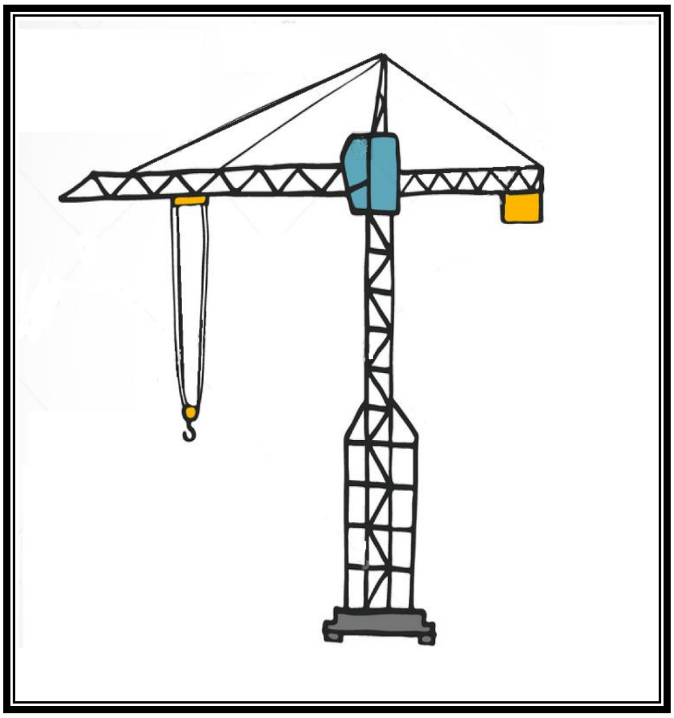
H. T. I.

THE HIGHER TECHNOLOGICAL INSTITUTE – TENTH OF RAMADAN CITY

Course Code: ENG 001 **Course Name: Engineering Mechanics 1**

Lecture notes in

ENGINEERING MECHANICS 1



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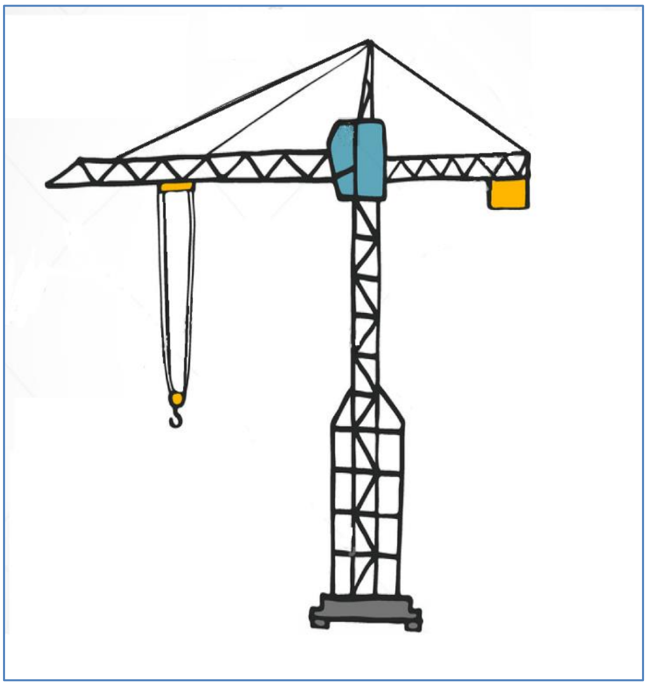

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Introduction

The physical science of mechanics is concerned with the condition of rest or motion of things when forces are acting on them. No subject is more important to engineering applications than mechanics. Current studies and innovations in the areas of machine constructions, rocket and spaceship designs, automated control, engine performance, fluid flow, electrical machines and equipment, as well as molecular, atomic, and subatomic behavior, are heavily reliant on the fundamentals of mechanics. The work in these and many other domains requires a solid comprehension of this topic.

In general, this field may be separated into three branches: fluid mechanics, deformable body mechanics, and rigid-body mechanics (bodies that do not change shape). As studying rigid-body mechanics is a prerequisite for learning about the mechanics of deformable bodies and fluids, it will be covered in this book. Rigid-body mechanics is also necessary for the design and analysis of many different structural member types, mechanical parts, and electrical devices that are used in engineering.

المقدمة

يهتم علم الفيزياء الميكانيكي بحالة سكون أو حركة الأشياء عندما تؤثر عليها قوى. لا يوجد موضوع أكثر أهمية للتطبيقات الهندسية من الميكانيكا. تعتمد الدراسات والابتكارات الحالية في مجالات إنشاءات الآلات، وتصميمات الصواريخ والسفن الفضائية، والتحكم الآلي، وأداء المحرك، وتدفق السوائل، والآلات والمعدات الكهربائية، وكذلك السلوك الجزيئي والذري ودون الذري، بشكل كبير على أساسيات الميكانيكا. يتطلب العمل في هذه المجالات والعديد من المجالات الأخرى فهمًا قويًا لهذا الموضوع.

بشكل عام، يمكن تقسيم هذا المجال إلى ثلاثة فروع: ميكانيكا الموائع، وميكانيكا الأجسام المشوهة، وميكانيكا الأجسام الصلبة (الأجسام التي لا يتغير شكلها). وبما أن دراسة ميكانيكا الأجسام الصلبة شرط أساسي للتعرف على ميكانيكا الأجسام والسوائل القابلة للتشوه، فسوف يتم تناولها في هذا الكتاب. تعد ميكانيكا الأجسام الصلبة ضرورية أيضًا لتصميم وتحليل العديد من أنواع الأعضاء الهيكلية المختلفة والأجزاء الميكانيكية والأجهزة الكهربائية المستخدمة في الهندسة.

