \times 79 \times (1.6 \times 10^{-19}\ C)^2}{(9.23 \times 10^{-15}\ m)}\right)$$

$$U_f = 3.94 \times 10^{-12}\ \text{J}$$

$$K_i = U_f = 3.94 \times 10^{-12}\ \text{J} = 24.6\ \text{MeV}$$

3.8 Electric potential due to a uniformly charged sphere

A uniformly charged insulating (non-conducting) sphere of radius (R) and has a total charge $(+Q)$ which distributed uniformly through its volume as shown in figure 3.8.

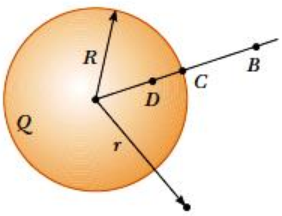


Figure 3.8: Potential due to charged sphere.

(i) The electric field (E) outside the sphere $(r > R)$ such as point (B) is calculated as $E = \mathrm{k}\frac{\mathrm{Q}}{\mathrm{r}^2}$ where the field is directed radially outward. to obtain the electric potential (V) at point (B), selecting an initially point (i) at infinity $(r_i = \infty)$ so its potential is zero $(V_i = 0)$;

$$V_B - V_\infty = kQ\left(\frac{1}{r_B} - \frac{1}{r_i}\right)$$

$$V_B - 0 = kQ\left(\frac{1}{r_B} - 0\right)$$

$$\Longrightarrow V_B = k\frac{Q}{r_B} \qquad (for\ r > R)$$

We can use this formula to obtain the potential (V) at the surface $(r = R)$ of the sphere. Then, the potential at a point such as (C) shown in figure 3.8 is

$$\Longrightarrow V_C = k\frac{Q}{R} \quad (for\ r = R)$$

(i) The electric field (E) inside an insulating uniformly charged sphere is

$$E = k\frac{Q}{R^3}r \qquad (for\ r < R)$$

To evaluate the potential difference $(V_D - V_C)$ at some interior point (D) (for r < R):

$$V_D - V_C = -\int_R^r E\,dr = -\frac{kQ}{R^3}\int_R^r r dr = \frac{kQ}{2R^3}(R^2 - r^2)$$

$$\Longrightarrow V_C = k\frac{Q}{R}$$

Substituting(V_C) into this expression and solving for (D), we obtain

$$\Longrightarrow V_D = \frac{kQ}{2R}\left(3 - \frac{r^2}{R^2}\right) \qquad (for\ r < R)$$

At $(r = R)$, this expression gives a result that agrees with that for the potential at the surface, that is, (V_C). when plots the potential (V) versus (r) for this charge distribution is given in this figure.

- Notice; The potential (V) inside conductor sphere $(r < R)$ is constant and inversely proportional with the distance outside the sphere $(r > R)$, as shown in figure 3.9.

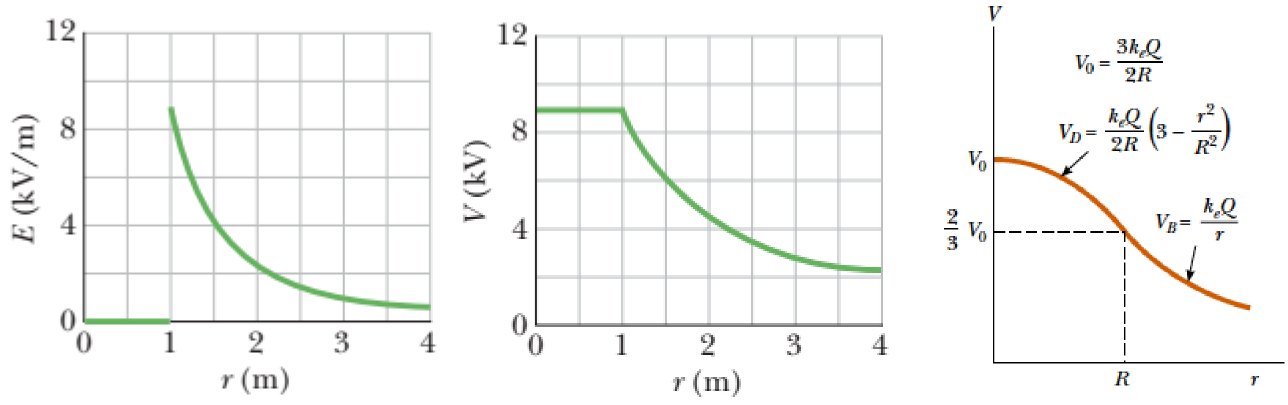


Figure 3.9: Relation between potential and the radius of sphere.

PROBLEMS

1. How much work is done (by a battery, generator, or some other source of potential difference) in moving Avogadro's number of electrons from an initial point where the electric potential is $\mathbf{9\ V}$ to a point where the potential is $\mathbf{-5\ V}$? (The potential in each case is measured relative to a common reference point.)

2. An ion accelerated through a potential difference of $\mathbf{115\ V}$ experiences an increase in kinetic energy of $\mathbf{7.37\ \times 10^{-17}\ J}$. Calculate the charge on the ion.

3. (a) Calculate the speed of a proton that is accelerated from rest through a potential difference of $\mathbf{120\ V}$. (b) Calculate the speed of an electron that is accelerated through the same potential difference.

4. What potential difference is needed to stop an electron having an initial speed of $\mathbf{4.2 \times 10^5\ m/s}$?

5. The difference in potential between the accelerating plates in the electron gun of a TV picture tube is about $\mathbf{25\ 000\ V}$. If the distance between these plates is $\mathbf{1.5\ cm}$, what is the magnitude of the uniform electric field in this region?

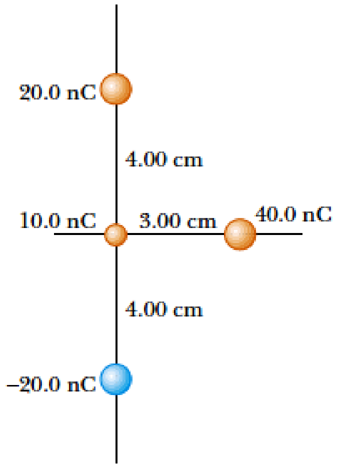
6. (i) Find the potential at a distance of $\mathbf{1\ cm}$ from a proton. (ii) What is the potential difference between two points that are $\mathbf{1\ cm}$ and $\mathbf{2\ cm}$ from a proton? (iii) What if? Repeat parts (a) and (b) for an electron.

7. At a certain distance from a point charge, the magnitude of the electric field is $\mathbf{500\ V/m}$ and the electric potential is $\mathbf{-3\ kV}$. (a) What is the distance to the charge? (b) What is the magnitude of the charge?

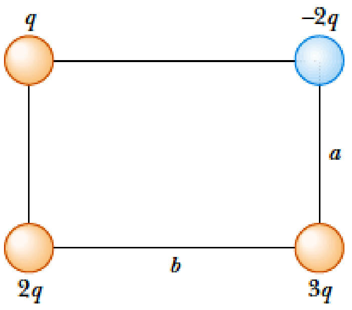
8. A charge **+q** is at the origin. A charge $\mathbf{-2q}$ is at $\mathbf{x\ =\ 2\ m}$ on the x axis. For what finite value(s) of x is (a) the electric field zero? (b) the electric potential zero?

9. Two-point charges, $\mathbf{Q_1\ =\ +5\ nC\ and\ Q_2\ =\ -3\ nC}$, are separated by $\mathbf{35\ cm}$. (a) What is the potential energy of the pair? What is the significance of the algebraic sign of your answer? (b) What is the electric potential at a point midway between the charges?

10. Two particles, with charges of $\mathbf{20\ nC\ and\ -20\ nC}$, are placed at the points with coordinates $(\mathbf{0, 4\ cm})$ $\mathbf{and}$ $(\mathbf{0, -4\ cm})$, as shown in figure. A particle with charge $\mathbf{10\ nC}$ is located at the origin. (i) Find the electric potential energy of the configuration of the three fixed charges. (ii) A fourth particle, with a mass of $\mathbf{2 \times 10^{-13}\ kg}$ and a charge of $\mathbf{40\ nC}$, is released from rest at the point $(\mathbf{3\ cm, 0})$. Find its speed after it has moved freely to a very large distance away.



11. Calculate the energy required to assemble the array of charges shown in figure, where $\boldsymbol{a} = \mathbf{0.2\ m}, \boldsymbol{b} = \mathbf{0.4\ m}$, and $\boldsymbol{q} = \mathbf{6}\ \mu\boldsymbol{C}$.



CHAPTER (4)
CAPACITANCE AND DIELECTRICS

4.1 Introduction

A capacitor is a device that stores electric charge. Capacitors vary in shape and size, but the basic configuration is two conductors carrying equal but opposite charges (figure 4.1). Capacitors have many important applications in electronics. Some examples include storing electric potential energy, delaying voltage changes when coupled with resistors, filtering out unwanted frequency signals, forming resonant circuits and making frequency-dependent and independent voltage dividers when combined with resistors.

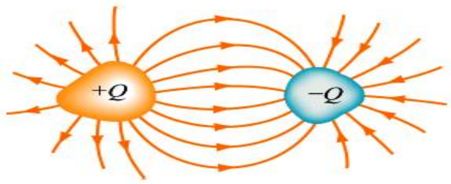


Figure 4.1: Two conductors carrying equal but opposite charges

In the uncharged state, the charge on either one of the conductors in the capacitor is $zero$. During the charging process, a charge (Q) is moved from one conductor to the other one, giving one conductor a charge, and the other one a charge $(-Q)$. A potential difference (V) is created, with the positively charged conductor at a higher potential than the negatively charged conductor. Note that whether charged or uncharged, the net charge on the capacitor as a whole is zero.

The simplest example of a capacitor consists of two conducting plates of area (A), which are parallel to each other, and separated by a distance (d), as shown in figure 4.2.

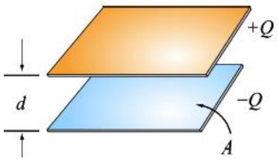


Figure 4.2: Two conducting plates make a capacitor

Experiments show that the amount of charge (Q) stored in a capacitor is linearly proportional to (V), the electric potential difference between the plates. Thus, we may write

$$Q = C\,V \qquad (4-1)$$

where, (C) is a positive proportionality constant called capacitance. Physically, capacitance is a measure of the capacity of storing electric charge for a given potential difference (V). The SI unit of capacitance is the farad (F):

$$1\,F = 1\,farad = 1coulomb/volt = 1\ C/V$$

A typical capacitance that one finds in a laboratory is in the picofarad

$$(1\,pF = 10^{-12}F)$$

to millifarad range,

$$(1\,mF = 10^{-3}\,F = 1000\,\mu F; 1\,\mu F = 10^{-6}\,F)$$

Example (4.1):

A storage capacitor on a random-access memory (RAM) Chip has a capacitance of $\mathbf{55}\ \boldsymbol{fF}$. It is charged to $\mathbf{5.3}\ \boldsymbol{V}$; how many excess electrons are there on its negative plate?

Solution:

$$n = \frac{q}{e} = \frac{CV}{e} = \frac{(55 \times 10^{-15}\ F)(5.3\ V)}{(1.6 \times 10^{-19}\ C)} = 1.8 \times 10^{6}\ electron$$

4.2 Calculation of capacitance

Let's see how capacitance can be computed in systems with simple geometry.

4.2.1 Parallel-plates capacitor

Consider two metallic plates of equal area (A) separated by a distance (d), as shown in figure 4.3. The top plate carries a charge $(+Q)$ while the bottom plate carries a charge $(-Q)$. The charging of the plates can be accomplished by means of a battery, which produces a potential difference. To find the capacitance (C), we first need to know the electric field between the plates. A real capacitor is finite in size. Thus, the electric field lines at the edge of the plates are not straight lines, and the field is not contained entirely between the plates. This is known as edge effects, and the non-uniform fields near the edge are called the fringing fields. In figure, the field lines are drawn incorporating edge effects. However, in what follows, we shall ignore such effects and assume an idealized situation, where field lines between the plates are straight lines, and zero outside.

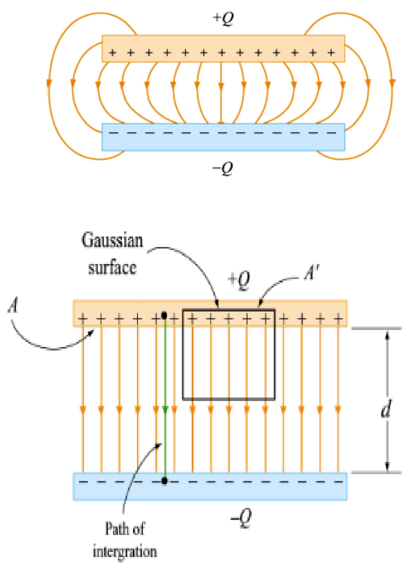


Figure 4.3: Parallel plates capacitor

In the limit where the plates are infinitely large, the system has planar symmetry and we can calculate the electric field everywhere using Gauss's law as:

$$\oint E.dA = \frac{Q}{\varepsilon_o}$$

By choosing a Gaussian "pillbox" with cap area (A) to enclose the charge on the positive plate as shown in figure, the electric field in the region between the plates is

$$\oint E.dA = Q/\varepsilon_o \Rightarrow EA = Q/\varepsilon_o \Rightarrow EA = \sigma A/\varepsilon_o \Rightarrow\ E = \sigma/\varepsilon_o$$

The potential difference between the plates is

$$V = -\int_{d}^{0} E.ds = -\int_{d}^{0} \frac{\sigma}{\varepsilon_o} ds = \int_{0}^{d} \frac{\sigma}{\varepsilon_o} ds = \frac{\sigma d}{\varepsilon_o}$$

From the definition of capacitance, we have

$$C = \frac{Q}{V} = \frac{\sigma A}{\sigma d/\varepsilon_o} = \varepsilon_o \frac{A}{d}$$

Note that (C) depends only on the geometric factors (A) and (d). The capacitance (C) increases linearly with the area (A) since for a given potential difference (V), a bigger plate can hold more charge. On the other hand, (C) is inversely proportional to (d), the distance of separation because the smaller the value of (d), the smaller the potential difference (V) for a fixed (Q).

Example (4.2):

A parallel-plate capacitor with air between the plates has an area $\boldsymbol{A = 2.00 \times 10^{-4}\ m^2}$ and a plate separation $\boldsymbol{d = 1.00\ mm}$. Find its capacitance.

Solution:

$$C = \varepsilon_o \frac{A}{d} = (8.85 \times 10^{-12}\ C^2/N.m^2)\frac{(2.00 \times 10^{-4}\ m^2)}{(1.00 \times 10^{-3}\ m)} = 1.77 \times 10^{-12}\ F = 1.77\ pF$$

Example (4.3):

The plates of a parallel plate capacitor are separated by a distance $\boldsymbol{d = 1\ mm}$. what must be the plate area if the Capacitance is to be $\boldsymbol{1\ F}$.

Solution:

$$A = \frac{Cd}{\varepsilon_0} = \frac{(1\ F)(0.001\ m)}{(8.854 \times 10^{-12}\ C^2/Nm^2)} = 1.1 \times 10^8\ m^2$$

4.2.2 Cylindrical capacitor

Consider next a solid cylindrical conductor of radius (a) surrounded by a coaxial cylindrical shell of inner radius (b), as shown in figure 4.4. The length of both cylinders is (L) and we take this length to be much larger than $(b - a)$, the separation of the cylinders, so that edge effects can be neglected. The capacitor is charged so that the inner cylinder has charge $(+Q)$ while the outer shell has a charge $(-Q)$.

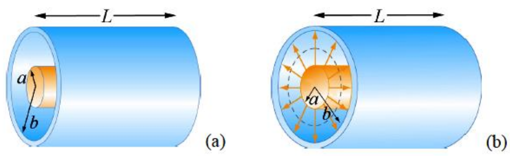


Figure 4.4: Cylindrical capacitor

To calculate the capacitance, we first compute the electric field everywhere. Due to the cylindrical symmetry of the system, we choose our Gaussian surface to be a coaxial cylinder with length $(\ell < L)$ and radius (r) where $(a < r < b)$. Using Gauss's law, we have

$$\oint E.dA = \frac{Q}{\varepsilon_o} \Rightarrow EA = \frac{Q}{\varepsilon_o} \Rightarrow E(2\pi r\ell) = \frac{\lambda\ell}{\varepsilon_o} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_o r}$$

$$V = -\int_b^a E.ds = \int_a^b E_r.dr = \frac{\lambda}{2\pi\varepsilon_o}\int_a^b \frac{dr}{r} = \frac{\lambda}{2\pi\varepsilon_o}[ln(r)]_a^b = \frac{\lambda}{2\pi\varepsilon_o}\,ln\left(\frac{b}{a}\right)$$

The capacitance is then

$$C = \frac{Q}{V} = \frac{\lambda L}{\lambda/2\pi\varepsilon_o\,ln\left(\frac{b}{a}\right)} = 2\pi\varepsilon_o\frac{L}{ln\left(\frac{b}{a}\right)} = \frac{1}{2KI}\frac{L}{ln\left(\frac{b}{a}\right)}$$

Once again, we see that the capacitance (C) depends only on the length (L), and the radii (a) and (b).

Example (4.4):

The inner and outer cylindrical conductors of a long coaxial cable, used to transmit $(\boldsymbol{TV})$ signals, have diameters $\boldsymbol{a = 0.15\,mm}$ and $\boldsymbol{b = 2.1\,mm}$. What is the capacitance per unit length of this cable?

Solution:

$$\frac{C}{L} = \frac{2\pi\varepsilon_0}{ln(b/a)} = \frac{2(3.14)(8.854\times10^{-12}\,C^2/Nm^2)}{ln(2.1\,m/0.15\,m)} = 21x10^{-12}\,F/m$$

4.2.3 Spherical capacitor

let's consider a spherical capacitor which consists of two concentric spherical shells of radii (a) and (b), as shown in figure 4.5. The inner shell has a charge $(+Q)$ uniformly distributed over its surface, and the outer shell an equal but opposite charge $(-Q)$.

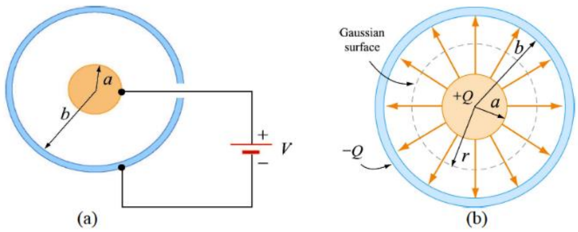


Figure 4.5: Spherical capacitor

The electric field is non-vanishing only in the region $(a < r < b)$. Using Gauss's law, we obtain

$$\oint E.dA = \frac{Q}{\varepsilon_o} \Rightarrow EA = \frac{Q}{\varepsilon_o} \Rightarrow E(4\pi r^2) = \frac{Q}{\varepsilon_o} = \frac{Q}{4\pi\varepsilon_o r^2}$$

Therefore, the potential difference between the two conducting shells is:

$$V = -\int_a^b E.ds = \int_b^a E_r.dr = \frac{Q}{4\pi\varepsilon_o}\int_b^a \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_o}\left[-\frac{1}{r}\right]_b^a = \frac{Q}{4\pi\varepsilon_o}\left(\frac{-1}{a} - \frac{-1}{b}\right) = \frac{Q}{4\pi\varepsilon_o}\left(\frac{b-a}{ab}\right)$$

which yields for the capacitance

$$C = \frac{Q}{V} = \frac{Q}{Q(b-a)/4\pi\varepsilon_o ab} = 4\pi\varepsilon_o\left(\frac{ab}{b-a}\right) = \frac{1}{K}\left(\frac{ab}{b-a}\right)$$

The capacitance (C) depends only on the radii (a) and (b).