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CHAPTER (1)

GENERAL PRINCIPLES

1. 1 Introduction

Mechanics is a branch of the physical sciences which deals with the state of rest or motion of bodies under the action of forces. There is no subject plays a greater role in engineering applications than mechanics. Modern researches and development in the fields of structures of machines, rockets and spacecraft designs, automatic control, engine performance, fluid flow, electrical machines and apparatus, and also molecular, atomic, and subatomic behaviour are highly dependent upon the basic principles of mechanics. A good understanding of this subject is very important for the work in these and many other fields.

In general, this subject can be subdivided into three branches: rigid-body mechanics (bodies that have no change in shape), deformable-body mechanics (bodies that have change in shape), and fluid mechanics. In this book we will study rigid-body mechanics since it is a basic requirement for the study of the mechanics of deformable bodies and the mechanics of fluids. Furthermore, rigid-body mechanics is essential for the design and analysis of many types of structural members, mechanical components, or electrical devices encountered in engineering.

Rigid-body mechanics is divided into two areas: statics and dynamics. Statics deals with the equilibrium of bodies. The bodies, those are either at rest or move with a constant velocity, whereas dynamics is concerned with the accelerated motion of bodies. We can consider statics as a special case of dynamics, in which the acceleration is zero; however, statics deserves separate treatment in engineering education since many objects are designed with the intention that they remain in equilibrium.

1.2 Fundamental concepts

Before we begin our study of engineering mechanics, it is important to understand the meaning of certain fundamental concepts and principles. The following quantities are used throughout mechanics.

Length. The length is used to locate the position of a point in space and there by describe the size of a physical system. Once a standard unit of length is defined, one can then use it to define distances and geometric properties of a body as multiples of this unit.

Time. The Time is the measure of the succession of events. Although the principles of statics are time independent, this quantity plays an important role in the study of dynamics.

Mass. The Mass is a measure of a quantity of matter that is used to compare the action of one body with that of another. It is a measure of the inertia of a body which provides a measure of the resistance of matter to a change in velocity.

Force. In general, force is considered as a “push” or “pull” exerted by one body on another. This interaction can occur when there is direct contact between the bodies, such as a person pushing on a wall, or it can occur through a distance when the bodies are physically separated. Examples of the latter type include gravitational, electrical, and magnetic forces. In any case, a force is completely characterized by its magnitude, direction, and point of application.

Particle. A particle has a mass, but a size that can be neglected. For example, the size of the earth is insignificant compared to the size of its orbit, and therefore the earth can be modeled as a particle when studying its orbital motion. When a body is idealized as a particle, the geometry of the body will not be involved in the analysis of the problem.

Rigid Body. A rigid body can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one two another, both before and after applying a load. This model is important because the body's shape does not change when a load is applied, and so we do not have to consider the type of a body material.

In most cases the actual deformations occurring in structures, machines, and mechanisms are relatively small, and the rigid-body assumption is suitable for analysis.

Newton's Three Laws of Motion. Engineering mechanics is formulated on the basis of Newton's three laws of motion, the validity of which is based on experimental observation. They may be stated as follows.

First Law. A particle originally at rest, or moving in a straight line with constant velocity, will remain in this state if the particle is not subjected to an unbalanced force, Fig. 1.1.

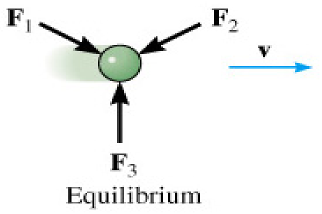


Fig. 1.1

Second Law. A particle acted upon by an unbalanced force **F** experiences an acceleration **a** that has the same direction as the force and a magnitude that is directly proportional to the force, Fig. 1.2. If **F** is applied to a particle of mass m, this law may be expressed mathematically as:

$$\mathbf{F} = m\mathbf{a} \tag{1.1}$$

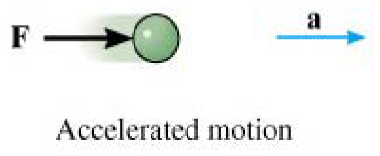


Fig. 1.2

Third Law. The mutual forces of action and reaction between two particles are equal, opposite, and collinear, Fig. 1.3.

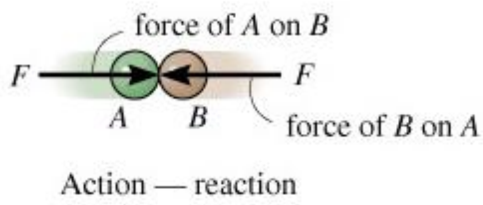


Fig. 1.3

Newton's Law of Gravitational Attraction. Shortly after formulating his three laws of motion, Newton postulated a law governing the gravitational attraction between any two particles. Stated mathematically,

$$F = G\,\frac{m_1 m_2}{r^2} \tag{1.2}$$

where

F = force of gravitation between the two particles

G = universal constant of gravitation; according to experimental evidence,

$G = 66.73(10^{-12})\ m^3/(kg\ .\ s^2)$

m_1, m_2 = mass of each of the two particles

r = distance between the two particles

Weight. According to Eq. 1.2, any two particles or bodies have mutual attractive (gravitational) force acting between them. In the case of a particle located at or near the surface of the earth, however, the only gravitational force having any sizable magnitude is that between the earth and the particle. Consequently, this force, termed the weight, will be the only gravitational force considered in our study of mechanics. From Eq. 1.2, we can develop an approximate expression for finding the weight W of a particle having a mass m_1 = m. If we assume the earth to be a non-rotating sphere of constant density and having a mass $m_2 = M_e$, then if **r** is the distance between the earth's center and the particle, we have

$$W = G\,\frac{mMe}{r^2}$$

Letting $g = G\,M_e / r^2$ yields

$$W = mg \qquad (1.3)$$

By comparison with **F** = m**a**, we can see that **g** is the acceleration due to gravity. Since it depends on r, then the weight of a body is not an absolute quantity. Instead, its magnitude is determined from where the measurement was made. For most engineering calculations, however, **g** is determined at sea level, which is considered the "standard location."