An "isolated" conductor (with the second conductor placed at infinity) also has a capacitance. In the limit where $b \longrightarrow \infty$, the above equation becomes

$$\lim_{b\to\infty} C = \lim_{b\to\infty} 4\pi\varepsilon_o \left(\frac{ab}{b-a}\right) = \lim_{b\to\infty} 4\pi\varepsilon_o \frac{a}{\left(1-\frac{a}{b}\right)} = 4\pi\varepsilon_o a$$

Thus, for a single isolated spherical conductor of radius (R), the capacitance is

$$C = 4\pi\varepsilon_o R = \frac{R}{K}$$

The above expression can also be obtained by noting that a conducting sphere of radius (R) with a charge (Q) uniformly distributed over its surface has $(V = Q/\,4\pi\varepsilon_o R)$, where infinity is the reference point at zero potential, $V\,(\infty) = 0$. Using our definition for capacitance,

$$C = \frac{q}{V} = \frac{Q}{Q/4\pi\varepsilon_o R} = 4\pi\varepsilon_o R$$

As expected, the capacitance of an isolated charged sphere only depends on the radius (R).

Example (4.5):

What is the capacitance of the earth, viewed as an Isolated conducting sphere of radius $\mathbf{6370\ km}$?

Solution:

$$C = 4\pi\varepsilon_o R = R/K = \frac{(6370\times 10^3\ m)}{(9\times 10^9\ Nm^2/C^2)} = 7.1\times 10^{-4}\ F$$

4.3 Capacitors with dielectric

A dielectric is a nonconducting material, such as rubber, glass, or waxed paper. When a dielectric is inserted between the plates of a capacitor (figure 4.6), the capacitance increases. If the dielectric completely fills the space between the plates, the capacitance increases by a dimensionless factor (***k***), which is called the dielectric constant of the material. The dielectric constant varies from one material to another. In this section, we analyze this change in capacitance in terms of electrical parameters such as electric charge, electric field, and potential difference.

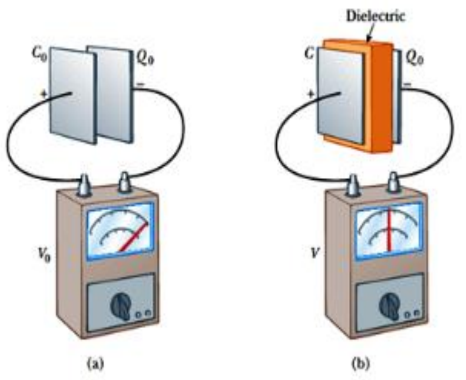


Figure 4.6: Capacitor a) without dielectric. b) with dielectric.

We can perform the following experiment to illustrate the effect of a dielectric in a capacitor. Consider a parallel-plate capacitor that without a dielectric has a charge (Q_o) and a capacitance (C_o). The potential difference across the capacitor is

$$V_o = \frac{Q_o}{C_o}$$

figure 4.6a illustrates this situation. The potential difference is measured by a voltmeter, note that no battery is shown in the figure; also, we must assume that no charge can flow through an ideal voltmeter. Hence, there is no path by which charge can flow and alter the charge on the capacitor. If a dielectric is now inserted between the plates, as in figure 4.6b, the voltmeter indicates that the voltage between the plates decreases to a value (V).The voltages with and without the dielectric are related by the factor ($\boldsymbol{k}$) as follows:

$$\mathrm{V} = \frac{\mathrm{V_o}}{\boldsymbol{k}}$$

Because ($\mathrm{V} < \mathrm{V_o}$), we see that ($\boldsymbol{k} > 1$).

Because the charge (Q_o) on the capacitor does not change, we conclude that thecapacitance must change to the value

$$C = \frac{Q_o}{V} = \frac{Q_o}{V_o/\boldsymbol{k}} = \boldsymbol{k}\frac{Q_o}{V_o} = \boldsymbol{k}\, C_o$$

That is, the capacitance increases by the factor ($\boldsymbol{k}$) when the dielectric completely fills theregion between the plates. For a parallel-plate capacitor, we can express the capacitance when the capacitor is filled with a dielectric as

$$\Longrightarrow \mathrm{C} = \boldsymbol{k}\frac{\varepsilon_o \mathrm{A}}{\mathrm{d}}$$

From above equation, it would appear that we could make the capacitance every large by decreasing (d), the distance between the plates. In practice, the lowest value of (d) is limited by the electric discharge that could occur through the dielectric medium separating the plates. For any given separation (d), the maximum voltage that can be applied to a capacitor without causing a discharge depends on the dielectric strength (maximum electric field) of the dielectric.

Physical capacitors have a specification called by a variety of names, including working voltage, breakdown voltage, and rated voltage. This parameter represents the largest voltage that can be applied to the capacitor without exceeding the dielectric strength of the dielectric material in the capacitor. Consequently, when selecting a capacitor for a given application, you must consider the capacitance of the device along with the expected voltage across the capacitor in the circuit, making sure that the expected voltage will be smaller than the rated voltage of the capacitor.

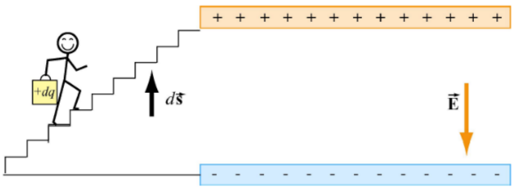
Insulating materials have values of ($\boldsymbol{k}$) greater than unity and dielectric strengthsgreater than that of air. Thus, we see that a dielectric provides the following advantages:

i) Increase in capacitance

ii) Increase in maximum operating voltage

iii) Possible mechanical support between the plates, which allows the plates to be close together without touching, thereby decreasing d and increasing (C).

4.4 Storing energy in a capacitor



As discussed in the introduction, capacitors can be used to stored electrical energy. The amount of energy stored is equal to the work done to charge it. During the charging process, the battery does work to remove charges from one plate and deposit them onto the other.

Let the capacitor be initially uncharged. In each plate of the capacitor, there are many negative and positive charges, but the number of negative charges balances the number of positive charges, so that there is no net charge, and therefore no electric field between the plates. We have a magic bucket and a set of stairs from the bottom plate to the top plate.

We start out at the bottom plate, fill our magic bucket with a charge $(+dq)$, carry the bucket up the stairs and dump the contents of the bucket on the top plate, charging it up positive to charge $(+dq)$. However, in doing so, the bottom plate is now charged to $(-dq)$. Having emptied the bucket of charge, we now descend the stairs, get another bucketful of charge $(+dq)$, go back up the stairs and dump that charge on the top plate. We then repeat this process over and over. In this way we build up charge on the capacitor, and create electric field where there was none initially.

Suppose the amount of charge on the top plate at some instant is $(+q)$, and the potential difference between the two plates is $(V = Q/C)$. To dump another bucket of charge $(+dq)$ on the top plate, the amount of work done to overcome electrical repulsion is $(dW = Vdq)$. If at the end of the charging process, the charge on the top plate is $(+Q)$, then the total amount of work done in this process is

$$W = \int_0^Q V dq = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C}$$

This is equal to the electrical potential energy (U) of the system:

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2$$

Energy density of the electric field

One can think of the energy stored in the capacitor as being stored in the electric field itself. In the case of a parallel-plate capacitor, with $(C = \varepsilon_o A/d)$ and $(V = \ Ed)$, we have

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(\varepsilon_o A/d)(Ed)^2 = \frac{1}{2}\varepsilon_o E^2(Ad)$$

• The SI unit of energy stored is the joule (J).

Because the quantity (Ad) represents the volume between the plates, we can define the electric energy density (u) as

$$u = \frac{stored\ energy}{Volume\ of\ capacitor} = \frac{\frac{1}{2}\varepsilon_o E^2(Ad)}{Ad} = \frac{1}{2}\varepsilon_o E^2$$

The energy density u is proportional to the square of the electric field. The SI unit of energy density stored is the joule (J/m^3).

Example (4.6):

An isolated conducting sphere whose radius $(\boldsymbol{R})$ is $\boldsymbol{6.85\ cm}$ has a charge $\boldsymbol{q\ =\ 1.25\ nC}$. (a) How much potential energy is stored in the electric field of this charged conductor (b) What is the energy density at the surface of the sphere?

Solution:

$$C\ = 4\pi\varepsilon_o R = R/k$$

$$\Rightarrow U = \frac{q^2}{2C} = k\frac{q^2}{2R} = (9\times 10^9\ Nm^2/C^2)\frac{(1.25\times 10^{-9}\ C)^2}{2(0.0685\ m)} = 1.03x10^{-7}J$$

$$E\ = k\frac{q}{R^2} = \frac{(9\times 10^9\ \ Nm^2/C^2)(1.25\times 10^{-9}\ C)}{(0.0685\ m)^2} = 2397.57N/C$$

$$\Rightarrow u = \frac{1}{2}\varepsilon_0 E^2\ = 0.5(8.854\times 10^{-12}\ C^2/Nm^2)(2397.57\ N/C)^2\ = 2.54\times 10^{-5}J/m^3$$

Example (4.7):

A parallel plate air filled capacitor having area $\boldsymbol{40\ cm^2}$ and plate spacing $\boldsymbol{1\ mm}$ is charged to a potential difference of $\boldsymbol{600\ V}$. Find (a) the capacitance (b) the magnitude of the charge of each plate (c) the stored energy (d) the electric field between the plates, and (e) the energy density between the plates?

Solution:

$$C =\ \varepsilon_o\frac{A}{d} = \frac{(8.854\times 10^{-12}\ C^2/Nm^2)(40\times 10^{-4}\ m^2)}{(10^{-3}\ m)} = 35.42\ PF$$

$$q\ = CV\ = (35.42\times 10^{-12}\ F)(600\ V) = 21.25\ nC$$

$$U = \frac{1}{2}QV\ = 0.5(21.25\times 10^{-9}\ F)(600\ V) = 6.4\ mJ$$

$$E\ = V/d = (600\ V)/(10^{-3}\ m) = 6\times 10^5\ N\ /C$$

$$u = \frac{1}{2}\varepsilon_o E^2 = \frac{1}{2}(8.854\times 10^{-12}\ C^2/Nm^2)(6\times 10^5\ N/C)^2 = 1.6\ J/m^3$$

4.5 Capacitors in electric circuits

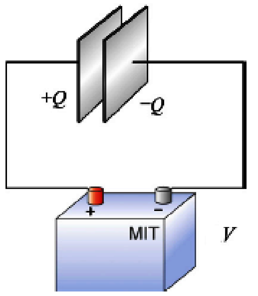


Figure 4.7: Capacitor in a circuit.

A capacitor can be charged by connecting the plates to the terminals of a battery in adjacent Figure, which are maintained at a potential difference V called the terminal voltage. The connection results in sharing the charges between the terminals and the plates. For example, the plate that is connected to the (positive) negative terminal will acquire some (positive) negative charge. The sharing causes a momentary reduction of charges on the terminals, and a decrease in the terminal voltage. Chemical reactions are then triggered to transfer more charge from one terminal to the other to compensate for the loss of charge to the capacitor plates, and maintain the terminal voltage at its initial level. The battery could thus be thought of as a charge pump that brings a charge (Q) from one plate to the other.

4.5.1 Parallel connection

Suppose we have two capacitors (C_1) with charge (Q_1) and (C_2) with charge (Q_2) that are connected in parallel, as shown in figure 4.8.

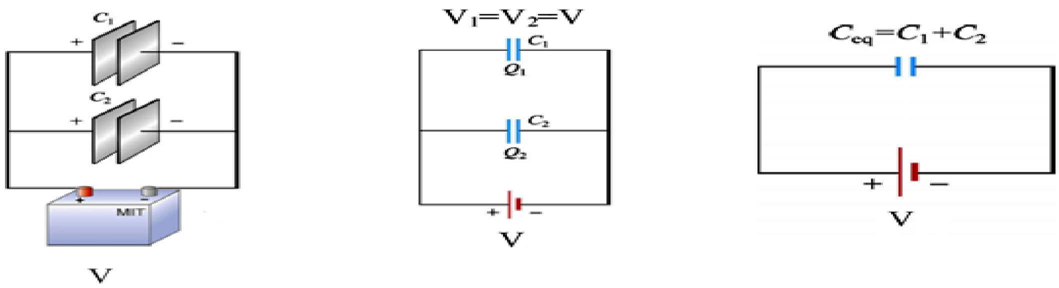


Figure 4.8: Parallel Connection of capacitor in a circuit.

The left plates of both capacitors (C_1) and (C_2) are connected to the positive terminal of the battery and have the same electric potential as the positive terminal. Similarly, both right plates are negatively charged and have the same potential as the negative terminal. Thus, the potential difference is the same across each capacitor. This gives

$$C_1 = \frac{Q_1}{V}. \qquad C_2 = \frac{Q_1}{V}$$

These two capacitors can be replaced by a single equivalent capacitor (C_{eq}) with a total charge (Q) supplied by the battery. However, since (Q) is shared by the two capacitors, we must have

$$Q = Q_1 + Q_2 = C_1V + C_2V = (C_1 + C_2)V$$

The equivalent capacitance is then seen to be given by

$$C_{eq} = \frac{Q}{V} = C_1 + C_2$$

Thus, capacitors that are connected in parallel add. The generalization to any number of capacitors is

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N = \sum_{i=1}^{N} C_i$$

4.5.2 Series connection

Suppose two initially uncharged capacitors (C_1) and (C_2) are connected in series, as shown in figure 4.9.

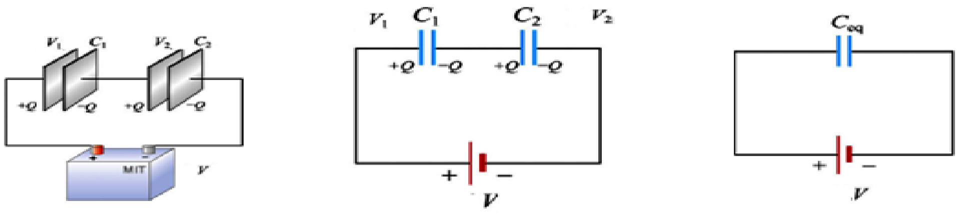


Figure 4.9: Series Connection of capacitor in a circuit.

A potential difference (V) is then applied across both capacitors. The left plate of capacitor (1) is connected to the positive terminal of the battery and becomes positively charged with a charge $(+Q)$, while the right plate of capacitor (2) is connected to the negative terminal and becomes negatively charged with charge $(-Q)$ as electrons flow in. What about the inner plates? They were initially uncharged; now the outside plates each attract an equal and opposite charge. So, the right plate of capacitor (1) will acquire a charge $(-Q)$ and the left plate of capacitor $(+Q)$.

The potential differences across capacitors (C_1) and (C_2) are

$$V_1 = \frac{Q}{C_1} \qquad and \qquad V_2 = \frac{Q}{C_2}$$

respectively. From figure, we see that the total potential difference is simply the sum of the two individual potential differences:

$$V = V_1 + V_2$$

In fact, the total potential difference across any number of capacitors in series connection is equal to the sum of potential differences across the individual capacitors. These two capacitors can be replaced by a single equivalent capacitor $\left(C_{eq} = Q/V\right)$. Using the fact that the potentials add in series,

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

and so, the equivalent capacitance for two capacitors in series becomes

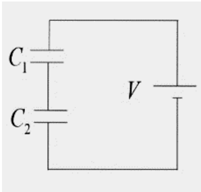
$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}$$

The generalization to any number of capacitors connected in series is

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots+\frac{1}{C_N}=\sum_{i=1}^{N}\frac{1}{C_i}$$

Example (4.8):

Capacitance $\boldsymbol{C_1 = 6\ \mu F}$ is connected in series with a capacitance $\boldsymbol{C_2 = 4\ \mu F}$, and a potential difference of $\boldsymbol{200\ V}$ is applied across the pair. (a) calculate the equivalent capacitance (b) what is the charge on each capacitor (c) what is the potential difference across each capacitor?



Solution:

$$C_{eq}=\frac{C_1C_2}{C_1+C_2}$$

$$C_{eq}=\frac{6\times 4}{6+4}=2.4\ \mu F$$

$$Q_t=C_{eq}V$$

$$Q_t=(2.4\times 10^{-6}\ F)(200\ V)=480\ \mu C$$

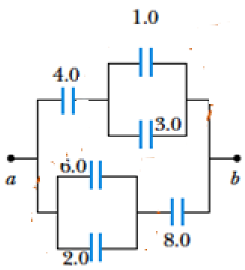
$$Q_t=q_1=q_2=480\ \mu C$$

$$V_1=\frac{q_1}{C_1}=\frac{(480\ \mu C)}{(6\ \mu F\ )}=80V$$

$$V_2=\frac{q_2}{C_2}=V-V_1=120V$$

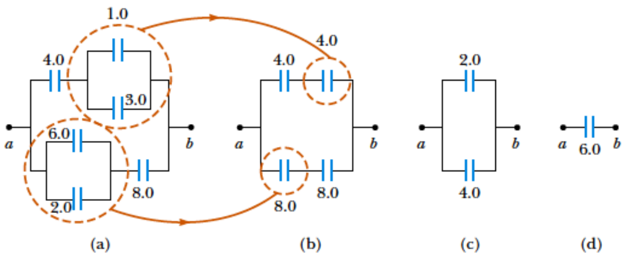
Example (4.9):

Find the equivalent capacitance between $(\boldsymbol{a})$ and $(\boldsymbol{b})$ for the combination of capacitors shown in figure. All capacitances are in microfarads.



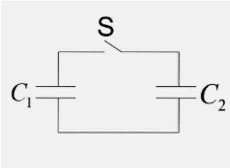
Solution:

We reduce the combination step by step as indicated in the figure. The $(1\ \mu F)$ and $(3\ \mu F)$ capacitors are in parallel and combine according to the expression $\left(C_{eq} = C_1 + C_2 = 4\ \mu F\right)$. The $(2\ \mu F)$ and $(6\ \mu F)$ capacitors also are in parallel and have an equivalent capacitance of $(8\ \mu F)$. Thus, the upper branch in figure (b) consists of two $(4\ \mu F)$ capacitors in series, which combine as follows; The lower branch in figure (b) consists of two $(8\ \mu F)$ capacitors in series, which combine



to yield an equivalent capacitance of $(4\ \mu F)$. Finally, the $(2\ \mu F)$ and $(4\ \mu F)$ capacitors in figure (c) are in parallel and thus have an equivalent capacitance of $(6\ \mu F)$.

Example (4.10):



A $\mathbf{3.55}\ \mu\boldsymbol{F}$ capacitor is charged to a potential difference $\mathbf{6.3}\ \boldsymbol{V}$, using a $\mathbf{6.3}\ \boldsymbol{V}$ battery. The battery is then removed and the capacitor is connected as shown to uncharged $\mathbf{8.95}\ \mu\boldsymbol{F}$ capacitor. When switch $(\boldsymbol{S})$ is closed charge flows from capacitor $(\mathbf{1})$ to capacitor $(\mathbf{2})$ until the capacitors have the same potential difference $(\boldsymbol{V})$. (i) What is this common potential difference? (ii) what is the potential energy of the two Capacitor system before and after switch $(\boldsymbol{S})$ is closed?

Solution:

(i)

$$Q = q_1 + q_2$$
$$C_1V_0 = C_1V + C_2V$$
$$V = V_o\left(\frac{C_1}{C_1 + C_2}\right)$$
$$V = (6.3\ V)\left(\frac{3.55\ \mu F}{3.55\ \mu F + 8.95\ \mu F}\right)$$
$$V = 1.79\ V$$

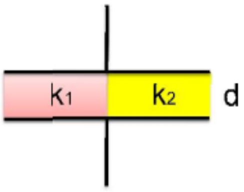
(ii)

$$U_i = \frac{1}{2}C_1{V_o}^2 = 0.5(3.55 \times 10^{-6}\ F)(6.3\ V)^2 = 7 \times 10^{-5} J$$
$$U_f = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 = 0.5\big((3.55 + 8.95) \times 10^{-6}F\big)(1.79\ V)^2 = 2 \times 10^{-5}\ J$$

This is missing energy appears as thermal energy in the connection wires.

Example (4.11):

A parallel plate capacitor of plate area $(\boldsymbol{A})$ is filled with two dielectrics as in the figure.



Show that the capacitance is given by

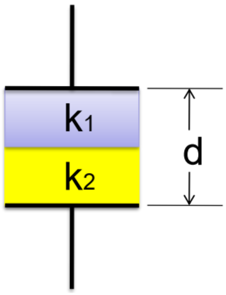
$$C = \frac{\varepsilon_o A}{d}\left(\frac{\boldsymbol{k_1} + \boldsymbol{k_2}}{2}\right)$$

Solution:

The two capacitors are in parallel

$$C_{eq} = C_1 + C_2$$
$$C_{eq} = \frac{\varepsilon_o \boldsymbol{k_1}(A/2)}{d} + \frac{\varepsilon_o \boldsymbol{k_2}(A/2)}{d}$$
$$C_{eq} = \frac{\varepsilon_o A}{d}\left(\frac{\boldsymbol{k_1} + \boldsymbol{k_2}}{2}\right)$$

Example (4.12):



A parallel plate capacitor of plate area $(\boldsymbol{A})$ is filled with two dielectrics as in the figure. Show that the capacitance is given by

$$C = \frac{2\varepsilon_o A}{d}\left(\frac{\boldsymbol{k_1 k_2}}{\boldsymbol{k_1} + \boldsymbol{k_2}}\right)$$

Solution:

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{\left(\frac{\varepsilon_o \boldsymbol{k_1} A}{d/2}\right)\left(\frac{\varepsilon_o \boldsymbol{k_2} A}{d/2}\right)}{\left(\frac{\varepsilon_o \boldsymbol{k_1} A}{d/2}\right) + \left(\frac{\varepsilon_o \boldsymbol{k_2} A}{d/2}\right)}$$

$$C_{eq} = \frac{\left(\frac{2\varepsilon_o A}{d}\right)^2 (\boldsymbol{k_1 k_2})}{\left(\frac{2\varepsilon_o A}{d}\right)(\boldsymbol{k_1} + \boldsymbol{k_2})} = \frac{2\varepsilon_o A}{d}\frac{(\boldsymbol{k_1 k_2})}{(\boldsymbol{k_1} + \boldsymbol{k_2})}$$

PROBLEMS

1. How much charge is on each plate of a $\mathbf{4}$ μ$\boldsymbol{F}$ capacitor when it is connected to a $\mathbf{12}\boldsymbol{V}$ battery?

2. (i) If an isolated metal drop of liquid has capacitance $\mathbf{1}\ \boldsymbol{pF}$, what is its radius? (ii) If another isolated metal drop has radius $\mathbf{2}\ \boldsymbol{mm}$, what is its capacitance? (iii) What is the charge on the smaller drop if its potential is $\mathbf{100}\ \boldsymbol{V}$?

3. An air-filled capacitor consists of two metal parallel plates of cross section area of $\mathbf{7.6}\ \boldsymbol{cm^2}$, they are separated by a distance of $\mathbf{1.8}\ \boldsymbol{mm}$. A $\mathbf{20}\ \boldsymbol{V}$ potential difference is applied to these plates. Calculate (i) the electric field between these plates, (ii) the surface charge density, (iii) the capacitance, and (iv) the charge on each plate.

4. An air-filled spherical capacitor is constructed with inner and outer shell radii of 7 and $\mathbf{14}\ \boldsymbol{cm}$, respectively. (i) Calculate the capacitance of the device. (ii) What potential difference between the spheres results in a charge of $\mathbf{4}$ μ$\boldsymbol{C}$ on the capacitor?

5. A $\mathbf{50}\ \boldsymbol{m}$ length of coaxial cable has an inner conductor that has a diameter of $\mathbf{2.58}\ \boldsymbol{mm}$ and carries a charge of $\mathbf{8.1}$ μ$\boldsymbol{C}$. The surrounding conductor has an inner diameter of $\mathbf{7.27}\ \boldsymbol{mm}$ and a charge of $-\mathbf{8.10}$ μ$\boldsymbol{C}$. (i) What is the capacitance of this cable? (ii) What is the potential difference between the two conductors? Assume the region between the conductors is air.

6. (i) A $\mathbf{3}$ μ$\boldsymbol{F}$ capacitor is connected to a $\mathbf{12}\ \boldsymbol{V}$ battery. How much energy is stored in the capacitor? (ii) If the capacitor had been connected to a $\mathbf{6}\ \boldsymbol{V}$ battery, how much energy would have been stored?

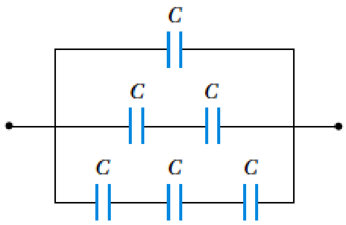
7. A wafer of titanium dioxide ($\boldsymbol{k} = \mathbf{173}$) of area $\mathbf{1}\ \boldsymbol{cm^2}$ has a thickness of $\mathbf{0.1}\ \boldsymbol{mm}$. Aluminum is evaporated on the parallel faces to form a parallel-plate capacitor. (a) Calculate the capacitance. (b) When the capacitor is charged with a $\mathbf{12}\ \boldsymbol{V}$ battery, what is the magnitude of charge delivered to each plate? (c) What is the charge on each ($\boldsymbol{Al}$) plate.

8. In open heart surgery, a much smaller amount of energy will defibrillate the heart. (i) What voltage is applied to the $\mathbf{8}$ μ$\boldsymbol{F}$ capacitor of a heart defibrillator that stores $\mathbf{40}\ \boldsymbol{J}$ of energy? (ii) Find the amount of stored charge.

9. Suppose you have a $\mathbf{9}\ \boldsymbol{V}$ battery, a $\mathbf{2}$ μ$\boldsymbol{F}$ capacitor, and a $\mathbf{7.4}$ μ$\boldsymbol{F}$ capacitor. (i) Find the charge and energy stored if the capacitors are connected to the battery in series. (ii) Do the same for a parallel connection.

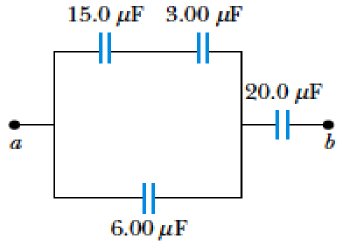
10. A $\mathbf{100}\,\boldsymbol{pF}$ capacitor is charged to a potential difference of $\mathbf{50}\,\boldsymbol{V}$ and the charging battery is disconnected. The capacitor is then connected in parallel with a second (initially uncharged) capacitor. If the measured potential difference drops to $\mathbf{35}\,\boldsymbol{V}$, what is the capacitance of this second capacitor?

11. What capacitance is required to store an energy of $\mathbf{10}\,\boldsymbol{kWh}$ at a potential difference of $\mathbf{1000}\,\boldsymbol{V}$?

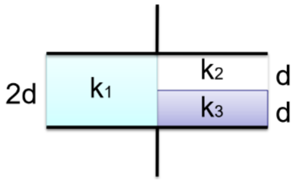


12. Evaluate the equivalent capacitance of the configuration shown in figure. All the capacitors are identical, and each has capacitance $(\boldsymbol{C})$.

13. Four capacitors are connected as shown in figure. (a) Find the equivalent capacitance between points $(\boldsymbol{a})$ and $(\boldsymbol{b})$. (b) Calculate the charge on each capacitor if $\boldsymbol{V_{ab}} = \mathbf{15}\,\boldsymbol{V}$.



14. What is the capacitance of the capacitor, of plate area $(\boldsymbol{A})$, shown in the figure?



CHAPTER (5)
ELECTRIC CURRENT AND RESISTANCE

5.1 Introduction

Thus far our treatment of electrical phenomena has been confined to the study of charges in equilibrium situations, or electrostatics. We now consider situations involving electric charges that are not in equilibrium. We use the term electric current to describe the rate of flow of charge through the conductors or in some region of space. Most practical applications of electricity deal with electric currents. For example, the battery in a flashlight produces a current in the filament of the bulb when the switch is turned on. Moreover, a variety of home devices operate on alternating current $(A.C)$. In these common situations, current exists in a conductor, such as a copper wire. It also is possible for currents to exist outside a conductor. For instance, a beam of electrons in a television picture tube constitutes a current.

5.2 Electric current

The electric current is defined as ***the rate of flow of charges across any cross-sectional area of a conductor in one direction***. Although an electric current is a stream of moving charges, not all moving charges constitute an electric current. If there is to be an electric current through a given surface, there must be a net flow of charge through this surface.

Furthermore, this example clarifies our meaning about the electric current. The free electrons (conduction electrons) in an isolated length of copper wire are in random motion at speeds of the order of $(10^6\ m/s)$. If you pass a hypothetical plane through such a wire, conduction electrons pass through it in both directions at the rate of many billions per second, but there is no net transport of charge and thus no current through the wire. However, if you connect the ends of the wire to a battery, you could motive the flow of charges in one direction, with the result that there now is a net transport of charges and thus an electric current move through the wire. In this chapter we limit ourselves to study within the scope of classical physics of steady currents of conduction electrons moving through metallic conductors such as copper wires

Figure 5.1a displays an isolated conducting loop regardless of whether it has an excess charge is all at the same electric potential $i.e.$ $(\Delta V = 0)$. No electric field can exist within it or along its surface. Although conduction electrons are available, no net electric force acts on them and thus there is no current.

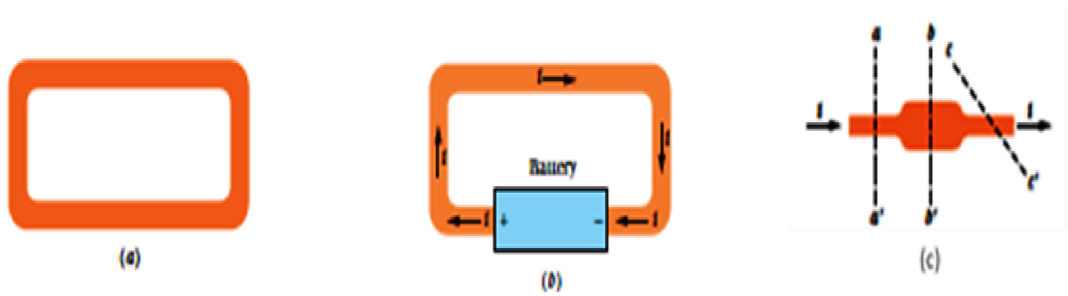


Figure 5.1: a) Wire only (hasn't current). b) Wire with battery (has a current). c)Wire has a current with different cross sectional area

In figure 5.1b, we insert a battery in the loop, the conducting loop is no longer at a single potential $(\Delta V \neq 0)$. Electric fields act inside the material to do electrical forces on the conduction electrons, causing them to move and thus generating an electric current. After a very short time, the electron flow reaches a constant value and the current is in its steady state. Figure 5.1c shows a section of a conductor, part of a conducting loop in which current has been passed. If charge (dq) passes through a hypothetical plane (e.g. aa' plane) in time (dt), then the current i passing through that plane is defined as

$$i = \frac{dq}{dt} \qquad (5-1)$$

We can find the electrical charges that passes through the plane in a time interval extending from (0) to (t) by integration:

$$q = \int dq = \int_0^t i dt \qquad (5-2)$$

in which the current (i) may vary with time. (i.e. current is function in time such as $A.C.$). Consequently, the S.I. unit for current is ampere (A) that equivalent to the coulomb per second.

$$1 Ampere = Coulomb/second$$

Under the steady-state conditions, ***the current is the same*** for planes aa', bb', and cc' and indeed for all planes that pass completely through the conductor, without knowing their location or orientation. This follows from the fact that charge is conserved. Under the steady-state conditions assumed here, an electron must pass through plane aa' for every electron that passes through plane cc'.

The electric current (i), as defined by equation (5-1), is a ***scalar*** because both charge and time in that equation are scalars. Yet, as in figure 5.1b, we often represent a current with an arrow to indicate that charge is moving. Such arrows are not vectors, however, and they do not require vector addition. Further, in figure 5.1b we drew the current arrows in the direction in which positively charged particles would be forced to move through the loop by the electric field. Such positive charge carriers, as they are often called, would move away from the positive battery terminal and toward the negative terminal. Actually, the charge carriers in the copper loop of figure 5.1b are electrons and thus are negatively charged. The electric field forces them to move in the direction opposite the current arrows, from the negative terminal to the positive terminal. For historical reasons, however, we use the following *convention directional:*

Current direction

''A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction''

We can use this convention because in most situations, the assumed motion of positive charge carriers in one direction has the same effect as the actual motion of negative charge carriers in the opposite direction. (When the effect is not the same, we shall drop the convention and describe the actual motion).

Figure 5.2 shows a conductor with current (I_1) splitting at a junction into two branches. Because charge is conserved, the magnitudes of the currents in the branches must add to yield the magnitude of the current in the original conductor, so that

$$I_1 = i_2 + i_3 \qquad (5-3)$$

Junction Rule:

"The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction."

$$\sum \boldsymbol{I_{in}} = \sum \boldsymbol{I_{out}}$$

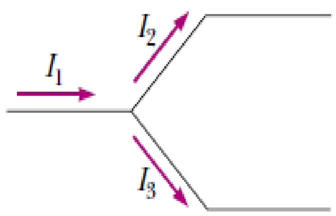


Figure 5.2: Junction Rule

Example (5.1):

A current of $\mathbf{5\ A}$ exists in a $\mathbf{10\ \Omega}$ resistor for $\mathbf{4\ min}$. How many (i) coulombs and (ii) electrons pass through any cross section of the resistor in this time?

Solution:

$$q = It = (5A)(4 \times 60\ s) = 1200\ C$$

$$N = \frac{q}{e} = \frac{(1200\ C)}{(1.6 \times 10^{-19}\ C)} = 7.5 \times 10^{21}\ electrons$$

Current density (J)

***"It is the quantity of charge ($\boldsymbol{Q}$) passing per unit time through normal unit cross-section area ($\boldsymbol{A}$) of conductor"* or; "*it is amount of electric current per normal unit area of conductor*"**

$$J = \frac{(Q/t)}{A} = \frac{I}{A} \tag{5-4}$$

Current density is a vector quantity. It is unit is expressed in (A/m^2).

Example (5.2):

A fuse in an electric circuit is a wire that is designed to melt, and thereby open the circuit, if the current exceeds a predetermined value. Suppose that the material composing the fuse melts once the current density rises to $\mathbf{440\ A/cm^2}$. What diameter of cylindrical wire should be used to limit the current to $\mathbf{0.5\ A}$?

Solution:

$$J = \frac{I}{A}$$

$$A = \frac{I}{J} = \frac{(0.5\ A)}{(440\ A/cm^2)} = 1.14 \times 10^{-3} cm^2$$

$$A = \frac{\pi}{4} d^2 \Rightarrow d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(1.14 \times 10^{-3}\ cm^2)}{\pi}} = 0.038\ cm$$

5.3 Relation between current density and drift velocity

Consider a conductor of length (L) and area of cross section (A). Electric field (E) is applied between its ends figure 5.3.

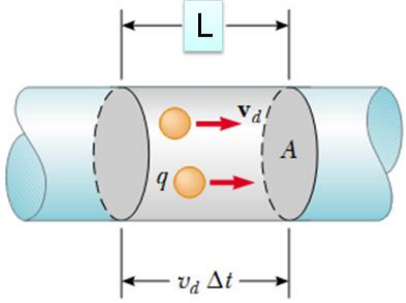


Figure 5.3: Conductor has a current

- Let (n) be the number of free electrons per unit volume. The free electrons move towards the right with a constant drift velocity (v_d).
- The number of conduction electrons in the considered conductor $N = nAL$.
- Then, the total charge passing through the conductor ($Q = Ne$) where (e) is charge of an electron.

$$Q = (nAL)\ e$$

- The time in which the charges pass through the conductor,

$$t = L/v_d$$

- The current flowing through the conductor is

$$I = \frac{Q}{t} = \frac{(nAL)e}{(L/v_d)} = nAev_d$$

- The (I) flowing through a conductor is directly proportional to the (v_d).

$$J = \frac{I}{A} = nev_d \qquad (5-5)$$

Example (5.3):

The copper wire in a typical residential building has a cross-sectional area of $\mathbf{3.31 \times 10^{-6}\ m^2}$. It carries a constant current of $\mathbf{10\ A}$. What is the drift speed of the electrons in the wire? Assume each copper atom contributes one free electron to the current. The density of copper is $\mathbf{8.92\ g/cm^3}$. $\mathbf{{}^{63.5}_{29}Cu}$ and Avogadro's number of atoms ($\mathbf{N_A = 6.02 \times 10^{23}\ mol^{-1}}$).

Solution:

$$\begin{pmatrix}\text{atoms}\\\text{per unit}\\\text{volume}\end{pmatrix} = \begin{pmatrix}\text{atoms}\\\text{per}\\\text{mole}\end{pmatrix}\begin{pmatrix}\text{moles}\\\text{per unit}\\\text{mass}\end{pmatrix}\begin{pmatrix}\text{mass}\\\text{per unit}\\\text{volume}\end{pmatrix}$$

$$n = (N_A)(\frac{1}{M})(\rho)$$

From the assumption that each copper atom contributes one free electron to the current, find the electron density in copper:

$$n = \frac{(6.02 \times 10^{23}\ \text{mol}^{-1})(8.92 \times 10^3\ kg/m^3)}{(0.0635\ kg/mol)} = 8.49\text{X}10^{28}\ electrons/m^3$$

$$v_d = \frac{I}{neA} = \frac{(10\ A)}{(8.49 \times 10^{28})(1.6 \times 10^{-19} C)(3.31 \times 10^{-6}\ m^2)} = 2.33 \times 10^{-4}\ m/s$$

Example (5.4):

A small but measurable current of $\mathbf{1.2 \times 10^{-10} A}$ exists in a copper wire whose diameter is $\mathbf{2.5\ mm}$. Calculate (i) the current density and (ii) the electron drift speed? ($\mathbf{n = 8.47x10^{28} electrons/m^3}$)

Solution:

$$J = \frac{I}{A} = \frac{(1.2 \times 10^{-10} A)}{\pi(1.25 \times 10^{-3}\ m)^2} = 2.44 \times 10^{-5} A/m^2$$

$$v_d = \frac{J}{ne} = \frac{(2.44 \times 10^{-5}\ A/m^2)}{(8.47 \times 10^{28}\ electrons/m^3)(1.6 \times 10^{-19}\ C)} = 1.8 \times 10^{-15}\ \ m/s$$

Example (5.5):

One end of an aluminum wires whose diameter is $\mathbf{2.5\ mm}$ is welded to one end of a copper wire whose diameter is $\mathbf{1.8\ mm}$. The composite wire carries a steady current $(\boldsymbol{I})$ of $\mathbf{1.3\ A}$. (i)What is the current density of each wire? (ii) What is the drift speed of the conduction electrons in the copper wire?