1.3 Units of Measurement

SI Units. The international system of units is a modern version of metric system, which has received world-wide recognition. As shown in the given table, the SI system defines length in meters (m), time in seconds (s), and mass in kilograms (kg). The unit of force, called a Newton (N), is derived from **F** = m**a**. Thus, 1 Newton is equal to a force required to give 1 kilogram of mass an acceleration of 1 m/s^2 (1 N = 1 $kg.m/s^2$). The weight of a body located at the "standard location" is determined in Newton according to equation W = mg, where g = 9.81 m/s^2 is the gravitational acceleration. Therefore, a body of mass 1 kg has a weight of 9.81 N.

U.S. Customary. In the U.S. Customary system of units (FPS), length is measured in feet (ft), time in seconds (s), and force in pounds (lb), In this system, the unit of mass, called a slug, is derived from **F** = m**a**. Hence,1 slug is equal to the amount of matter accelerated at 1 ft/s^2 when acted upon by a force of 1 lb (1 slug = 1 $lb.s^2/ft$). To determine the mass of a body having a weight in pounds, we must apply equation **W = mg**, where g = 32.2 ft/s^2, at the "standard location". So, a body weighing 32.2 Ib has a mass of 1 slug.

Systems of Units

Name	SI	FPS
Length	meter (m)	Foot (ft)
Time	second (s)	second (s)
Mass	kilogram (kg)	Slug ($Ib.s^2/ft$)
Force	Newton (N)	pound (Ib)

CHAPTER (2)

FORCE VECTOR

2.1 Scalars and Vectors

Scalar. A scalar is any positive or negative physical quantity that can be completely specified by its magnitude. Examples of scalar quantities include length, mass, time, volume, and energy.

Vector. A vector is any physical quantity that requires both a magnitude and a direction for its complete description. Examples of vectors encountered in statics are force, position, and moment; other vectors are displacement, velocity and acceleration. A vector quantity will be denoted by a capital letter with an arrow; e.g. $\vec{\mathbf{A}}, \vec{\mathbf{B}}, \vec{\mathbf{F}}, \vec{\mathbf{V}}$...... etc. The magnitude of the vector is denoted by **A, B, F, V…..**etc. A vector quantity $\vec{\mathbf{V}}$ is represented by a straight line Fig. 2.1, having the direction of the vector (angle θ between the vector and certain axis) and having an arrow head to indicate the sense. The length of the directed line segment represents, to some convenient scale, the magnitude of the vector $\vec{\mathbf{V}}$.

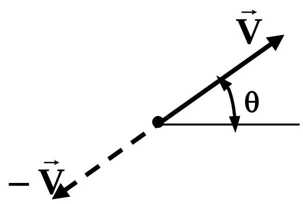


Fig. 2.1

2.2. Vector Addition

Vectors may be added and subtracted according to the triangle or parallelogram laws. The two vectors $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ in Fig. 2.2(a) may be added head-to-tail to obtain their sum as shown in Fig.2.2(b) by the triangle law. The order of their combination does not affect their sum. The identical result in magnitude and direction is obtained by completing the parallelogram as shown in Fig. 2.2(c). In each case this vector addition is expressed symbolically by the vector equation

$$\vec{V} = \vec{V}_1 + \vec{V}_2 \tag{2.1}$$

where the plus sign used in conjunction with the vector quantities means

vector and not scalar addition.

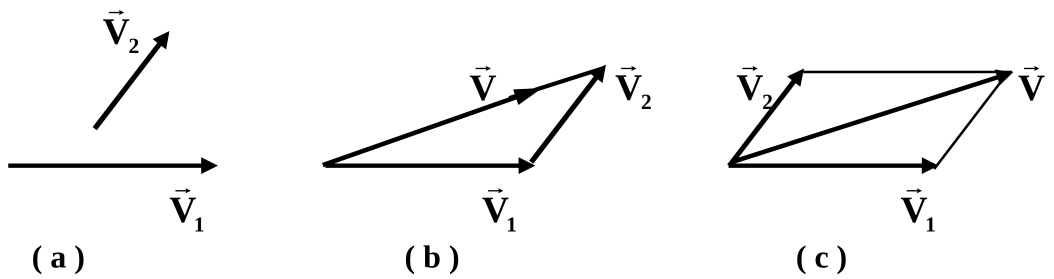


Fig. 2.2

The difference $\vec{\mathbf{V}}'$ between the vectors $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ may be obtained by either the triangle or the parallelogram procedure as shown in Fig. 2.3. It is necessary only to add the negative of $\vec{\mathbf{V}}_2$ to $\vec{\mathbf{V}}_1$ to obtain the vector difference. This difference is indicated by the equation:

$$\vec{V}' = \vec{V}_1 - \vec{V}_2 \tag{2.2}$$

where the minus sign is used to denote vector subtraction.

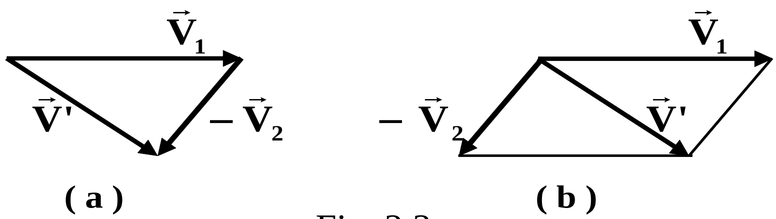
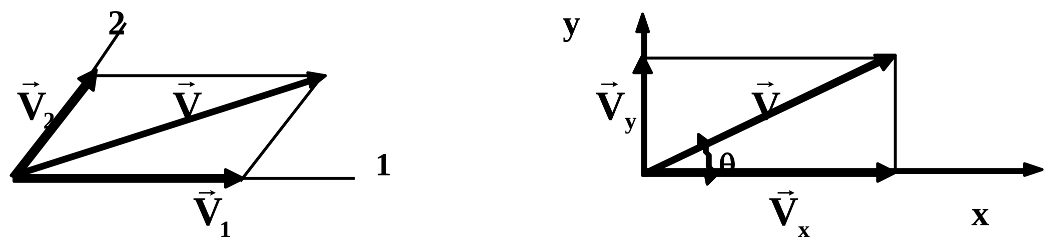