$[\boldsymbol{N_A = 6.02 \times 10^{23}\ atoms/mol, M = 0.064\ kg/mol, \rho = 9000\ Kg/m^3, e = 1.6 \times 10^{-19} C}]$

Solution:

(i)

$$J_{AL} = \frac{I}{A_{AL}} = \frac{I}{\pi {r_{AL}}^2} = \frac{(1.3\ A)}{\pi(1.25 \times 10^{-3}\ m)^2} = 2.6 \times 10^5\ A/m^2$$

$$J_{CU} = \frac{I}{A_{CU}} = \frac{I}{\pi {r_{CU}}^2} = \frac{(1.3\ A)}{\pi(0.9 \times 10^{-3}\ m)^2} = 5.1 \times 10^5\ A/m^2$$

(ii)

$$V = \frac{M}{\rho} = \frac{(0.064\ kg/mol)}{(9000\ Kg/m^3)} = 7.11 \times 10^{-6}\ m^3$$

$$n = \frac{N}{V} = \frac{N_A}{V} = \frac{(6.02 \times 10^{23}\ atoms/mol)}{(7.11 \times 10^{-6}\ m^3)} = 8.47 \times 10^{28}\ electrons/m^3$$

$$v_d = \frac{J}{ne} = \frac{(5.1 \times 10^5\ A/m^2)}{(8.47 \times 10^{28}\ electrons/m^3)(1.6 \times 10^{-19}\ C)} = 3.8 \times 10^{-5}\ m/s$$

5.4 Relation between drift velocity and mobility (μ)

Consider a conductor XY connected to a battery (figure 5.4). A steady electric field (E) is established in the conductor in the direction X to Y. In the absence of an electric field, the free electrons in the conductor move randomly in all possible directions. They do not produce current. But, as soon as an electric field is applied, the free electrons at the end Y experience a force $(F = eE)$ in a direction opposite to the electric field. The electrons are accelerated and, in the process, they collide with each other and with the positive ions in the conductor. Then, due to collisions, a backward force acts on the electrons and they are slowly drifted with a constant average drift velocity (v_d) in a direction opposite to electric field.

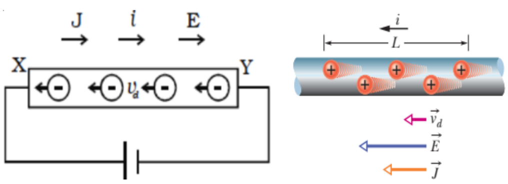


Figure 5.4: Current carrying conductor

Drift velocity

"It is the velocity of free electrons get drifted towards the positive terminal, when an electric field is applied".

If (Δt) is the average time between two successive collisions and the acceleration experienced by the electron be (a), then the drift velocity is given by,

$$v_d = a\, \Delta t$$

The force experienced by the electron of mass (m) is

$$F = m\, a = eE \Rightarrow a = \frac{eE}{m} \therefore$$

$$v_d = \frac{e\Delta t}{m} E \qquad (5-6)$$

$$\Rightarrow v_d = \mu E \qquad (5-7)$$

where $\left(\mu = \frac{e\Delta t}{m}\right)$ is the mobility and is defined as the drift velocity acquired per unit electric field**.** It takes the unit $(m^2/V.s)$. The drift velocity of electrons is proportional to the electric field intensity. It is very small and is of the order of $(0.1\ cm/s)$.

5.5 Resistance and electrical resistivity

The Impedance of the flow electric charge in conductor is named electric resistance (R). If the geometrical dimensions of resistance are considered named electric resistivity (ρ) which is electrical property for each material. The originality of electric resistance sources can be summarized in two factors; (i) the strange particles within material which is named *impurities*. They help to scatter the paths of free electrons and causing lower mean free paths. (ii) Atomic thermal oscillation of materials which rise up with increasing the ambient temperature. Theses more thermal vibrations help to resist the uniform motion of free electrons and be producing more electric resistance. If we apply the electric potential difference between the ends of geometrically similar rods of copper and of glass, very different currents result. The characteristic of the conductor that enters here is its electrical resistance. We determine the resistance between any two points of a conductor by applying a potential difference (V) between those points and measuring the current (i) that results. Therefore, electric resistance (R) usually measures using Ohm's law to detriment its value and/or the electric resistivity for any substance experimentally. It is worth to note that, the resistance is a property of an object but Resistivity is a property of a material. If we know the resistivity of a substance such as copper, we can calculate the resistance of a length of wire made of that substance.

5.6 Ohm's Law

Let (A) is the cross-sectional area of the conducting wire and (L) its length. when a potential difference (V) applied between its ends (figure 5.5). The current density (J) travel throughout the wire, the electric field and the current density will be constant for all points within the wire. The relation between the electric field (E) and the potential difference (V) is given by.

$$V = EL$$

This equation can be applied only to a homogeneous isotropic conductor of uniform cross section. The important macroscopic quantities e.g. potential difference (V), electric current (I) and resistance (R) are effective parameters during electrical measurements.

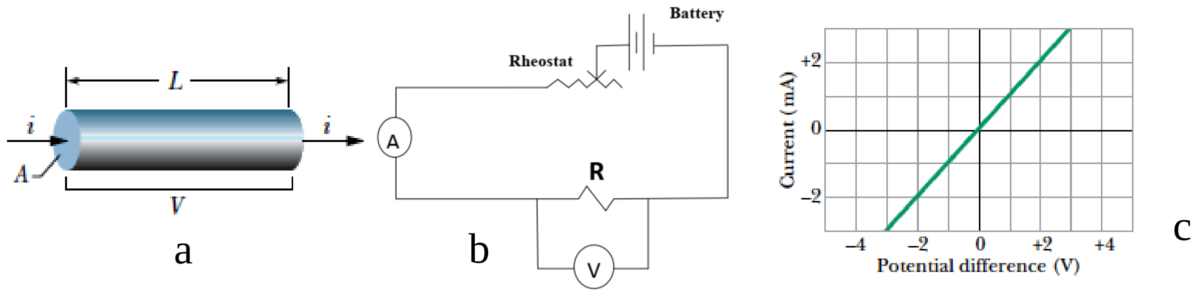


Figure 5.5: A potential difference (V) is applied between the ends of a wire of length (L) and cross section

Figure 5.5b shows circuit to verify ohm's law, where a potential difference (V) is applied across the device being tested, and the resulting current (I) through the resistor is measured as (V) is varied in both magnitude and polarity. The polarity of $(V)$is arbitrarily taken to be positive when the left terminal of the device is at a higher potential than the right terminal. The direction of the resulting current is arbitrarily assigned a plus sign. When reverse the polarity of (V) is then negative; the current it causes is assigned a minus sign. Furthermore, figure 5.5c is a plot of (I) versus (V) for a resistor. This plot is a straight line passing through the origin, so the ratio (I/V) (which is the slope of the straight line) is the same for all values of (V). This means that the resistance $R = V/I$ of the device is independent of the magnitude and polarity of the applied potential difference (V).

The important macroscopic quantities e.g. potential difference (V), electric current (I) and resistance (R) are effective parameters during electrical measurements. The Ohm's universal law is related between theses macroscopic quantities which state that;

"At a constant temperature, the electric current (I) pass through a conductor is directly proportional to the electric potential difference (V) between the two ends of the conductor and this relation not denpend on the polarity of the circuit"

$$V \propto I \quad at \quad T = constant$$

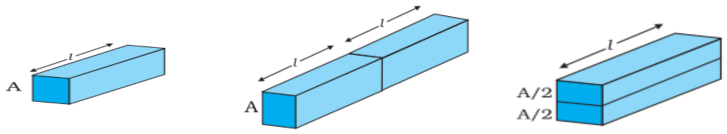
$$\Rightarrow V = RI \quad at \quad T = constant$$

Therefore, the electric resistance can be measured by dividing the value of applying potential difference (V) and the current that pass through the resistor. Also, the S.I. unit for resistance is (ohm) $(1\,\Omega = volt/Ampere)$

$$R = \frac{V}{I} \qquad (5-8)$$

Electrical resistivity $(\boldsymbol{\rho})$

Electrical resistivity (ρ) is a fundamental property of a material that quantifies the nature of impedance and obstructing the flow of electric current within conductor. A low resistivity indicates a material that readily allows electric current. The electric resistivity can be calculated if we take the dimension of resistance. [i.e. its cross-section area (A) and length (L)]. It is well known that; the electric resistance (R) of conductor is direct proportional with its length (L) and inversely proportional with its cross-section area (A). Therefore, the proportionality constant is considered as electric resistivity.



$$R \propto L/A \Rightarrow R = \rho\frac{L}{A} \qquad \Rightarrow \rho = R\frac{A}{L} \qquad (5-9)$$

<u>Example (5.6):</u>

A rectangular block of iron has dimensions $\mathbf{1.2 \times 1.2 \times 15\ cm^3}$ (i) What is the resistance of the block measured between the two square ends? (ii) What is the resistance between two opposite Rectangular faces? $\boldsymbol{\rho}_{\mathbf{iron}} = \mathbf{9.68 \times 10^{-8}\ \Omega.m}$

<u>Solution:</u>

(i) $$R = \frac{\rho L}{A} = \frac{(9.68\times10^{-8}\ \Omega.m)(0.15\,m)}{(0.012\,m)(0.012\,m)} = 1\times10^{-4}\Omega$$

(ii) $$R = \frac{\rho L}{A} = \frac{(9.68\times10^{-8}\,\Omega.m)(0.012\,m)}{(0.012\,m)(0.15\,m)} = 6.5\times10^{-7}\Omega$$

<u>Example (5.7):</u>

A steel trolley car rail has a cross sectional area of $\mathbf{56\ cm^2}$. What is the resistance of $\mathbf{10\ km}$ of rail? The resistivity of the steel is $\mathbf{3\ \times 10^{-7}\ \Omega.m}$.

Solution:

$$R = \frac{\rho L}{A} = \frac{(3 \times 10^{-7}\ \Omega.m)(10 \times 10^{3}\ m)}{(56 \times 10^{-4}\ m^{2})} = 0.54\ \Omega$$

Example (5.8):

A wire $\boldsymbol{4\ m}$ long and 6 mm in diameter has a resistance of $\boldsymbol{15\ \Omega.m}$ If a potential difference of $\boldsymbol{23\ V}$ is applied between the ends (a) what is the current in the wire? (b) what is the current density? (c) calculate the resistivity of the wire material.

Solution:

$$I = \frac{V}{R} = \frac{(23\ V)}{(15 \times 10^{-3}\ \Omega)} = 1533.3\ A$$

$$J = \frac{I}{A} = \frac{I}{\pi r^{2}} = \frac{1533.3\ A}{\pi(3 \times 10^{-3}\ m)^{2}} = 54.2 \times 10^{6}\ A/m^{2}$$

$$\rho = R\frac{A}{L} = \frac{(15 \times 10^{-3}\ \Omega)(\pi(3 \times 10^{-3}\ m)^{2})}{(4\ m)} = 0.06 \times 10^{-7}\ \Omega.m$$

Relation between resistivity $(\boldsymbol{\rho})$ and electric field $(\boldsymbol{E})$

We wish to take a general view and deal not with particular (R) of objects but with materials and their electrical characteristics. Here we do so by focusing not on the potential difference (V) across a particular resistor but on the electric field at a point in a resistive material. Also, instead of dealing with the current (I) through the resistor, we deal with the current density at the point in question. Instead of the resistance (R) of an object, we deal with the resistivity (ρ) of the material.

$$\rho = R\frac{A}{L}$$

Using ohm' s law and replace the value of resistance

$$R = \frac{V}{I} \quad and \quad \rho = \left(\frac{V}{I}\right)\left(\frac{A}{L}\right)$$

$$\Rightarrow \rho = \left(\frac{A}{I}\right)\left(\frac{V}{L}\right) = \left(\frac{1}{J}\right)\left(\frac{E}{1}\right)$$

As we known, the current density ($J = I/A$) and the electric field ($E = V/L$). Therefore, we can be finding

$$\Rightarrow \rho = \frac{E}{J}$$

The SI unit of (ρ) is the ohm-meter $(\Omega.m)$. We can write equation (4-10) in vector form as

$$E = \rho J \qquad (5-10)$$

equation (5-10) hold only for ***isotropic materials*** whose electrical properties are the same in all directions.

The conductivity (σ) of a material

It is simply defined as the reciprocal of its resistivity (ρ), and its unit is $(\Omega.m)^{-1}$.

$$\sigma = \frac{1}{\rho} \qquad or \qquad \sigma = \frac{J}{E} \tag{5-11}$$

The variation of electrical resistivity with temperature

The values of most physical properties vary with temperature, and resistivity is no exception. Figure 5.6, for example, shows the variation of this property for copper over a wide temperature range. The relation between temperature and resistivity for copper and for metals in general is fairly linear over a rather broad temperature range. For such linear relations we can write an empirical approximation that is good enough for most engineering purposes:

$$\rho - \rho_o = \rho_o \alpha \tag{5-12}$$

Here (T_o) is a selected reference temperature and ρ_o is the resistivity at that temperature. Usually $(T_o = 293\,K)$ (room temperature), for which $(\rho_o = 1.69 \times 10^{-8}\,\Omega.m)$ for copper. Because temperature enters equation (5-12) as a difference, it does not matter whether you use the Celsius or Kelvin scale in that equation because the sizes of degrees on these scales are identical. The quantity (α) in equation (5-12), called the **temperature coefficient of resistivity**, is chosen so that the equation gives good agreement with experiment for temperatures in the chosen range. Some values of (α) for metals are listed in **Table** 5.1. Because resistance is proportional to resistivity (equation (5-9)), we can write the variation of resistance as

$$R - R_o = \alpha R_o (T - T_o) \tag{5-13}$$

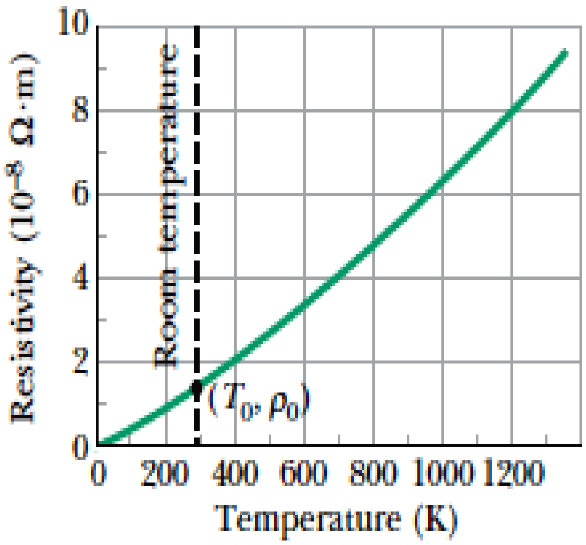


Figure 5.6: The resistivity of copper as a function of temperature. The dot on the curve marks a convenient reference point at temperature ($T_o = 293\ K$) and resistivity ($\rho_o = 1.69 \times 10^{-8}\ \Omega . m$).

Table 5.1: Resistivities and Temperature Coefficients of Resistivity for Various Materials at Room Temperature (20 °C)

Material	**ρ (Ωm)**	**α [(°C)$^{-1}$]**
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome	1×10^{-6}	0.4×10^{-3}
Carbon	$3.5 \times 10 - 5$	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	2.3×10^{3}	-75×10^{-3}

5.7 Classification of materials in terms of resistivity

The resistivity of a material is the characteristic of that particular material. The materials can be broadly classified into conductors (have a great number of charge carriers) and insulators (have not charge carriers). The metals and alloys *which have low resistivity of the order of* $(10^{-6} - 10^{-8}\ \Omega m)$ *are good conductors* of electricity. They carry current without appreciable loss of energy. Such as: silver, aluminum, copper, iron, tungsten, nichrome (chromium, iron, nickel), manganin (copper, manganese, nickel) and constantan (copper, nickel).

The resistivity of metals increases with increase in temperature. **Insulators** are *substances which have very high resistivity of the order of* $(10^{8} - 10^{14}\ \Omega\ m)$. They offer very high resistance to the flow of current and are termed non–conductors. Such as: glass, mica, amber, quartz, wood, Teflon, Bakelite. In between these two classes of materials lie the **semiconductors** (have a few number of charge carriers) *They are partially conducting. The resistivity of semiconductor is* $(10^{-2} - 10^{4}\ \Omega\ m)$. Such as: germanium, silicon

Example (5.9):

(a) Calculate the resistance per unit length of a nichrome wire of radius $\mathbf{0.321\ mm}$. **(b)** If a potential difference of $\mathbf{10\ V}$ is maintained across a $\mathbf{1\ m}$ length of the nichrome wire, what is the current in the wire? **(c)** The wire is melted down and recast with twice its original length. Find the new resistance $(\boldsymbol{R_N})$ as a multiple of the old resistance $(\boldsymbol{R_o})$.

Solution:

(a) Calculate the resistance per unit length. Find the cross-sectional area of the wire: $A = \pi r^2 = \pi(0.321 \times 10^{-3}\ m)^2 = 3.24 \times 10^{-7}\ m^2$

Obtain the resistivity of nichrome from **Table** 4-1, solve equation (5-9) for (R/l), and substitute:

$$\frac{R}{l} = \frac{\rho}{A} = \frac{(1.5 \times 10^{-6}\ \Omega.\mathrm{m})}{(3.24 \times 10^{-7} m^2)} = 4.6\ \Omega/\mathrm{m}$$

(b) Find the current in a $(1\ m)$ segment of the wire if the potential difference across it is $(10\ V)$. Substitute given values into Ohm's law:

$$I = \frac{\Delta V}{R} = \frac{(10\ V)}{(4.6\ V)} = 2.2\ A$$

(c) If the wire is melted down and recast with twice its original length, find the new resistance as a multiple of the old. Find the new area (A_N) in terms of the old area (A_o), using the fact the volume doesn't change and $(l_N = 2l_o)$:

$$V_N = V_o \rightarrow A_N l_N = A_o l_o \rightarrow A_N = A_o(l_o / l_N)$$

$$A_N = A_o(l_o / 2l_o) = A_o/2$$

Substitute into equation (5-9):

$$R_N = \frac{\rho l_N}{A_N} = \frac{\rho(2l_o)}{(A_o/2)} = 4\frac{\rho l_o}{A_o} = 4R_o$$

5.8 Power in electric circuits

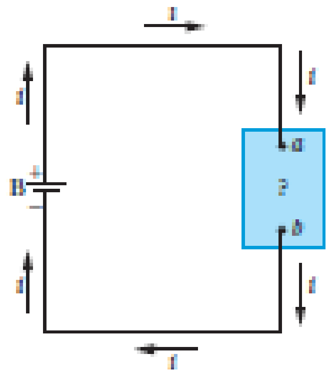


Figure 5.7: A simple circuit has two batteries, wires and resistor.

Figure 5.7 shows a circuit consisting of a battery (B) that is connected by wires, which we assume have negligible resistance, to an unspecified conducting device. The device might be a resistor, a storage battery (a rechargeable battery), a motor, or some other electrical device. The battery maintains a potential difference of magnitude (V) across its own terminals and thus (because of the wires) across the terminals of the unspecified device, with a greater potential at terminal a of the device than at terminal (b).