Fig. 2.3

2.3 Vector Components

Any two or more vectors whose sum equals a certain vector $\vec{\mathbf{V}}$ are said to be the components of that vector. Hence the vectors $\vec{\mathbf{V}}_1$ and $\vec{\mathbf{V}}_2$ in Fig. 2.4(a) are the components of $\vec{\mathbf{V}}$ in the directions 1 and 2, respectively. It is usually more convenient to deal with vector components that are mutually perpendicular, and these are called rectangular components. The vectors $\vec{\mathbf{V}}_x$ and $\vec{\mathbf{V}}_y$ in Fig. 2.4(b) are the x and y components, respectively, of $\vec{\mathbf{V}}$. When expressed in rectangular components, the direction of the vector with respect to, say, the x-axis is clearly specified by:

$$\theta = \tan^{-1}\frac{V_y}{V_x} \tag{2.3}$$



(a) (b)

Fig. 2.4

2.4 Vector Addition of Forces

Experimental evidence has shown that a force is a vector quantity since it has a specified magnitude, direction, and sense so it can be added according to the parallelogram law or by the triangle law. The two forces $\vec{\mathbf{P}}$ and $\vec{Q}$ can be added together to form the resultant force $\vec{R} = \vec{P} + \vec{Q} = \vec{Q} + \vec{P}$ as shown in Fig. 2.5a. From this construction, or using the triangle rule, Fig. 2.5b, we can apply the law of cosines or the law of sines to the triangle in order to obtain the magnitude of the resultant force and its direction.

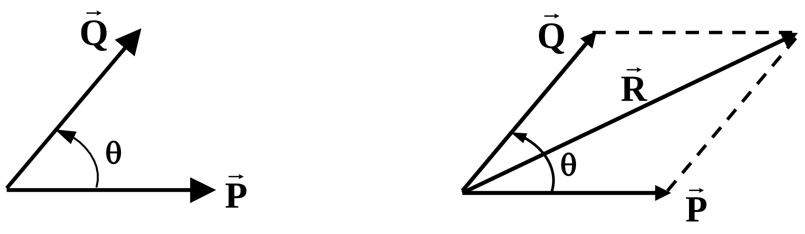


Fig. 2.5a

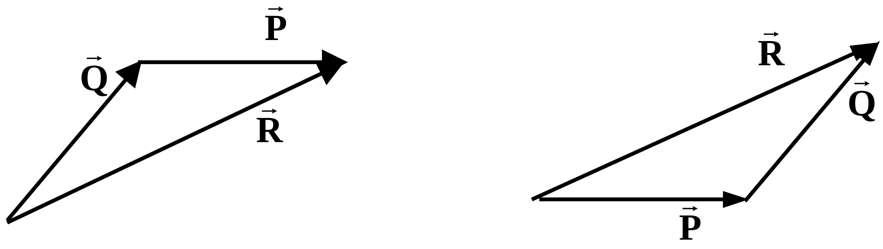
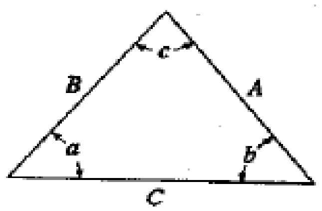


Fig. 2.5b

The sides and angles of a non-right triangle are determined from

Sine law $\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$

Cosine law $C = \sqrt{A^2 + B^2 - 2AB\cos c}$



If more than two forces are to be added, successive applications of the parallelogram law or triangle law can be carried out, Fig. 2.6, in order to obtain the resultant force, $\vec{F}_R = \vec{P} + \vec{Q} + \vec{S} = (\vec{P} + \vec{Q}) + \vec{S}$. Vector addition is associative.

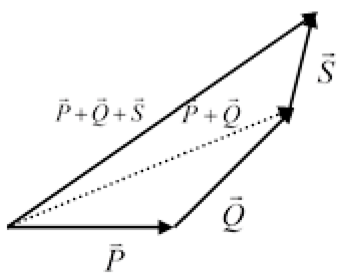


Fig. 2.6

2.5 Addition of a System of Coplanar Forces

When a force is resolved into two components along the x and y axes, the components are then called rectangular components. For analytical work we can represent these components in one of two ways, using either scalar or Cartesian vector notation.

Scalar Notation. The rectangular components of force $\vec{F}$ shown in Fig. 2.7 are found using the parallelogram law, so that $\vec{F} = \vec{F}_x + \vec{F}_y$ because these components form a right triangle, they can be determined from

$$F_x = F \cos\theta \text{ and } F_y = F \sin\theta$$

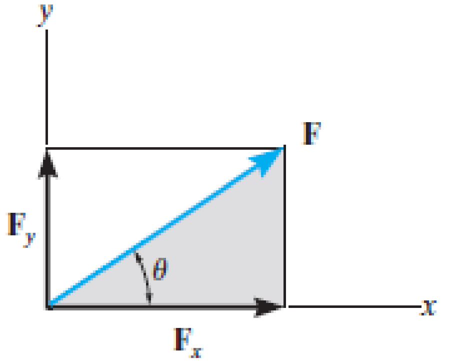


Fig. 2.7

Cartesian Vector Notation. It is also possible to represent the x and y components of a force in terms of Cartesian unit vectors $\vec{\mathbf{i}}$ and $\vec{\mathbf{j}}$. They are

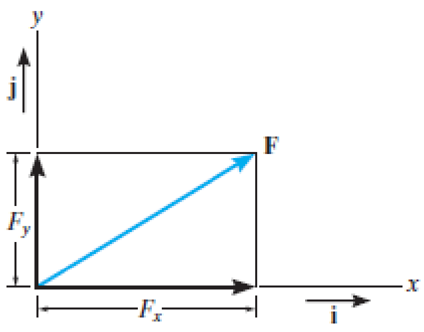


Fig. 2.8

called unit vectors because they have a dimensionless magnitude of 1, and so they can be used to designate the directions of the x and y axes, respectively, Fig. 2.8, we can express $\vec{\mathbf{F}}$ as a Cartesian vector,

$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

Coplanar Force Resultants. We can use either of the two methods just described to determine the resultant of several coplanar forces, i.e., forces that all lie in the same plane. To do this, each force is first resolved into its **x** and **y** components, and then the respective components are added using scalar algebra since they are collinear. The resultant force is then formed by adding the resultant components using the parallelogram law. For example, consider the three concurrent forces in Fig. 2.9a, which have **x** and **y** components shown in Fig. 2.9b. Using Cartesian vector notation, each force is first represented as a Cartesian vector, i.e.,

$$\vec{F}_1 = F_{1x}\vec{i} + F_{1y}\vec{j}$$

$$\vec{F}_2 = -F_{2x}\vec{i} + F_{2y}\vec{j}$$

$$\vec{F}_3 = F_{3x}\vec{i} - F_{3y}\vec{j}$$

The vector resultant is therefore

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\vec{F}_R = F_{1x}\vec{i} + F_{1y}\vec{j} - F_{2x}\vec{i} + F_{2y}\vec{j} + F_{3x}\vec{i} - F_{3y}\vec{j}$$

$$\vec{F}_R = (F_{1x} - F_{2x} + F_{3x})\vec{i} + (F_{1y} + F_{2y} - F_{3y})\vec{j}$$

$$\vec{F}_R = (F_{Rx})\vec{i} + (F_{Ry})\vec{j}$$

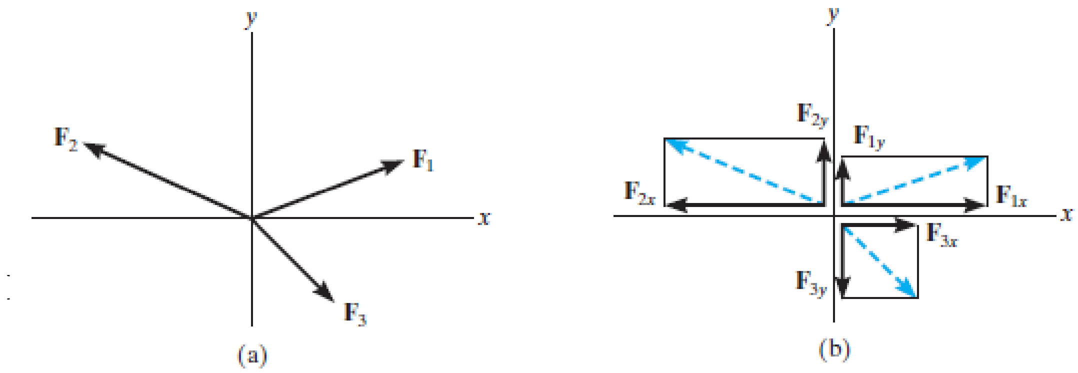


Fig. 2.9

Indicating the positive directions of components along the x and y axes with symbolic arrows, we have

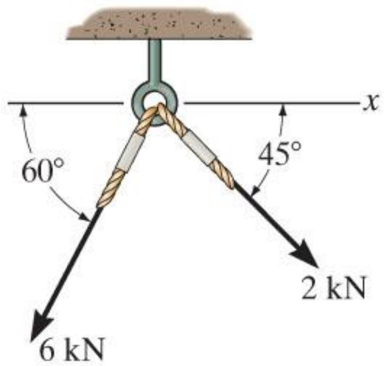
$$\xrightarrow{+} F_{Rx} = F_{1x} - F_{2x} + F_{3x}$$

$$\uparrow^{+} F_{Ry} = F_{1y} + F_{2y} - F_{3y}$$

These are the same results as the $\vec{i}$ and $\vec{j}$ components of $\vec{F}_R$ determined above.

Example (2.1)

Determine the magnitude of the resultant force acting on the screw eye?



Solution

(1) Force $(F_1 = 2\ \text{kN})$

$$F_{1x} = F_1 \cos\theta = 2\cos 45 = 1.4142\ \text{kN}$$

$$F_{1y} = -F_1 \sin\theta = -2\sin 45 = -1.4142\ \text{kN}$$

$$\vec{F_1} = F_{1x}\ \vec{i} + F_{1y}\ \vec{j}$$

$$\vec{F_1} = 1.4142\ \vec{i} - 1.4142\ \vec{j}$$

(2) Force $(F_2 = 6\ \text{kN})$

$$F_{2x} = -F_2 \cos\theta = -6\cos 60 = -3\ \text{kN}$$

$$F_{2y} = -F_2 \sin\theta = -6\sin 60 = -5.196\ \text{kN}$$

$$\vec{F_2} = F_{2x}\,\vec{i} + F_{2y}\,\vec{j} \qquad \vec{F_2} = -3\,\vec{i} - 5.196\,\vec{j}$$

(3) Resultant Force F_R

$$\vec{F_R} = \vec{F_1} + \vec{F_2}$$

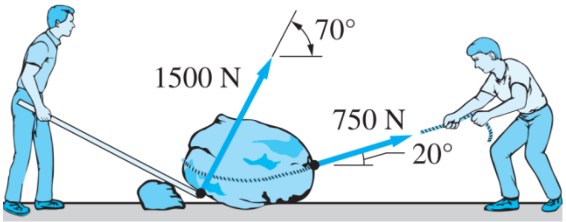
$$\vec{F_R} = \left(1.4142\,\vec{i} - 1.4142\,\vec{j}\right) + \left(-3\,\vec{i} - 5.196\,\vec{j}\right)$$

$$\vec{F_R} = -1.5858\,\vec{i} - 6.6102\,\vec{j} \qquad F_R = \sqrt{(1.5858)^2 + (6.6102)^2}$$

$$F_R = 6.8\ \text{kN}$$

Example (2.2)

Two men are trying to roll the boulder by applying the forces shown. Determine the magnitude and direction of the resultant force?