Because there is an external conducting path between the two terminals of the battery, and because the potential differences set up by the battery are maintained, a steady current (I) is produced in the circuit, directed from terminal (a) to terminal (b). When the amount of charge (dq) moves between those terminals in time interval (dt) is equal to $(I\,dt)$. This charge (dq) moves due to the potential difference of magnitude (V), and thus its electric potential energy decreases in magnitude by the amount

$$dU = dq\,V = (I\,dt)\,V$$

The principle of conservation of energy tells us that the decrease in electric potential energy from (a) to (b) is accompanied by a transfer of energy to some other form. The power (P) associated with that transfer is the rate of dissipation electric energy (dU/dt), which is given by

$$P = \frac{dU}{dt} = IV \qquad (5-14)$$

Moreover, this power (P) is also the rate at which energy is transferred from the battery to the unspecified device. If that device is a motor connected to a mechanical load, the energy is transferred as work done on the load. If the device is a storage battery that is being charged, the energy is transferred to stored chemical energy in the storage battery. If the device is a resistor, the energy is transferred to internal thermal energy, tending to increase the resistor's temperature. Moreover, the unit of power is the $(volt \times ampere)$ which is equivalent to the watt.

$$1\,V.A = (1\,J/C)(1\,C/s) = 1\,J/s = 1W.$$

As an electron moves through a resistor at constant drift speed, its average kinetic energy remains constant and its lost electric potential energy appears as thermal energy in the resistor and the surroundings. On a microscopic scale this energy transfer is due to collisions between the electron and the molecules of the resistor, which leads to an increase in the temperature of the resistor lattice.

The mechanical energy thus transferred to thermal energy is dissipated (lost) because the transfer cannot be reversed. For a resistor or some other device with resistance (R), we can combine equations (5-8) $(R = V/I)$ and (5-14) to obtain, for the rate of electrical energy dissipation due to a resistance, either

$$P = i^2R \qquad (5-15)$$

or

$$P = \frac{V^2}{R} \qquad (5-16)$$

Example (5.10):

A student kept his $\mathbf{9\,V, 7\,W}$ portable radio turned on from $\mathbf{9\,pm}$. until $\mathbf{2\,am}$. How much charge went through it?

Solution:

$$q = It = (\frac{P}{V})t = (7/9)(5 \times 3600) = 14000\ C$$

Example (5.11):

A certain $\boldsymbol{X\,ray}$ tube operates at a current of $\mathbf{7\,mA}$ and a potential difference of $\mathbf{80\,kV}$. What power in watts is dissipated?

Solution:

$$P = VI = (80 \times 10^3\,V)(7 \times 10^{-3}\,A) = 560\,W$$

Example (5.12):

The resistance $(\boldsymbol{R})$ of a uniform heating wire made of Nichrome; has a of $\mathbf{72\,\Omega}$. At what rate is energy dissipated in each of the following situations? $(\mathbf{1})$ A potential difference of $\mathbf{120\,V}$ is applied across the full length of the wire. $(\mathbf{2})$ The wire is cut in half, and a potential difference of $\mathbf{120\ V}$ is applied across the length of each half.

Solution:

Because we know the potential (V) and resistance (R), we use equation (5-16), which yields, for situation (1),

$$P = \frac{V^2}{R} = \frac{120^2}{72} = 200\ \text{W}$$

In situation (2), the resistance of each half of the wire is $(72/2)$, or (36). Thus, the dissipation rate for each half is

$$P' = \frac{V^2}{R} = \frac{(120\ V)^2}{(36\ \Omega)} = 400\ \text{W}$$

PROBLEMS

1. A certain conductor has $\mathbf{7.5 \times 10^{28}}$ **free electrons per cubic meter**, a cross-sectional area of $\mathbf{4 \times 10^{-6}\ m^2}$, and carries a current of $\mathbf{2.5\ A}$. Find the drift speed of the electrons in the conductor.

2. A $\mathbf{1\ V}$ potential difference is maintained across a $\mathbf{10}\ \Omega$ resistor for a period of $\mathbf{20\ s}$. What total charge passes through the wire in this time interval?

3. An aluminum wire carrying a current of $\mathbf{5\ A}$ has a cross-sectional area of $\mathbf{4 \times 10^{-6}\ m^2}$. Find the drift speed of the electrons in the wire. The density of aluminum is $\mathbf{2.7\ g/cm^3}$. (Assume that one electron is supplied by each atom.)

4. If the current carried by a conductor is doubled, what happens to (a) the charge carrier density? (b) the electron drift velocity?

5. A potential difference of $\mathbf{12\ V}$ is found to produce a current of $\mathbf{0.4\ A}$ in a $\mathbf{3.2\ m}$ length of wire with a uniform radius of $\mathbf{0.4\ cm}$. What is (a) the resistance of the wire? (b) the resistivity of the wire?

6. A rectangular block of copper has sides of length $\mathbf{10\ cm}$, $\mathbf{20\ cm}$, and $\mathbf{40\ cm}$. If the block is connected to a $\mathbf{6\ V}$ source across two of its opposite faces, what are (a) the maximum current and (b) the minimum current that the block can carry?

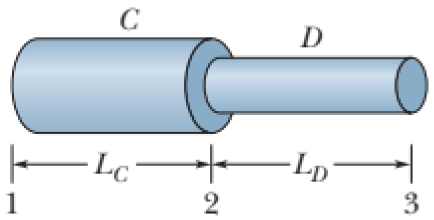
7. A metal wire has a resistance of $\mathbf{10}\ \Omega$ at a temperature of $\mathbf{20\ °C}$. If the same wire has a resistance of $\mathbf{10.55}\ \Omega$ at $\mathbf{90\ °C}$, what is the resistance of the wire when its temperature is $\mathbf{-20\ °C}$?

8. A toaster is rated at $\mathbf{600\ W}$ when connected to a $\mathbf{120\ V}$ source. What current does the toaster carry, and what is its resistance?

9. The power supplied to a typical black-and-white television set is $\mathbf{90\ W}$ when the set is connected to $\mathbf{120\ V}$. (a) How much electrical energy does this set consume in $\mathbf{1\ hour}$? (b) A color television set draws about $\mathbf{2.5\ A}$ when connected to $\mathbf{120\ V}$. How much time is required for it to consume the same energy as the black-and-white model consumes in $\mathbf{1\ hour}$?

10. A common flashlight bulb is rated at $\mathbf{0.3\ A}$ and $\mathbf{2.9\ V}$ (the values of the current and voltage under operating conditions). If the resistance of the tungsten bulb filament at room temperature ($\mathbf{20\ °C}$) is $\mathbf{1.1\ \Omega}$, what is the temperature of the filament when the bulb is on?

11. Wire $(\boldsymbol{C})$ and wire $(\boldsymbol{D})$ are made from different materials and have length $(\boldsymbol{L_C} = \boldsymbol{L_D} = \mathbf{1.0}\ \boldsymbol{m})$. The resistivity and diameter of wire $\boldsymbol{C}$ are $\mathbf{2.0} \times \mathbf{10^{-6}}\ \boldsymbol{\Omega.m}$ and $\mathbf{1}\ \boldsymbol{mm}$, and those of wire $\boldsymbol{D}$ are $\mathbf{1.0} \times \mathbf{10^{-6}}\ \boldsymbol{\Omega.m}$ and $\mathbf{0.5}\ \boldsymbol{mm}$. The wires are joined as shown in figure. and a current of $\mathbf{2}\ \boldsymbol{A}$ is set up in them. What is the electric potential difference between (a) points $(\mathbf{1})$ and $(\mathbf{2})$ and (b) points $(\mathbf{2})$ and $(\mathbf{3})$? What is the rate at which energy is dissipated between (c) points $(\mathbf{1})$ and $(\mathbf{2})$ and (d) points $(\mathbf{2})$ and $(\mathbf{3})$?



CHAPTER (6)
ELECTRIC CIRCUITS

6.1 Introduction

Batteries, resistors, and capacitors can be used in various combinations to construct electric circuits, which direct and control the flow of electricity and the energy it carries. Such circuits make possible all the modern conveniences in a home electric lights, electric stove tops and ovens, washing machines, and a host of other appliances and tools. Electric circuits are also found in our cars, in tractors that increase farming productivity, and in all types of medical equipment that saves so many lives every day.

6.2 Sources of electromotive force (EMF)

A current is maintained in a closed circuit by a source of (emf). (The term (emf) comes from the outdated phrase *electromotive force*) Among such sources are any devices (for example, batteries and generators) that increase the potential energy of the circulating charges. A source of (emf) can be thought of as a “charge pump” that forces electrons to move in a direction opposite the electrostatic field inside the source. The (emf) of a source is the work done per unit charge; hence the SI unit of (emf) is the $(volt)$.

Consider the circuit in figure 6.1a consisting of a battery connected to a resistor. We assume that the connecting wires have no resistance. If we neglect the internal resistance of the battery, the potential drop across the battery (the terminal voltage) equals the (emf) (ε) of the battery (this is called an ideal Battery).

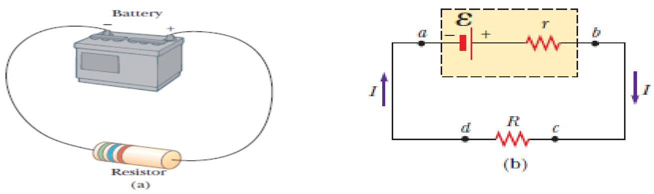


Figure 6.1: (a) A circuit consisting of a resistor connected to the terminals of a battery. (b) A circuit diagram of a source of (emf) having internal resistance (r) connected to an external resistor (R).

Because a Real Battery always has some internal resistance (r), however, the terminal voltage is not equal to the (emf). The circuit of figure 6.1a can be described schematically by the diagram in figure 6.1b. The battery, represented by the dashed rectangle, consists of a source of emf (ε) in series with an internal resistance (r). Now imagine a positive charge moving through the battery from (a) to (b)in the figure. As the charge passes from the negative to the positive terminal of the battery, the potential of the charge increases by (ε). As the charge moves through the resistance (r), however, its potential decreases by the amount (I_r), where (I) is the current in the circuit. The terminal voltage of the battery, $(\Delta V = V_b - V_a)$, is therefore given by

$$\Delta V = \varepsilon - I_r \qquad (6-1)$$

From this expression, we see that (ε) is *equal to the terminal voltage when the current is zero*, called the open-circuit voltage. By inspecting figure 6.1b, we find that the terminal voltage (ΔV) must also equal the potential difference across the external resistance (R), often called the load resistance; that is, $(\Delta V = IR)$. Combining this relationship with equation (6-1), we arrive at

$$\varepsilon = IR + I_r \qquad (6-2)$$

6.3 Resistors in series

When two or more resistors are connected end to end as in figure 6.2, they are said to be in *series*. The resistors could be simple devices, such as lightbulbs or heating elements. When two resistors (R_1) and (R_2) are connected to a battery as in figure 6.2, the current is the same in the two resistors, because any charge that flows through (R_1) must also flow through (R_2). This is analogous to water flowing through a pipe with two constrictions, corresponding to (R_1) and (R_2). Whatever volume of water flows in one end in a given time interval must exit the opposite end.

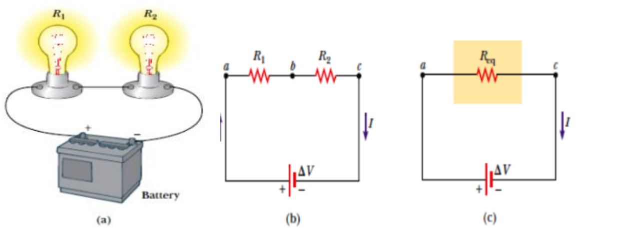


Figure 6.2: A series connection of two resistors, (R_1) and (R_2). The currents in the resistors are the same, and the equivalent resistance of the combination is given by $(R_{eq} = R_1 + R_2)$

Because the potential difference between (a) and (b) in figure 6.2b equals (IR_1) and the potential difference between (b) and (c) equals (IR_2), the potential difference between (a) and (c) is

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

Regardless of how many resistors we have in series, the sum of the potential differences across the resistors is equal to the total potential difference across the combination. As we will show later, this is a consequence of the conservation of energy.

Figure 6.2c shows an equivalent resistor (R_{eq}) that can replace the two resistors of the original circuit. The equivalent resistor has the same effect on the circuit because it results in the same current in the circuit as the two resistors. Applying Ohm's law to this equivalent resistor, we have

$$\Delta V = IR_{eq}$$

Equating the preceding two expressions, we have

$$IR_{eq} = I(R_1 + R_2)$$

or

$$R_{eq} = R_1 + R_2 \text{ (series combination)} \qquad (6-3)$$

An extension of the preceding analysis shows that the equivalent resistance of three or more resistors connected in series is

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \qquad (6-4)$$

Therefore, *the equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances and is always greater than any individual resistance.*

Note that if the filament of one lightbulb in figure 6.2 were to fail, the circuit would no longer be complete (an open-circuit condition would exist) and the second bulb would also go out.

6.4 Resistors in parallel

Now consider two resistors connected in parallel, as in figure 6.3. In this case, the potential differences across the resistors are the same because each is connected directly across the battery terminals. The currents are generally not the same. When charges reach point a (called a junction) in figure 6.3b, the current splits into two parts: (I_1), flowing through (R_1) and (I_2), flowing through (R_2). If (R_1) is greater than (R_2), then (I_1) is less than (I_2). In general, more charge travels through the path with less resistance. Because charge is conserved, the current (I) that enters point (a) must equal the total current ($I_1 + I_2$) leaving that point. Mathematically, this is written

$$I = I_1 + I_2$$

The potential drop must be the same for the two resistors and must also equal the potential drop across the battery. Ohm's law applied to each resistor yields

$$I_1 = \frac{\Delta V}{R_1} \qquad I_2 = \frac{\Delta V}{R_2}$$

Ohm's law applied to the equivalent resistor in figure (6-3c) gives

$$I = \frac{\Delta V}{R_{eq}}$$

When these expressions for the currents are substituted into the equation ($I = I_1 + I_2$)

and the ($\Delta V's$) are cancelled, we obtain

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{(pallel combination)} \qquad (6-5)$$

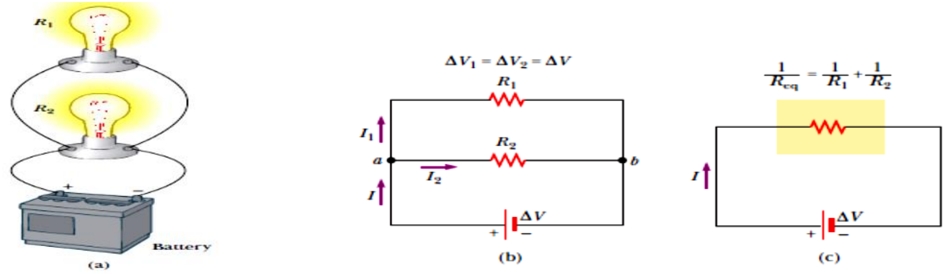


Figure 6.3: (a) A parallel connection of two lightbulbs with resistances (R_1) and (R_2). (b) Circuit diagram for the two-resistor circuit. The potential differences across (R_1) and (R_2) are the same. (c) The equivalent resistance of the combination is given by the reciprocal relationship $(1/R_{eq} = 1/R_1 + 1/R_2)$.

An extension of this analysis to three or more resistors in parallel produces the following general expression for the equivalent resistance:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (6-6)$$

From this expression, we see that *the inverse of the equivalent resistance of two or more resistors connected in parallel is the sum of the inverses of the individual resistances and is always less than the smallest resistance in the group.*

6.5 Calculating the current

We discuss here two equivalent ways to calculate the current in the simple *single loop* circuit of figure 6.4; one method is based on energy conservation considerations, and the other on the concept of potential. The circuit consists of an ideal battery (B) with emf, (ε), a resistor of resistance (R), and two connecting wires.

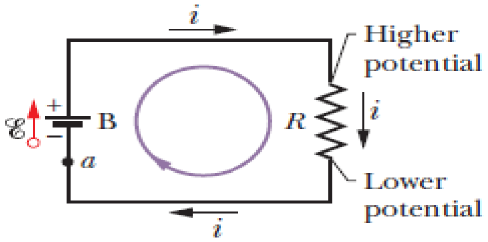


Figure 6.4: A single-loop circuit in which a resistance (R) is connected across an ideal battery (B) with emf (ε).The resulting current (i) is the same throughout the circuit.

Energy Method

This equation ($P = i^2 R$) tells us that in a time interval (dt) an amount of energy given by ($i^2 R\ dt$) will appear in the resistor of figure 6.1b as thermal energy. As noted in previous chapter, this energy is said to be *dissipated.* (Because we assume the wires to have negligible resistance, no thermal energy will appear in them.) During the same interval, a charge ($dq = i\ dt$) will have moved through battery, and the work that the battery will have done on this charge, is

$$dW = \varepsilon\, dq = \varepsilon\, i\, dt.$$

From the principle of conservation of energy, the work done by the (ideal) battery must equal the thermal energy that appears in the resistor:

$$\varepsilon\, i\, dt = i^2 R dt.$$

This gives us

$$\varepsilon = iR.$$

The emf (ε) is the energy per unit charge transferred to the moving charges by the battery. The quantity (iR) is the energy per unit charge transferred from the moving charges to thermal energy within the resistor. Therefore, this equation means that the energy per unit charge transferred to the moving charges is equal to the energy per unit charge transferred from them. Solving for (i), we find

$$I = \frac{\sum \varepsilon}{R_{eq}} \qquad (6-7)$$

Potential Method

Suppose we start at any point in the circuit of figure 6.4 and mentally proceed around the circuit in either direction, adding algebraically the potential differences that we encounter. Then when we return to our starting point, we must also have returned to our starting potential. Before actually doing so, we shall formalize this idea in a statement that holds not only for single-loop circuits such as that of figure 6.4 but also for any complete loop in a multi loop circuit.

LOOP RULE: *The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be zero.*

$$\sum \Delta V = 0$$

This is often referred to as *Kirchhoff's loop rule* (or *Kirchhoff's voltage law*), after German physicist Gustav Robert Kirchhoff. This rule is equivalent to saying that each point on a mountain has only one elevation above sea level. If you start from any point and return to it after walking around the mountain, the algebraic sum of the changes in elevation that you encounter must be zero.

In figure 6.4, let us start at point (a), whose potential is (V_a), and mentally walk clockwise around the circuit until we are back at (a), keeping track of potential changes as we move. Our starting point is at the low-potential terminal of the battery. Because the battery is ideal $(r = 0)$, so the potential difference between its terminals is equal to (ε).When we pass through the battery to the high-potential terminal, the change in potential is $(+\varepsilon)$.

As we walk along the top wire to the top end of the resistor, there is no potential change because the wire has negligible resistance. When we pass through the resistor, however, the potential changes as we can rewrite as $(V = iR)$. The potential must decrease because we are moving from the higher potential side of the resistor to low potential side. Thus, the change in potential is negative $(-iR)$. Then, we return to point (a) by moving along the bottom wire. Because this wire also has negligible resistance, we again find no potential change. Beyond we traversed a complete loop, our initial potential, as modified for potential changes along the way, must be equal to our final potential; that is,

$$V_a + \varepsilon - iR = V_a$$

The value of (V_a) cancels from this equation, which becomes

$$\varepsilon - iR = 0$$

Solving this equation for $(\boldsymbol{i})$ gives us the same result, $(i = \varepsilon/R)$**,** as the energy method equation (6-6). If we apply the loop rule to a complete *counterclockwise* walk around the circuit, the rule gives us

$$-\varepsilon + iR = 0$$

and we again find that $(i = \varepsilon/R)$. Thus, you may mentally circle a loop in either direction to apply the loop rule. To prepare for circuits more complex than that of figure 6.4, let us set down two rules for finding potential differences as we move around a loop:

RESISTANCE RULE: *For a move through a resistance in the direction of the current, the change in potential is* $(-iR)$*; in the opposite direction it is* $(+\,iR)$.