Solution

(1) Force $(F_1 = 750\ \text{N})$

$$F_{1x} = F_1\cos\theta = 750\cos 20 = 704.77\ \text{N}$$

$$F_{1y} = F_1\sin\theta = 750\sin 20 = 256.52\ \text{N}$$

$$\vec{F_1} = F_{1x}\,\vec{i} + F_{1y}\,\vec{j} \qquad \vec{F_1} = 704.77\,\vec{i} + 256.52\,\vec{j}$$

(2) Force $(F_2 = 1500\ \text{N})$

$$F_{2x} = F_2\cos\theta = 1500\cos 70 = 513\ \text{N}$$

$$F_{2y} = F_2\sin\theta = 1500\sin 70 = 1409.54\ \text{N}$$

$$\vec{F_2} = F_{2x}\,\vec{i} + F_{2y}\,\vec{j} \qquad \vec{F_2} = 513\,\vec{i} + 1409.54\,\vec{j}$$

(4) Resultant Force F_R

$$\vec{F_R} = \vec{F_1} + \vec{F_2}$$

$$\vec{F_R} = \left(704.77\,\vec{i} + 256.52\,\vec{j}\right) + \left(513\,\vec{i} + 1409.54\,\vec{j}\right)$$

$$\overrightarrow{F_R} = 1217.77\,\vec{i} + 1666.06\,\vec{j}$$

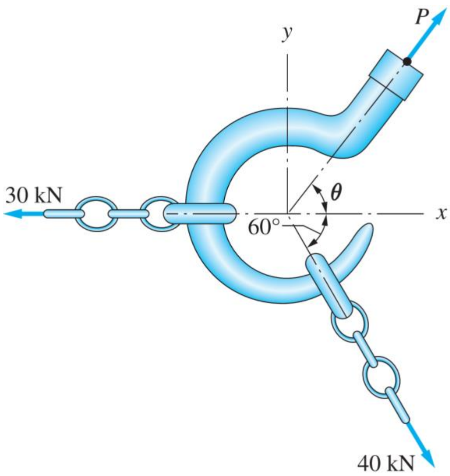
$$F_R = \sqrt{(1217.77)^2 + (1666.06)^2}$$

$$F_R = 2063.67 \text{ N}$$

$$\tan\theta_R = \frac{F_{Ry}}{F_{Rx}} = \frac{1666.06}{1217.77} = 1.3681237 \qquad \theta_R = 53.83^0$$

Example (2.3)

Determine P and θ so that the resultant of the three forces shown is $\overrightarrow{F_R} = 85\,\vec{i} + 20\,\vec{j}$ (kN).



Solution

(1) Force $(F_1 = 30\text{ kN})$

$$\vec{F_1} = -F_1\,\vec{i} = -30\,\vec{i}$$

(2) Force $(F_2 = 40\text{ kN})$

$$F_{2x} = F_2\cos\theta = 40\cos 60 = 20\text{ kN}$$

$$F_{2y} = -F_2\sin\theta = -40\sin 60 = -34.64\text{ kN}$$

$$\vec{F_2} = F_{2x}\,\vec{i} + F_{2y}\,\vec{j} \qquad \vec{F_2} = 20\,\vec{i} - 34.64\,\vec{j}$$

(3) Resultant Force F_R

$$\overrightarrow{F_R} = \vec{F_1} + \vec{F_2} + \vec{P}$$

$$85\,\vec{i} + 20\,\vec{j} = \left(-30\,\vec{i}\right) + \left(20\,\vec{i} - 34.64\,\vec{j}\right) + \vec{P}$$

$$85\,\vec{i} + 20\,\vec{j} = -10\,\vec{i} - 34.64\,\vec{j} + \vec{P}$$

$$\vec{P} = 95\,\vec{i} + 54.64\,\vec{j}$$

(4) Force P

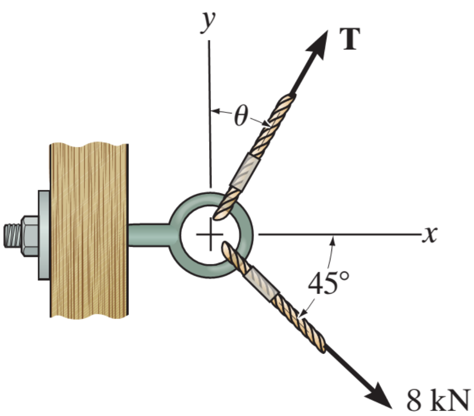
$$\vec{P} = 95\,\vec{i} + 54.64\,\vec{j}$$

$$P = \sqrt{(95)^2 + (54.64)^2} \qquad P = 109.6 \text{ kN}$$

$$\tan\theta = \frac{P_y}{P_x} = \frac{54.64}{95} \qquad \theta = 30^0$$

Example (2.4)

If the magnitude of the resultant force is to be **9 kN** directed along the positive x–axis, determine the magnitude of the force **T** acting on the eyebolt and its angle $\boldsymbol{\theta}$.



Solution

(1) Force $(F_1 = 8\text{ kN})$

$$F_{1x} = F_1\cos\theta = 8\cos45 = 5.66\text{ kN}$$

$$F_{1y} = -F_1\sin\theta = -8\sin45 = -5.66\text{ kN}$$

$$\vec{F_1} = F_{1x}\,\vec{i} + F_{1y}\,\vec{j} \qquad \vec{F_1} = 5.66\,\vec{i} - 5.66\,\vec{j}$$

(2) Force (T)

$$T_x = T\sin\theta \qquad T_y = T\cos\theta$$

$$\vec{T} = T_x\,\vec{i} + T_y\,\vec{j} \qquad \vec{T} = (T\sin\theta)\,\vec{i} + (T\cos\theta)\,\vec{j}$$

(3) Resultant Force F_R

$$\vec{F_R} = \vec{F_1} + \vec{T} \qquad 9\,\vec{i} = 5.66\,\vec{i} - 5.66\,\vec{j} + \vec{T}$$

$$\vec{T} = 3.34\,\vec{i} + 5.66\,\vec{j}$$

$$T = \sqrt{(3.34)^2 + (5.66)^2} \qquad T = 6.572 \text{ N}$$

$$T_x = T\sin\theta$$

$$3.34 = 6.572\sin\theta \qquad \sin\theta = 0.5082 \qquad \theta = 30.5^0$$

2. 6 FORCES IN SPACE

2.6.1 Rectangular Components of a Force Vector

Consider a force $\vec{F}$ acting at the origin O, Fig. 2.10. We may construct a rectangular parallelogram with the vector $\vec{F}$ as a main diagonal and with sides along the direction of coordinate axes. The force $\vec{F}$ may be resolved into two perpendicular components (in vertical plane containing z axis and $\vec{F}$). These two components are denoted $\vec{F}_z$ along z axis, and $\vec{F}_h$ laying in horizontal plane x-y.

$$\vec{F} = \vec{F}_z + \vec{F}_h \qquad (2.4a)$$

Where: $$F^2 = F_z^2 + F_h^2 \qquad (2.4b)$$

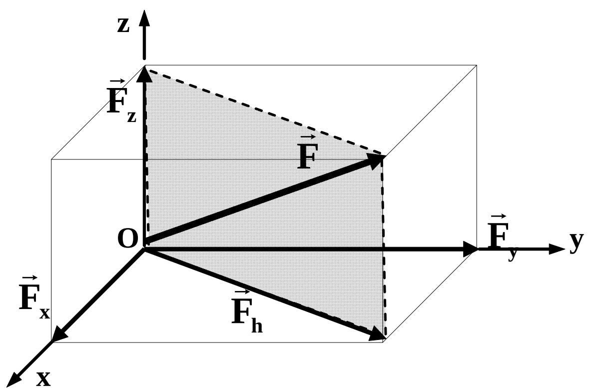


Fig. 2.10

In turn, the horizontal component $\vec{F}_h$ can be resolved by another rectangular parallelogram into two perpendicular components, $\vec{F}_x$ along x-direction and $\vec{F}_y$ along y-direction. They are related with the vector equation:

$$\vec{F}_h = \vec{F}_x + \vec{F}_y \qquad (2.5a)$$

Where: $$F_h^2 = F_x^2 + F_y^2 \qquad (2.5b)$$

Substituting $\vec{F}_h$ from equation (2.5a) into equation (2.4a) we have:

$$\vec{F} = \vec{F}_x + \vec{F}_y + \vec{F}_z \quad (2.6)$$

And from equation (2.5b) into equation (2.4b) we obtain

$$F^2 = F_x^2 + F_y^2 + F_z^2 \quad (2.7)$$

The vectors $\vec{F}_X$, $\vec{F}_Y$ and $\vec{F}_Z$ are called the rectangular vector components of the force $\vec{F}$ along the coordinate axes x, y and z, respectively.

2.6.2 Fundamental Unit Vectors

In three dimensions, the set of a Cartesian unit vectors $\vec{\mathbf{i}}$, $\vec{\mathbf{j}}$, $\vec{\mathbf{k}}$ is used to designate the directions of the x-, y- and z-axes, respectively. The positive fundamental unit vectors $\vec{\mathbf{i}}$, $\vec{\mathbf{j}}$, $\vec{\mathbf{k}}$ are shown in Fig. 2.11.

Using fundamental unit vectors, the vector components of Eq. (2.8) may be written as follows:

$$\vec{F}_x = F_x \vec{i}, \qquad \vec{F}_y = F_y \vec{j}, \qquad \vec{F}_z = F_z \vec{k} \quad (2.8)$$

So, the force can be written in the "Cartesian vector form" as:

$$\vec{F} = F_X \vec{i} + F_Y \vec{j} + F_Z \vec{k} \quad (2.9)$$

The magnitude of the force vector from equation (2.7) is given by:

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \quad (2.10)$$

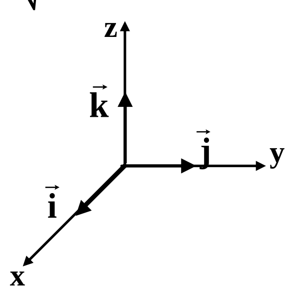


Fig. 2.11

2.6.3 Direction Angles of a Force Vector

The orientation of a force vector in space $\vec{F}$ is defined by the coordinate direction angles **α** (**alpha**), **β** (**beta**) and **γ** (**gamma**), measured between the vector $\vec{F}$ and the positive direction x-, y-, z-axes, Fig. 2.12. (Note that each of these angles will be between 0° and 180°). Consider the projection of $\vec{F}$ on to x-, y-, z-axes, we have:

$$\cos\alpha = \frac{F_x}{F}, \qquad \cos\beta = \frac{F_y}{F}, \qquad \cos\gamma = \frac{F_z}{F} \qquad (2.11)$$

These numbers are known as the direction cosines of $\vec{F}$. Once they have been obtained, the coordinate direction angles **α**, **β** and **γ** can then be determined.