EMF RULE: *For a move through an ideal* emf *device in the direction of the* emf *arrow, the change in potential is* $(+\varepsilon)$*; in the opposite direction it is* $(-\varepsilon)$.

Figure 6.5 shows a circuit containing more than one loop. For simplicity, we assume the batteries are ideal. There are two *junctions* in this circuit, at (b) and (d), and there are three branches connecting these junctions. The branches are the left branch (bad), the right branch (bcd), and the central branch (bd). What are the currents in the three branches?

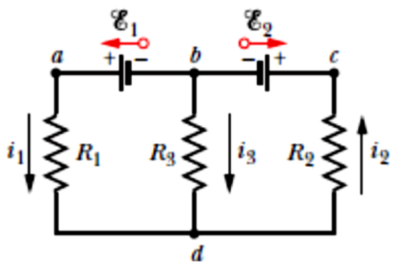


Figure 6.5: A multi loop circuit consisting of three branches: left-hand branch (bad), right hand branch (bcd), and central branch (bd). The circuit also consists of three loops: left hand loop $(badb)$, right-hand loop $(bcdb)$, and big loop $(badcb)$.

We arbitrarily label the currents, using a different subscript for each branch. Current (i_1) has the same value everywhere in branch (bad), (i_2) has the same value everywhere in branch (bcd), and (i_3) is the current through branch (bd). The directions of the currents are assumed arbitrarily. Consider junction (d) for a moment: Charge comes into that junction via incoming currents (i_1) and (i_3), and it leaves via outgoing current (i_2). Because there is no variation in the charge at the junction, the total incoming current must equal the total outgoing current:

$$\boldsymbol{i_1 + i_3 = i_2}$$

You can easily check that applying this condition to junction b leads to exactly the same equation. The above equation suggests a general principle:

JUNCTION RULE: *The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.*

This rule is often called *Kirchhoff's junction rule* (or *Kirchhoff's current law*). It is simply a statement of the conservation of charge for a steady flow of charge there is neither a buildup nor a depletion of charge at a junction. Thus, our basic tools for solving complex circuits are the *loop rule* (based on the conservation of energy) and the *junction rule* (based on the conservation of charge).

6.6 Potential Difference Between Two Points

We often want to find the potential difference between two points in a circuit. For example, in figure 6.6, what is the potential difference $(V_b - V_a)$ between points (a) and (b)? To find out, let's start at point (a) (at potential Va) and move through the battery to point (b) (at potential (V_b) while keeping track of the potential changes we encounter. When we pass through the battery's (emf), the potential increases by (ε). When we pass through the battery's internal resistance (r), we move in the direction of the current and thus the potential decreases by (ir). We are then at the potential of point (b) and we have

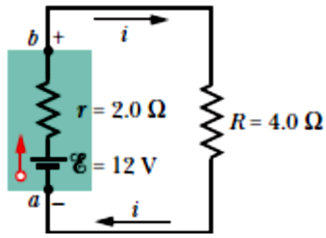


Figure 6.6: Points (a) and (b), which are at the terminals of a real battery, differ in potential.

$$V_a + \varepsilon - ir = V_b \Longrightarrow V_a - V_b = \varepsilon - ir$$

To evaluate this expression, we need the current (i), then, the current is

$$i = \frac{\varepsilon}{R + r}$$

Substituting this equation into above equation gives us

$$V_a - V_b = \varepsilon - \frac{\varepsilon}{R + r} r$$

$$\therefore V_a - V_b = \frac{\varepsilon}{R + r} R$$

Now substituting the data given in figure 6.6, we have

$$V_a - V_b = \frac{12V}{4\Omega + 2\Omega} 4\Omega = 8\text{V}$$

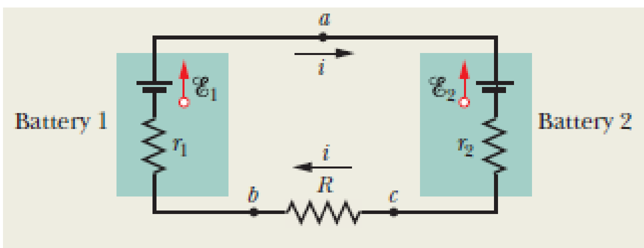
Suppose, instead, we move from (a) to (b) counterclockwise, passing through resistor (R) rather than through the battery. Because we move opposite the current, the potential increases by (iR)**.** Thus,

$$V_a = iR + V_b \Longrightarrow V_a - V_b = iR$$

Substituting for (i) from considered equation, we again find the same value as in first case. In general, *"To find the potential between any two points in a circuit, start at one point and traverse the circuit to the other point, following any path, and add algebraically the changes in potential you encounter".*

<u>Example (6.1)</u>

The $(\boldsymbol{emfs})$ and resistances in the circuit of Figure have the following values: $\boldsymbol{\varepsilon_1} = \mathbf{4.4\,V}$, $\boldsymbol{\varepsilon_1} = \mathbf{2.1\,V}, \boldsymbol{r_1} = \mathbf{2.3}\,\Omega, \boldsymbol{r_2} = \mathbf{1.8}\,\Omega, \boldsymbol{R} = \mathbf{5.5}\,\Omega$. a) What is the current $(\boldsymbol{i})$ in the circuit?



Solution:

We can get an expression involving the current (i) in this single-loop circuit by applying the energy method and the loop rule, in which we sum the potential changes around the full loop. Although knowing the direction of (i) is not necessary, we can easily determine it from the $(emfs)$ of the two batteries. Because (ε_1) is greater than (ε_2), battery (1) controls

the direction of (i), so the direction is clockwise. Let us then apply the loop rule by going counterclockwise against the current and starting at point (a). (These decisions about where to start and which way you go are arbitrary but, once made, you must be consistent with decisions about the plus and minus signs.) We find

$$-\varepsilon_1 + ir_1 + iR + ir_2 + \varepsilon_2 = 0$$

$$i = \frac{\varepsilon_1 - \varepsilon_2}{R + r_1 + r_2} = \frac{4.4V - 2.1V}{5\Omega + 2.3\Omega + 1.8\Omega} = 240\, mA$$

If we solve by applying the energy method, we will get the same equation and the same result

$$i = \frac{\varepsilon_1 - \varepsilon_2}{R + r_1 + r_2} = \frac{4.4V - 2.1V}{5\Omega + 2.3\Omega + 1.8\Omega} = 240\, mA$$

b) What is the potential difference between the terminals of battery (1) in the figure?

We need to sum the potential differences between points (a) and (b). Calculations: Let us start at point (b) (effectively the negative terminal of battery (1)) and travel clockwise through battery (1) to point (a) (effectively the positive terminal), keeping track of potential changes. We find that

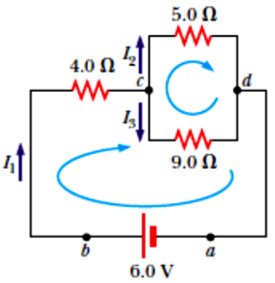
$$V_b - ir_1 + \varepsilon_1 = V_a$$

$$V_a - V_b = -ir_1 + \varepsilon_1$$

$$= (0.24A)(2.3\Omega) + 4.4V = 3.8\, V$$

Example (6.2):

Find the currents in the circuit shown in figure by using Kirchhoff's rules.



Solution:

Apply the junction rule to point (c). (I_1) is directed into the junction, (I_2) and (I_3) are directed out of the junction.

$$I_1 = I_2 + I_3 \quad (1)$$

Select the bottom loop, and traverse it clockwise starting at point (a), generating an equation with the loop rule:

$$\varepsilon - 4\Omega I_1 - 9\Omega I_3 = 0$$

$$4I_1 + 9I_3 = 6 \quad (2)$$

$$-5I_2 + 9I_3 = 0 \quad (3)$$

Solve equation (3) for (I_2) and substitute into equation (1): $I_2 = 1.8\,I_3$

$$I_1 = I_2 + I_3 = 1.8I_3 + I_3 = 2.8I_3$$

Substitute the latter expression into equation (2) and solve for (I_3):

$$4(2.8I_3) + 9I_3 = 6.0 \implies I_3 = 0.3\,A$$

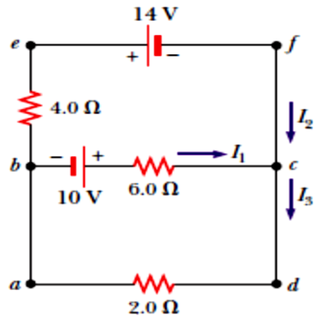
Substitute (I_3) back into equation (3) to get (I_2):

$$-5I_2 + 9.0(0.30\,A) = 0 \implies I_2 = 0.54\,A$$

Substitute (I_3) into equation (2) to get (I_1):

$$4I_1 + 9(0.3\,A) = 6.0 \implies I_3 = 0.83\,A$$

Example (6.3):



Find the currents in a circuit with three currents and two batteries when some current directions are chosen wrongly.

Solution:

Apply Kirchhoff's junction rule to junction (c). Because of the chosen current directions, (I_1) and (I_2) are directed into the junction and (I_3) is directed out of the junction.

Apply Kirchhoff's loop rule to the loops ($abcda$) and ($befcb$). (Loop ($aefda$) gives no new information.) In loop ($befcb$), a positive sign is obtained when the 6Ω resistor is traversed, because the direction of the path is opposite the direction of the current (I_1).

$$I_3 = I_1 + I_2 \quad (1)$$

Loop ($abcda$): $10V - 6\Omega I_1 - 2\Omega I_3 = 0 \quad (2)$

Loop ($befcb$): $-14V + 6\Omega I_1 - 10 - 4\Omega I_2 = 0 \quad (3)$

Using equation (1), eliminate (I_3) from equation (2)

$$10 - 6I_1 - 2(I_1+I_2) = 0$$

$$10 = 8I_1 + 2I_2 = 0 \quad (4)$$

Divide each term in equation (3) by (2) and rearrange the equation so that the currents are on the right side:

$$-12 = -3I_1 + 2I_2 \quad (5)$$

Subtracting equation (5) from equation (3) (4) eliminates (I_2) and gives (I_1):

$$22 = 11I_1 \implies I_1 = 2\,A$$

Substituting this value of (I_1) into equation (5) gives (I_2):

$$2I_2 = 3I_1 - 12 = 3(2) - 12 = -6\,A \quad \rightarrow \quad I_2 = -3A$$

Finally, substitute the values found for (I_1) and (I_2) into equation (1) to obtain (I_3):

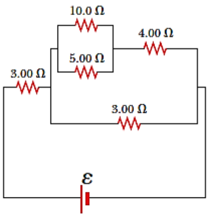
$$I_3 = I_1 + I_2 = 2\,A - 3\,A = -1\,A$$

PROBLEMS

1. A battery having an (emf) of $9\ V$ delivers $117\ mA$ when connected to a $72\ \Omega$ load. Determine the internal resistance of the battery.

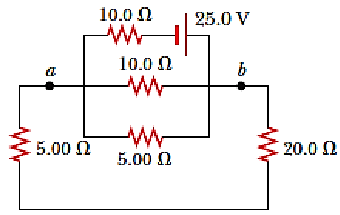
2. A $4\ \Omega$ resistor, an $8\ \Omega$ resistor, and a $12\ \Omega$ resistor are connected in series with a $24\ V$ battery. What are (a) the equivalent resistance and (b) the current in each resistor? (c) Repeat for the case in which all three resistors are connected in parallel across the battery.

3. (a) Find the equivalent resistance of this circuit (b) If the total power supplied to the circuit is $4\ W$, find the (emf) of the battery.

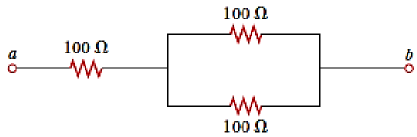


4. A $9\ \Omega$ resistor and a $6\ \Omega$ resistor are connected in series with a power supply. (a) The voltage drops across the $6\ \Omega$ resistor is measured to be $12\ V$. Find the voltage output of the power supply. (b) The two resistors are connected in parallel across a power supply, and the current through the $9\ \Omega$ resistor is found to be $0.25\ A$. Find the voltage setting of the power supply.

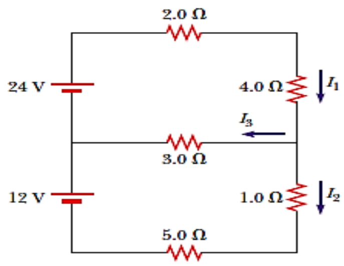
5. Consider the circuit shown in figure. Find (a) the current in the $20\ \Omega$ resistor and (b) the potential difference between points (a) and (b).



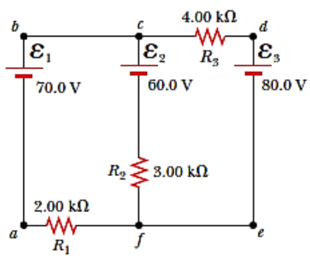
6. Three $100\ \Omega$ resistors are connected as shown in figure. The maximum power that can safely be delivered to any one resistor is $25\ W$. (a) What is the maximum voltage that can be applied to the terminals (a) and (b)? (b) For the voltage determined in part (a), what is the power delivered to each resistor? What is the total power delivered?



7. Calculate each of the unknown currents (I_1), (I_2), and (I_3) for the circuit in the figure.



8. Using Kirchhoff's rules, (a) find the current in each resistor shown in figure and (b) find the potential difference between points (c) and (f).



CHAPTER (7)
MAGNETIC FIELDS

7.1 Introduction

In terms of applications, magnetism is one of the most important fields in physics. Large electromagnets are used to pick up heavy loads. Magnets are used in such devices as meters, motors, and loudspeakers. Magnetic tapes and disks are used routinely in sound- and video-recording equipment and to store computer data. Although digital technology has largely replaced magnetic recording, the industry still depends on the magnets that control (CD) and (DVD) players and computer hard drives; magnets also drive the speaker cones in headphones, (TVs), computers, and telephones. A modern car comes equipped with dozens of magnets because they are required in the motors for engine ignition, automatic window control, sunroof control, and windshield wiper control. Most security alarm systems, doorbells, and automatic door latches employ magnets. Intense magnetic fields are used in magnetic resonance imaging (MRI) devices to explore the human body with better resolution and greater safety than x-rays can provide. Giant superconducting magnets are used in the cyclotrons that guide particles into targets at nearly the speed of light. In short, you are surrounded by magnets. The science of magnetic fields is physics; the application of magnetic fields is engineering.

7.2 Magnetism

Magnetic phenomena were first observed at least 2500 years ago in fragments of magnetized iron ore found near the ancient city of Magnesia (now Manisa, in western Turkey). These fragments were examples of what are now called permanent magnets; you probably have several permanent magnets on your refrigerator door at home.

Permanent magnets were found to exert forces on each other as well as on pieces of iron that were not magnetized. It was discovered that when an iron rod is brought in contact with a natural magnet, the rod also becomes magnetized. When such a rod is floated on water or suspended by a string from its center, it tends to line itself up in a north-south direction. The needle of an ordinary compass is just such a piece of magnetized iron. Before the relationship of magnetic interactions to moving charges was understood, the interactions of permanent magnets and compass needles were described in terms of *magnetic poles.* If a bar- shaped permanent magnet, or bar magnet, is free to rotate, one end points north. This end is called a *North Pole*; the other end is a *South Pole. Opposite poles attract each other, and like poles repel each other* (figure 7.1).

(a) Opposite poles attract. **(b) Like poles repel.**

Figure 7.1: (a) Two bar magnets attract when opposite poles (N and S, or S and N) are next to each other. (b) The bar magnets repel when like poles (N and N, or S and S) are next to each other.

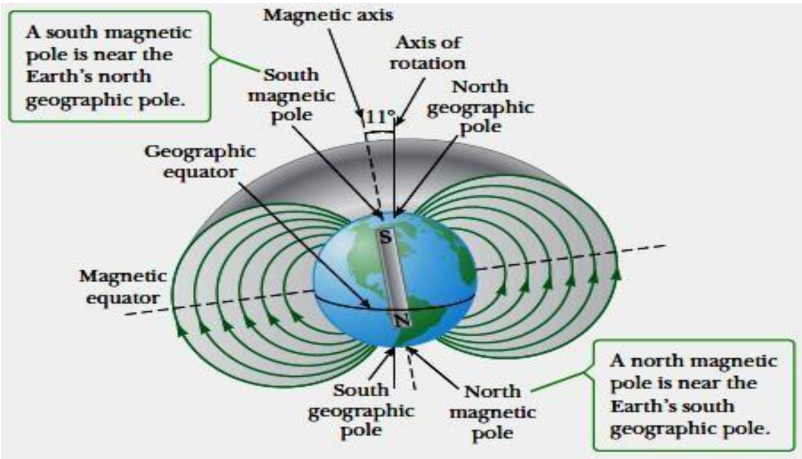
An object that contains iron but is not itself magnetized (that is, it shows no tendency to point north or south) is attracted by *either* pole of a permanent magnet. This is the attraction that acts between a magnet and the unmagnetized steel door of a refrigerator. By analogy to electric interactions, we describe the interactions by saying that a bar magnet sets up a *magnetic field* in the space around it and a second body responds to that field. A compass needle tends to align with the magnetic field at the needle's position.

In 1600, William Gilbert (1540–1603) extended de Maricourt's experiments to a variety of materials. He knew that a compass needle orients in preferred directions, so he suggested that the Earth itself is a large, permanent magnet. In 1750, experimenters used a torsion balance to show that magnetic poles exert attractive or repulsive forces on each other and that these forces vary as the inverse square of the distance between interacting poles. Although the force between two magnetic poles is otherwise similar to the force between two electric charges, electric charges can be isolated (witness the electron and proton), whereas a single magnetic pole has never been isolated. That is, magnetic poles are always found in pairs. All attempts thus far to detect an isolated magnetic pole have been unsuccessful. No matter how many times a permanent magnet is cut in two, each piece always has a north and a south pole.

Magnetism is closely linked with electricity. This relationship was discovered in 1819 when, during a lecture demonstration, Hans Christian Oersted found that an electric current in a wire deflected a nearby compass needle. In the 1820s, further connections between electricity and magnetism were demonstrated independently by Faraday and Joseph Henry (1797–1878). They showed that an electric current can be produced in a circuit either by moving a magnet near the circuit or by changing the current in a nearby circuit. These observations demonstrate that a changing magnetic field creates an electric field. Years later, theoretical work by Maxwell showed that the reverse is also true: a changing electric field creates a magnetic field.

Earth has a magnetic field similar to that of a bar magnet. The north and south poles of a small bar magnet are correctly described as the "north-seeking" and "south-seeking" poles. This description means that if a magnet is used as a compass,

the north pole of the magnet will seek, or point to, a location near the geographic North Pole of Earth. Because unlike poles attract, we can deduce that the geographic North Pole of Earth corresponds to the magnetic south pole and the geographic South Pole of Earth corresponds to the magnetic north pole. Note that the configuration of Earth's magnetic field, pictured in figure 7.2, resembles the field that would be produced if a bar magnet were buried within Earth. The earth's magnetic axis is not quite parallel to its geographic axis (the axis of rotation), so a compass reading deviates somewhat from geographic north. This deviation, which varies with location, is called *magnetic declination* or *magnetic variation.*



Also, the magnetic

Figure 7.2: Earth's magnetic field lines. The lines leading away from the immediate vicinity of the north magnetic pole and entering the vicinity of the south magnetic pole have been left out for clarity.

field is not horizontal at most points on the earth's surface; its angle up or down is called *magnetic inclination.* At the magnetic poles the magnetic field is vertical.

Figure 7.2 is a sketch of the earth's magnetic field. The lines, called *magnetic field lines,* show the direction that a compass would point at each location. The direction of the field at any point can be defined as the direction of the force that the field would exert on a magnetic north pole. In the next section we'll describe a more fundamental way to define the direction and magnitude of a magnetic field.

Labeling Airport Runways: is used as the application of the Earth magnetic field. The magnetic field of Earth is used to label runways at airports according to their direction. A large number is painted on the end of the runway so that it can be read by the pilot of an incoming airplane. This number describes the direction in which the airplane is traveling, expressed as the magnetic heading, in degrees measured clockwise from magnetic north divided by (10). A runway marked (9) would be directed toward the east (90° divided by 10), whereas a runway marked 18 would be directed toward magnetic south.

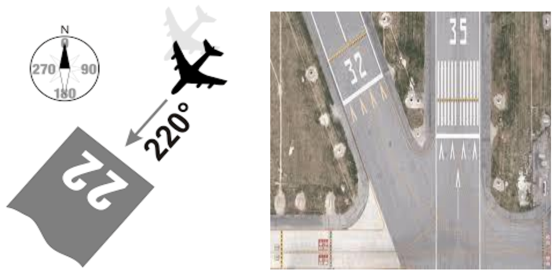


Figure 7.3: Labeling Airport Runways

7.3 Magnetic Field

To introduce the concept of magnetic field properly, let's review our formulation of electric interactions, where we introduced the concept of electric field. We represented electric interactions in two steps:

1. A distribution of electric charge at rest creates an electric field $(\vec{E})$ in the surrounding space.

2. The electric field exerts a force $(\vec{F} = q\vec{E})$ on any other charge that is present in the field.

In similar way the magnetic interactions can be describe as:

1. A moving charge or a current creates a magnetic field $(\vec{B})$ in the surrounding space (in addition to its *electric* field).

2. The magnetic field exerts a force $(\vec{F})$ on any other moving charge or current that is present in the field.

Like electric field, ***magnetic field*** is a vector quantity associated with each point in space. The symbol $(\vec{B})$ is used for magnetic field. At any position the direction of $(\vec{B})$ is defined as the direction in which the north pole of a compass needle tends to point.

The magnetic field $(\vec{B})$ can be define at some point in space in terms of the magnetic force $(\vec{F}_B)$ acting on a test particle with charge (q) moving through the field with velocity $(\vec{v})$. Then we can define the magnitude of the magnetic field according to the following equation:

$$F_B = \pm q\left(\vec{v} \times \vec{B}\right) \Rightarrow B = \frac{F_B}{|q| v sin\theta} \qquad (7-1)$$

The **SI** unit of magnetic field strength is $newton\ per\ coulomb - meter\ per\ second$.

For convenience, this is called the **tesla** (T).

$$\boldsymbol{tesla(T) = (N.s/C.m) = (N/A.m)}$$

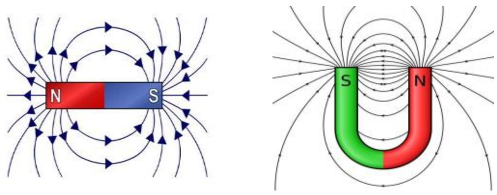
Magnetic field lines

We can represent magnetic fields with field lines, as we did for electric fields.

Similar rules apply:

1. The direction of the tangent to a magnetic field line at any point gives the direction of at that point.

2. The spacing of the lines represents the magnitude of the magnetic field is stronger where the lines are closer together, and conversely.



(a) **(b)**

Figure 7.4: Magnetic field lines for: a) A bar magnet, and b) A horseshoe magnet. which can be visualized as closed loops leaving the north pole of a magnet and entering the south pole of the same magnet.

Figure 7.4a shows how the magnetic field near a bar magnet (a permanent magnet in the shape of a bar) can be represented by magnetic field lines. The lines all pass through the magnet, and they all form closed loops (even those that are not shown closed in the figure). The external magnetic effects of a bar magnet are strongest near its ends, where the field lines are most closely spaced. The (closed) field lines enter one end of magnet and exit the other end. The end of a magnet from which the field lines emerge is called the *north pole* of the magnet; the other end, where field lines enter the magnet, is called the *South Pole.* Because a magnet has two poles, it is said to be a **magnetic dipole.** The magnets we use to fix notes on refrigerators are short bar magnets. Figure 7.4b shows one other common shape for magnets: a *horseshoe magnet.*

Regardless of the shape of the magnets, if we place two of them near each other we find that: *Opposite magnetic poles attract each other, and like magnetic poles repel each other*

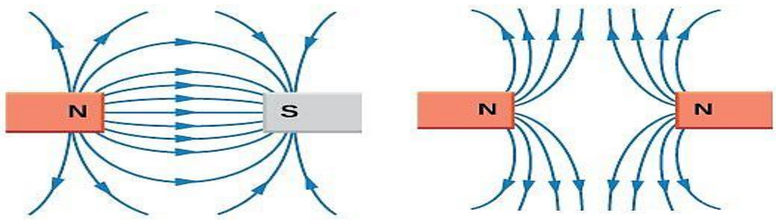
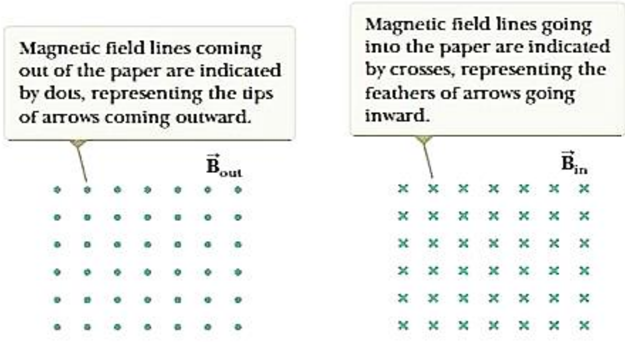


Figure 7.5: Magnetic field lines between unlike poles and between like poles.

To indicate the direction of $(\vec{B})$ in illustrations. If $(\vec{B})$ lies in the plane of the page or is present in a perspective drawing, we use green vectors or green field lines with arrowheads. In no perspective illustrations, we depict a magnetic field perpendicular to and directed out of the page with a series of green dots, which represent the tips of arrows coming toward you (as shown in figure (7.6a). In this case, the field is labeled $(\vec{B})$ out. If $(\vec{B})$ is directed perpendicularly into the page, we use green crosses, which represent the feathered tails of arrows fired away from you, as in figure (7.6b). In this case, the field is labeled $(\vec{B})$ in, where the subscript "in" indicates "into the page." The same notation with crosses and dots is also used for other quantities that might be perpendicular to the page such as forces and current directions.



(a) **(b)**

Figure 7.6: Representations of magnetic field lines perpendicular to the page.

7.4 Magnetic Forces on Moving Charges

There are four key characteristics of the magnetic force on a moving charge in a magnetic field:

1. The magnitude of the force is proportional to the magnitude of the charge.

2. The magnitude of the force is also proportional to the magnitude, or "strength," of the field.

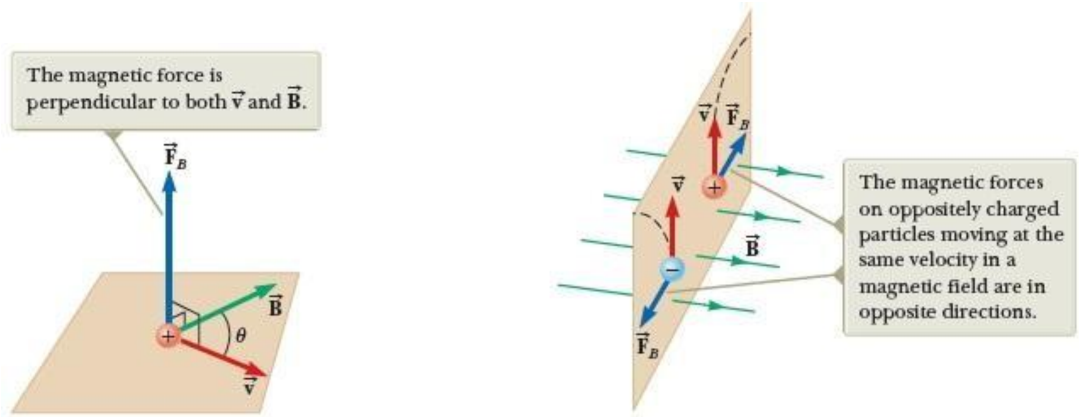
3. The magnetic force depends on the particle's velocity. This is quite different from the electric-field force, which is the same whether the charge is moving or not. A charged particle at rest experiences *no* magnetic force.

4. The magnetic force does not have the same direction as the magnetic field but instead is always perpendicular to both $(\vec{B})$ and velocity $(\vec{v})$.

5. Despite this complicated behavior, these observations can be summarized in a compact way by writing the magnetic force in the form:

$$\overrightarrow{F_B} = \pm q(\vec{v} \times \vec{B}) \qquad (7-2)$$

Figure 7.7 shows these relationships. The direction of is force always perpendicular to the plane containing $(\vec{v})$ and $(\vec{B})$.



(a) (b)

Figure 7.7: (a) The direction of the magnetic force acting on a charged particle moving with a velocity in the presence of a magnetic field. (b) Magnetic forces on positive and negative charges. The dashed lines show the paths of the particles.

The magnitude of the magnetic force on a charged particle is given by:

$$F_B = \pm q\left(\vec{v} \times \vec{B}\right) \Longrightarrow F_B = |q|vB\, sin\theta \qquad (7-3)$$

where $||q||$ is the magnitude of the charge and (θ) is the angle measured from the direction of (v) to the direction of (B) as shown in the figure 7.7. The magnitude (F_B) of the force is found to be proportional to the component of perpendicular to the field; when that component is zero (i.e. when (v) and (B) are parallel $(\theta = 0)$ or antiparallel $(\theta = \pi)$ the force is zero.

To determine the direction of the force, we employ right-hand rule. Figure 7.9 reviews two right-hand rules for determining the direction of the cross product $\left(\vec{v} \times \vec{B}\right)$ and determining the direction of (F_B). The rule in figure 7.9.a depends on our right- hand rule for the cross product:

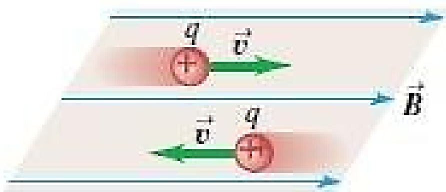
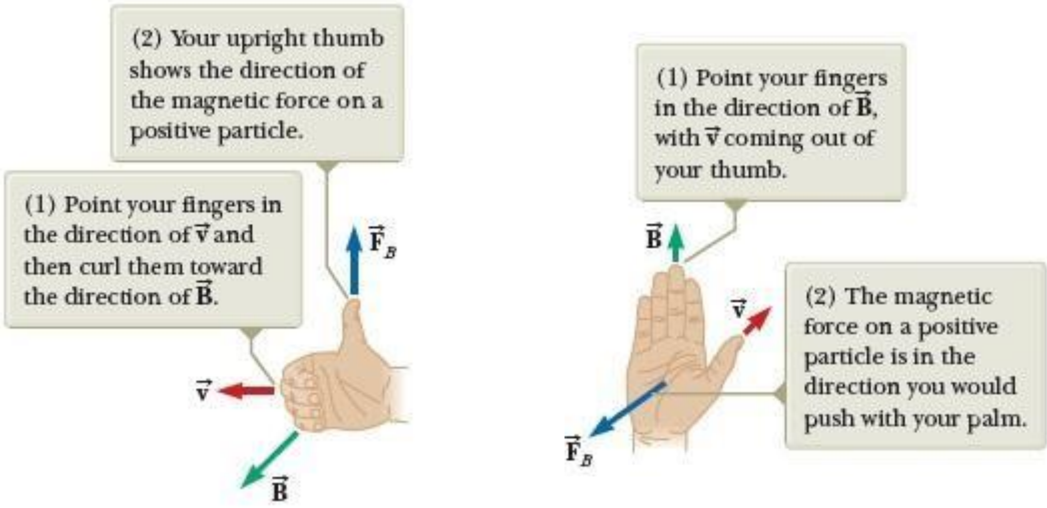


Fig. 7.8; charge moving parallel or antiparallel to a magnetic field

1. Point the fingers of your right hand in the direction of the velocity.

2. Curl the fingers in the direction of the magnetic field (B), moving through the smallest angle.

3. Your thumb is now pointing in the direction of the magnetic force (F_B) exerted on a positive charge.



(a) **(b)**

Figure 7.9: Two right-hand rules for determining the direction of the magnetic force acting on a particle with charge moving with a velocity in a magnetic field. (a) In this rule, the magnetic force is in the direction in which your thumb points. (b) In this rule, the magnetic force is in the direction of your palm, as if you are pushing the particle with your hand.

An alternative rule is shown in figure 7.9.b. Here the thumb points in the direction of $v$and the extended fingers in the direction of (B). Now, the force (F_B)on a positive charge extends outward from the palm. The advantage of this rule is that the force on the charge is in the direction you would push on something with your hand:

outward from your palm. The force on a negative charge is in the opposite direction. You can use either of these two right-hand rules.

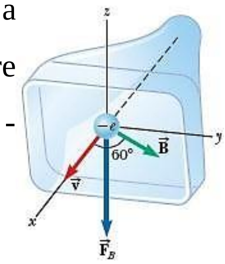
Let's compare the important differences between the electric and magnetic versions of the particle in a field model:

1. The electric force acts along the direction of the electric field, whereas the magnetic force acts perpendicular to the magnetic field.

2. The electric force acts on a charged particle regardless of whether the particle is moving, whereas the magnetic force acts on a charged particle only when the particle is in motion.

3. The electric force does work in displacing a charged particle, whereas the magnetic force associated with a steady magnetic field does no work when a particle is displaced because the force is perpendicular to the displacement.

Example (7.1):



An electron in an old-style television picture tube moves toward the front of the tube with a speed of $\mathbf{8 \times 10^6 \ m/s}$ along the *x* axis. Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude $\mathbf{0.025 \ T}$, directed at an angle of $\mathbf{60°}$ to the *x* -axis and lying in the $(\boldsymbol{x - y})$ plane. Calculate the magnetic force on the electron.

Solution:

By using equation (7-3);

$$F_B = |q|vB \ sin\theta = (1.6 \times 10^{-19} \ C)(8 \times 10^6 \ m/s)(0.025 \ T)(sin \ 60^o) = 2.8 \times 10^{-14} \ N$$

7.5 Motion of a Charged Particle in a Uniform Magnetic Field

When a charged particle moves in a magnetic field, it is acted on by the magnetic force, and the motion is determined by Newton's laws. Figure 7.10 shows a simple example. A particle with positive charge (q) is at point (O), moving with velocity (v) in a uniform magnetic field $(\vec{B})$ directed into the plane of the figure. The vectors $(\vec{v})$ and $(\vec{B})$ are perpendicular to each other, so the magnetic $\vec{F_B} = |q|(\vec{v} \times \vec{B})$ force has magnitude $(F_B = qvB)$ and a direction as shown in the figure. The force is always perpendicular to (v) so it cannot change the magnitude of the velocity, only its direction. To put it differently, the magnetic force never has a component parallel to the particle's motion, so the magnetic force can never do work on the particle. This is true even if the magnetic field is not uniform.

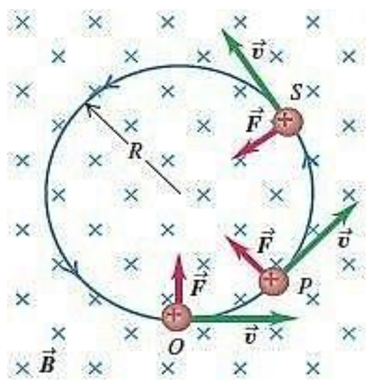


Figure 7.10: when the velocity of a charged particle is perpendicular to a uniform magnetic field, the particle moves in a circular path in a plane perpendicular to (B).

Using this principle, we see that in the situation shown in figure 7.10 the magnitudes of both (F_B) and (v) are constant. At points such as (P) and (S) the directions of force and velocity have changed as shown, but their magnitudes are the same. The particle therefore moves under the influence of a constant-magnitude force that is always at right angles to the velocity of the particle. As seen in figure 7.10, that the particle's path is a ***circle***, traced out with constant speed (v). The centripetal acceleration is (v^2/R) and only the magnetic force acts, so from Newton's second law:

$$F_B = F_c$$

$$qvB = m\frac{v^2}{R} \qquad (7-4)$$

This expression leads to the following equation for the radius of the circular path:

$$R = \frac{mv}{|q|\ B} \qquad (7-5)$$

This equation says that the radius of the path is proportional to the momentum (mv) of the particle and is inversely proportional to the charge and the magnetic field. Equation (7-5) is often called the ***cyclotron equation*** because it's used in the design of these instruments. The angular speed of the particle is:

$$\omega = \frac{v}{R} = \frac{|q|B}{m} \qquad (7-6)$$

The period (T) (the time of complete cycle) is equal to the circumference $((2\pi R))$ divided by the linear speed (v):

$$T = \frac{2\pi R}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{|q|B} \qquad (7-7)$$

The frequency (f) (the number of revolutions per unit time) is reciprocal of (T):

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \qquad (7-8)$$

Example 7.2:

A magnetron in a microwave oven emits electromagnetic waves with frequency $(\boldsymbol{f = 2450\ MHz})$. What magnetic field strength is required for electrons to move in circular paths with this frequency?

Solution:

$$f = \frac{|q|B}{2\pi m} \Longrightarrow B = \frac{2\pi mf}{|q|} = \frac{2\pi(9.11\times10^{-31}kg)(2450\times10^{6}\ Hz)}{(1.6\times10^{-19}\ C)} = 0.088\ T$$

Example 7.3:

A proton is moving in a circular orbit of radius $(\boldsymbol{14\ cm})$ in a uniform $(\boldsymbol{0.35\ T})$ magnetic field perpendicular to the velocity of the proton. Find the speed of the proton.

Solution:

$$|q|vB = m\frac{v^2}{r} \Longrightarrow v = \frac{|q|rB}{m_p} = \frac{(1.6\times10^{-19}\ C)\ (0.14\ m)\ (0.35\ T)}{(1.67\times10^{-27}\ kg)} = 4.7\times10^{6}\ m/s$$

7.6 The Hall Effect

When a current-carrying conductor is placed in a magnetic field, a potential difference is generated in a direction perpendicular to both the current and the magnetic field. This phenomenon, first observed by Edwin Hall (1855–1938) in 1879, is known as the ***Hall effect.*** The arrangement for observing the Hall effect consists of a flat conductor carrying a current (I) in the x-direction as shown in Figure 7.11 uniform magnetic field $(\vec{B})$ is applied in the *y*-direction. If the charge carriers are electrons moving in the negative *x*-direction with drift velocity (v_d). they experience an upward magnetic force $\left(\overrightarrow{F_B} = |q|(\vec{v} \times \vec{B})\right)$ are deflected upward, and accumulate at the upper edge of the flat conductor, leaving an excess of positive charge at the lower edge (figure 7.12a) This accumulation of charge at the edges establishes an electric field in the conductor and increases until the electric force on carriers remaining in the bulk of the conductor balances the magnetic force acting on the carriers.

The electrons can now be described by the particle in equilibrium model, and they are no longer deflected upward. A sensitive voltmeter connected across the sample as shown in figure 7.12 can measure the potential difference, known as the **Hall voltage** (ΔV_H), generated across the conductor If the charge carriers are positive and hence move in the positive *x* direction (for rightward current) as shown in figures 7.11 and 7.12b, they also experience an upward magnetic force $\left(|q|(\vec{v} \times \vec{B})\right)$, which produces a buildup of positive charge on the upper edge and leaves an excess of negative charge on the lower edge. Hence, the sign of the Hall voltage generated in the sample is opposite the sign of the Hall voltage resulting from the deflection of electrons. The sign of the charge carriers can therefore be determined from measuring the polarity of the Hall voltage

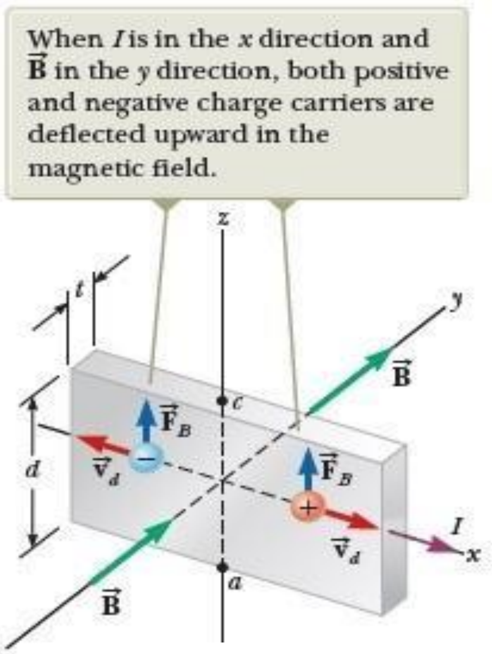


Figure 7.11: To observe the Hall effect, a magnetic field is applied to a current-carrying conductor.

$$F_B = F_e$$

$$qv_dB = qE_H$$

$$\Rightarrow E_H = v_dB$$

If (d) is the width of the conductor, the Hall voltage is:

$$\Delta V_H = E_H d = v_d B d \qquad (7-9)$$

Therefore, the measured Hall voltage gives a value for the drift speed of the charge carriers if (d) and (B) are known.

We can obtain the charge-carrier density (n) by measuring the current in the sample. The drift speed can be expressed as:

$$\Rightarrow v_d = \frac{j}{nq} = \frac{i}{nqA} \qquad (7.10)$$

where (A) is the cross-sectional area $(area = (width)(thickness))$ of the conductor. Then, substituting equation (7-10) into equation (7-9) we obtain

$$\Rightarrow \Delta V_H = \frac{iBd}{nqA} \qquad (7.11)$$

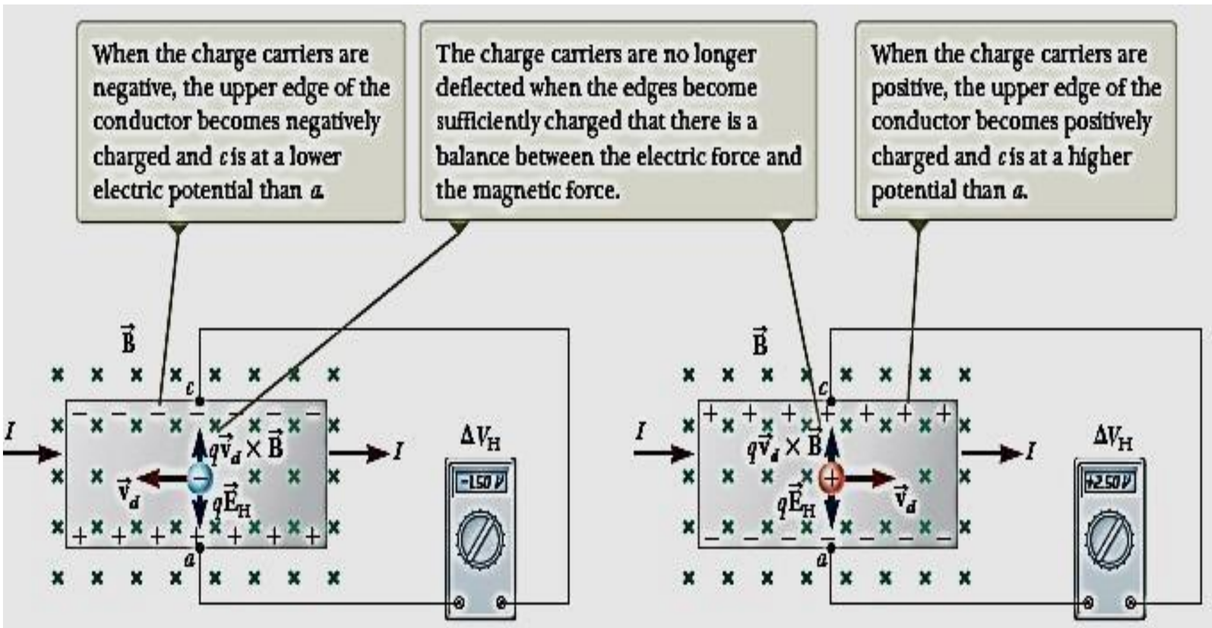


Figure 7.12; In deriving an expression for the Hall voltage, first note that the magnetic force exerted on the carriers has magnitude $(qv_d B)$. In equilibrium, this force is balanced by the electric force (qE_H), where (E_H) is the magnitude of the electric field due to the charge separation (sometimes referred to as the *Hall field*).

Because $(A = d\ t)$, where(t) is the thickness of the conductor, we can also express equation (7-11) as:

$$\Delta V_H = \frac{iB}{nqt} \qquad (7-12)$$

Drift Speed. It is also possible to use the Hall effect to measure directly the drift speed (v_d) of the charge carriers, which you may recall is of the order of centimeters per hour. In this clever experiment, the metal strip is moved mechanically through the magnetic field in a direction opposite that of the drift velocity of the charge carriers. The speed of the moving strip is then adjusted until the Hall potential difference vanishes. At this condition, with no Hall effect, the velocity of the charge carriers *with respect to the laboratory frame* must be zero, so the velocity of the strip must be equal in magnitude but opposite the direction of the velocity of the negative charge carriers.

Example 7.4:

A strip of copper $\mathbf{150}$ μ$\boldsymbol{m}$ thick is placed in magnetic field of magnitude $\mathbf{0.65}\ \boldsymbol{T}$, and a current $\mathbf{23}\ \boldsymbol{A}$ is sent through the strip. What Hall Potential difference will appear across the width of the trip? where the number of charge carriers per unit volume for copper is $\mathbf{8.47 \times 10^{28}\ electrons/m^3}$.

Solution:

$$V_H = \frac{iB}{nqt} = \frac{(23\ A)(0.65\ T)}{(8.47 \times 10^{28}\ electrons/m^3)(1.6 \times 10^{-19}\ C)(150 \times 10^{-6}\ m)} = 7.4 \times 10^{-6}\ V$$
$$= 7.4\ \mu V$$

7.7 Magnetic Force on a Current-Carrying Wire

We have already seen (in connection with the Hall effect) that a magnetic field exerts a sideways force on electrons moving in a wire. This force must then be transmitted to the wire itself, because the conduction electrons cannot escape sideways out of the wire.

In figure 7.13*a*, a vertical wire, carrying no current and fixed in place at both ends, extends through the gap between the vertical pole faces of a magnet. The magnetic field between the faces is directed outward from the page.

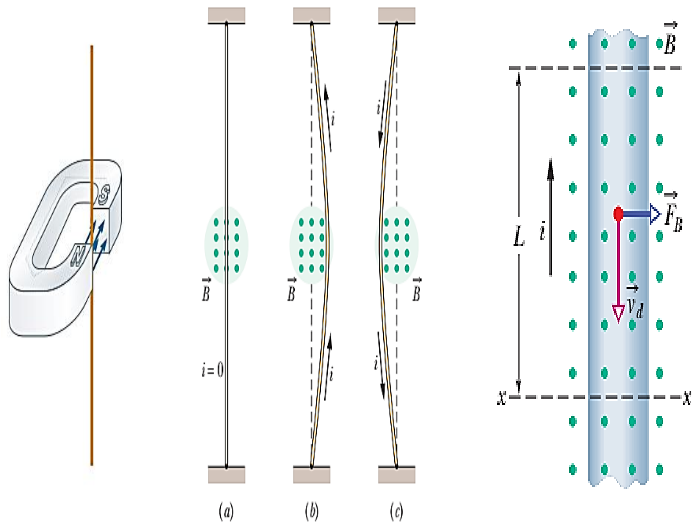


Figure 7.13: A flexible wire passes between the pole faces of a magnet (a) Without current in the wire, the wire is straight. (b) With upward current, the wire is deflected rightward. (c) With downward current, the deflection is leftward.

In figure 7.13*b*, a current is sent upward through the wire; the wire deflects to the right. In figure 7.13*c*, we reverse the direction of the current and the wire deflects to the left.

Figure 7.14 shows what happens inside the wire of figure 7.13*b*.We see one of the conduction electrons, drifting downward with an assumed drift speed (v_d).

Equation (7-3), in which we must put $\theta = 90^{\circ}$, tells us that a force (F_B) of magnitude (ev_dB) must act on each such electron. From equation (7-2) we see that this force must be directed to the right. We expect then that the wire as a whole will experience a force to the right, in agreement with figure 7.13*b*.

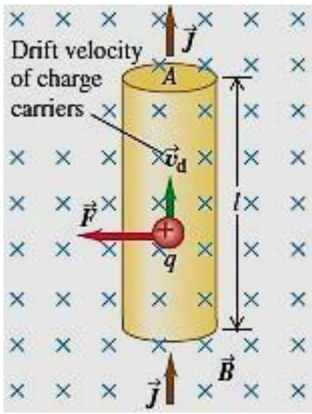


Figure 7.14: Forces on a moving positive charge in a current-carrying wire.

If, in figure 7.14, we were to reverse *either* the direction of the magnetic field *or* the direction of the current, the force on the wire would reverse, being directed now to the left. Note too that it does not matter whether we consider negative charges drifting downward in the wire (the actual case) or positive charges drifting upward. The direction of the deflecting force on the wire is the same. We are safe then in dealing with a current of positive charge, as we usually do in dealing with circuit.

The magnetic Force that act on wire.

Consider a length (L) of the wire in figure 7.13d. A magnetic field that emerges from the plane of the page causes the electrons and the wire to be deflected to the right.

1. The magnetic force act on charge (q) is given by

$$F_B = qv_dB\ sin\theta \quad if\ \ \theta = 90^o$$

$$\therefore F_B = qv_dB$$

2. Consider a conductor wire of length (L), the conduction electrons travel this length in time (t) with drift speed (v_d).

$$L = v_dt$$

$$\Rightarrow t = L/v_d$$

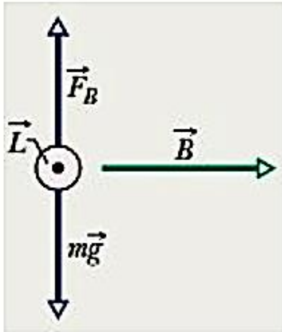
$$q = it = \frac{iL}{v_d}$$

$$\Rightarrow F_B = q\ v_dB\ sin\theta = \frac{iL}{v_d} v_dB\ sin\theta$$

$$\Rightarrow F_B = BiL\sin\theta \qquad (7-14)$$

This equation gives the magnetic force that acts on a length (L) of straight wire carrying a current $\boldsymbol{i}$ and immersed in a uniform normal magnetic field (B).

Example 7.4:



A straight, horizontal length of copper wire has a current $\boldsymbol{i = 28\ A}$ through it. What are the magnitude and direction of the minimum magnetic field $(\boldsymbol{B})$ needed to suspend the wire, that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is $\boldsymbol{46.6\ g/m}$.

Solution:

$$F_B = BiL \quad that\ equal \quad F_g = mg$$

$$B = \frac{mg}{i\,L} = \left(\frac{m}{L}\right)\left(\frac{g}{i}\right) = (46.6 \times 10^{-3}\ kg/m)\frac{(9.8\ m/s^2)}{(28\ A)} = 1.6 \times 10^{-2}\ T$$

7.8 Magnetic Flux and Gauss's Law for Magnetics

We define the *magnetic flux* through a surface just as we defined electric flux in connection with Gauss's law in chapter (5). We can divide any surface into elements

of area (figure 7.15). For each element we determine $(B_\perp)$, the component of $(\vec{B})$ normal to the surface at the position of that element.

From the figure, $B_\perp = B\cos\phi$ where (ϕ) is the angle between the direction of $(\vec{B})$ and a line perpendicular to the surface. (Be careful not to confuse (ϕ) with (Φ_B). In general, this component varies from point to point on the surface. We define the magnetic flux $(d\Phi_B)$ through this area as:

$$d\Phi_B = B_\perp dA = B\cos\phi = \vec{B}.\overrightarrow{dA}$$

The total magnetic flux through the surface is the sum of the contributions from the individual area elements:

$$\Phi_B = \int B_\perp dA = \int B\cos\phi = \int \vec{B}.\overrightarrow{dA} \qquad (7-16)$$

Magnetic flux is a *scalar* quantity. If $(\vec{B})$ is uniform over a plane surface with total area (A), then $(B_\perp)$ and (ϕ) are the same at all points on the surface,

$$\Phi_B = B_\perp dA = B\cos\phi \qquad (6-17)$$

The **SI** unit of magnetic flux is equal to the unit of magnetic field $(1T)$ times the unit of area $(1m^2)$. This unit is called the ***weber*** $(1\,Wb)$, in honor of the German physicist Wilhelm Weber $(1804-1891)$:

$$1\,Wb = Tm^2 = Nm/A$$

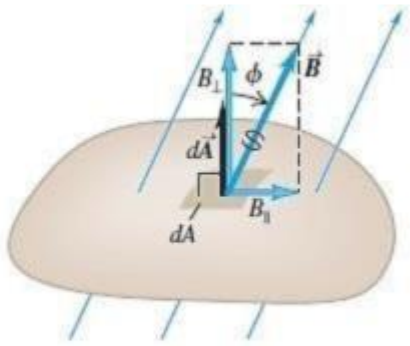


Figure 7.15: The magnetic flux through an area element is defined to be $(d\Phi_B = B_\perp dA)$.

In Gauss's law the total *electric* flux through a closed surface is proportional to the total electric charge enclosed by the surface. For example, if the closed surface encloses an electric dipole, the total electric flux is zero because the total charge is zero. By analogy, if there were such a thing as a single magnetic charge (magnetic monopole), the total *magnetic* flux through a closed surface would be proportional to the total magnetic charge enclosed. But we have mentioned that no magnetic monopole has ever been observed, despite intensive searches.

"The total magnetic flux through a closed surface is always zero".

Symbolically,

$$\Phi_B = \oint \vec{B}.\overrightarrow{dA} = 0 \qquad (7-18)$$

This equation is sometimes called ***Gauss's law for magnetism***.

PROBLEMS

1. In a $\mathbf{1.25\ T}$ magnetic field directed vertically upward, a particle having a charge of magnitude $\mathbf{8.5\ \mu C}$ and initially moving northward at $\mathbf{4.75\ Km/s}$ is deflected toward the east. (a) What is the sign of the charge of this particle? Make a sketch to illustrate how you found your answer. (b) Find the magnetic force on the particle.

2. An electron experiences a magnetic force of magnitude $\mathbf{4.6 \times 10^{15}\ N}$ when moving at an angle of $\mathbf{60°}$ with respect to a magnetic field of magnitude $\mathbf{3.5 \times 10^{3}\ T}$. Find the speed of the electron.

3. A horizontal rectangular surface has dimensions $\mathbf{2.8\ cm}$ by $\mathbf{3.2\ cm}$ and is in a uniform magnetic field that is directed at an angle of $\mathbf{30°}$ above the horizontal. What must the magnitude of the magnetic field be in order to produce a flux of $\mathbf{4.2 \times 10^{-4}\ Wb}$ through the surface?

4. An open plastic soda bottle with an opening diameter of $\mathbf{2.5\ cm}$ is placed on a table. A uniform $\mathbf{1.75\ T}$ magnetic field directed upward and oriented $\mathbf{25°}$ from vertical encompasses the bottle. What is the total magnetic flux through the plastic of the soda bottle?

5. An electron in the beam of a $(\mathbf{TV})$ picture tube is accelerated by a potential difference of $\mathbf{2\ kV}$. Then it passes through a region of transverse magnetic field, where it moves in a circular arc with radius $\mathbf{0.18\ m}$. What is the magnitude of the field?

6. A horizontal power line carries a current of $\mathbf{5000\ A}$ from south to north. Earth's magnetic field ($\mathbf{60\ \mu T}$) is directed toward the north and inclined downward at $\mathbf{70°}$ to the horizontal. Find the (a) magnitude and (b) direction of the magnetic force on $\mathbf{100\ m}$ of the line due to Earth's field.

7. An alpha particle can be produced in certain radioactive decays of nuclei and consists of two protons and two neutrons. The particle has a charge of $\mathbf{q = +2e}$ and a mass of $\mathbf{4\ u}$, where $\mathbf{u}$ is the atomic mass unit, with $\mathbf{1\ u = 1.661 \times 10^{27}\ kg}$. Suppose an alpha particle travels in a circular path of radius $\mathbf{4.5\ cm}$ in a uniform magnetic field with $\mathbf{B = 1.2\ T}$. Calculate (a) its speed, (b) its period of revolution, (c) its kinetic energy, and (d) the potential difference through which it would have to be accelerated to achieve this energy.

8. A Hall-effect probe operates with a $\mathbf{120\ mA}$ current. When the probe is placed in a uniform magnetic field of magnitude $\mathbf{0.08\ T}$, it produces a Hall voltage of $\mathbf{0.7\ \mu V}$. (a) When it is used to measure an unknown magnetic field, the Hall voltage is $\mathbf{0.33\ \mu V}$. What is the magnitude of the unknown field?

(b) The thickness of the probe in the direction of $(\vec{B})$ is $\mathbf{2\ mm}$. Find the density of the charge carriers, each of which has charge of magnitude e.

9. In an experiment designed to measure the Earth's magnetic field using the Hall effect, a copper bar $\mathbf{0.5\ cm}$ thick is positioned along an east–west direction. Assume the plane of the bar is rotated to be perpendicular to the direction of $(\vec{\mathrm{B}})$ and $\boldsymbol{n = 8.46 \times 10^{28}\ electrons/m^3}$**.** If a current of $\mathbf{8\ A}$ in the conductor results in a Hall voltage of $\mathbf{5.1 \times 10^{12}\ V}$, what is the magnitude of the Earth's magnetic field at this location?

10. A straight wire carrying a $\mathbf{3\ A}$ current is placed in a uniform magnetic field of magnitude $\mathbf{0.28\ T}$ directed perpendicular to the wire. (a) Find the magnitude of the magnetic force on a section of the wire having a length of $\mathbf{14\ cm}$. (b) Explain why you can't determine the direction of the magnetic force from the information given in the problem.