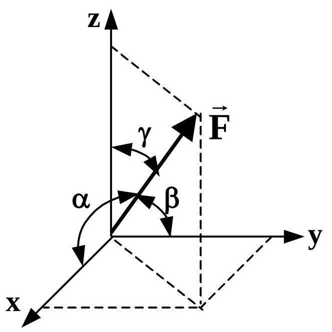


Fig. 2.12

An easy way for obtaining these direction cosines is to form a unit vector. A unit vector is a vector having a magnitude of 1 and having the same direction of the vector. If $\vec{F}$ is a vector having a magnitude F ≠ 0, then a unit vector having the same direction of $\vec{F}$ is represented by:

$$\vec{u}_F = \frac{\vec{F}}{F} \qquad (2.12)$$

Substituting about $\vec{F}$ from equation (2.9) we obtain:

$$\vec{u}_F = \frac{F_x}{F}\vec{i} + \frac{F_y}{F}\vec{j} + \frac{F_z}{F}\vec{k} \qquad (2.13)$$

By comparison with equation (2.11), it is seen that the Cartesian components of the unit vector represent the direction cosines of the vector, i.e.,

$$\vec{u}_F = \cos\alpha\,\vec{i} + \cos\beta\,\vec{j} + \cos\gamma\,\vec{k} \qquad (2.14)$$

Since the magnitude of a vector is equal to the positive square root of the sum of the squares of the magnitudes of its components, and $\vec{\mathbf{u}}_{\mathbf{F}}$ has a magnitude of one, then from the above equation an important relation among the direction cosines can be formulated as

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \qquad (2.15)$$

Finally by using equations (2.12) and (2.14), the force vector may be expressed in Cartesian vector form as:

$$\vec{F} = F\cos\alpha\,\vec{i} + F\cos\beta\,\vec{j} + F\cos\gamma\,\vec{k} \qquad (2.16)$$

Example 2.5

Given a force $\vec{\mathbf{F}}$ of magnitude **F= 200 N** with direction angles $\boldsymbol{\beta = 60°, \gamma = 45°}$. If the x-component of $\vec{\mathbf{F}}$ acts along the positive x-axis, express $\vec{\mathbf{F}}$ in the Cartesian vector form.

Solution

Since: $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Then: $\cos^2\alpha + \cos^2(60°) + \cos^2(45°) = 1$

$\cos\alpha = \pm 0.5$

Hence, $\alpha = \cos^{-1}(0.5) = 60°$ or $\alpha = \cos^{-1}(-0.5) = 120°$

Since F_x is in the positive x-axis, it is necessary that $\alpha = 60°$.

With F = 200 N**,** we have:

$F_x = F\cos\alpha = 200\cos 60° = 100$ N

$$F_y = F\cos\beta = 200\cos 60° = 100 \text{ N}$$

$$F_z = F\cos\gamma = 200\cos 45° = 141.4 \text{ N}$$

So the expression of $\vec{\mathbf{F}}$ in Cartesian vector form is as follows:

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} = 100\vec{i} + 100\vec{j} + 141.4\vec{k} \quad \text{N}$$

Example 2.6

A force $\vec{\mathbf{F}}$ has components $\mathbf{F_x = 200\ N}$, $\mathbf{F_y = -300\ N}$ and $\mathbf{F_z = 600\ N}$. Determine its magnitude F and its direction angles $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$.

Solution

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(200)^2 + (-300)^2 + (600)^2} = 700 \text{ N}$$

The direction angles of the vector can be determined as:

$$\cos\alpha = \frac{F_x}{F} = \frac{200}{700} = \frac{2}{7} \qquad \text{then,} \qquad \alpha = 73.4°$$

$$\cos\beta = \frac{F_y}{F} = \frac{-300}{700} = -\frac{3}{7} \qquad \text{then,} \qquad \beta = 115.4°$$

$$\cos\gamma = \frac{F_z}{F} = \frac{600}{700} = \frac{6}{7} \qquad \text{then,} \qquad \gamma = 31.0°$$

2.6.4 Position Vector

The position vector $\vec{\mathbf{r}}$ is defined as a fixed vector which locates a point in space relative to another point. For example, if $\vec{\mathbf{r}}$ extends from the origin of coordinates, **O**, to point **P (x, y, z)**, Fig. 2.13(a) then $\vec{\mathbf{r}}$ can be expressed in Cartesian vector form as:

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \qquad (2.17)$$

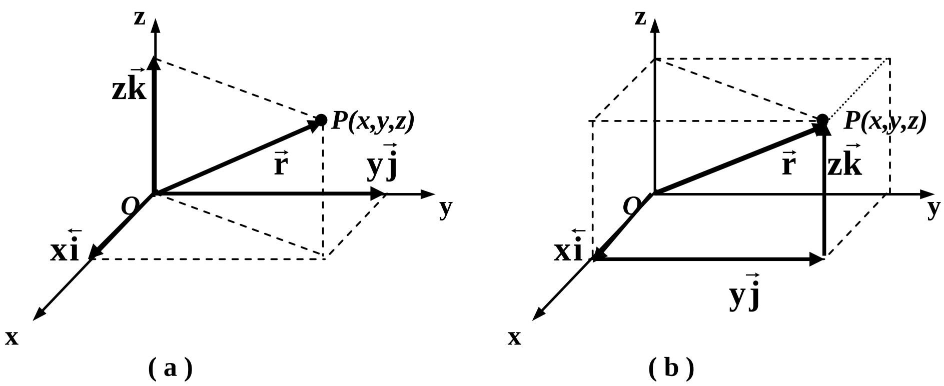


Fig. 2.13

In particular, note how the head-to-tail vector addition of the three components yields vector $\vec{\mathbf{r}}$, Fig. 2.13(b). Starting at the origin **O**, one travels **x** in the $+\vec{\mathbf{i}}$ direction, then *y* in the $+\vec{\mathbf{j}}$ direction, and finally **z** in the $+\vec{\mathbf{k}}$ direction to arrive at point **P (x, y, z).**

In more general case, the position vector may be directed from point **A** to point **B** in space, Fig. 2.14(a). As noted, this vector is also designated by the symbol $\vec{\mathbf{r}}$. As a matter of convention, however, we will sometimes refer to this vector with two subscripts to indicate from and to the point where it is directed, thus, $\vec{\mathbf{r}}$ can also be designated as $\vec{\mathbf{r}}_{\mathbf{AB}}$ or $\overrightarrow{AB}$. Also, note that $\vec{\mathbf{r}}_{\mathbf{A}}$ and $\vec{\mathbf{r}}_{\mathbf{B}}$ referenced with only one subscript since they extend from the origin of coordinates.

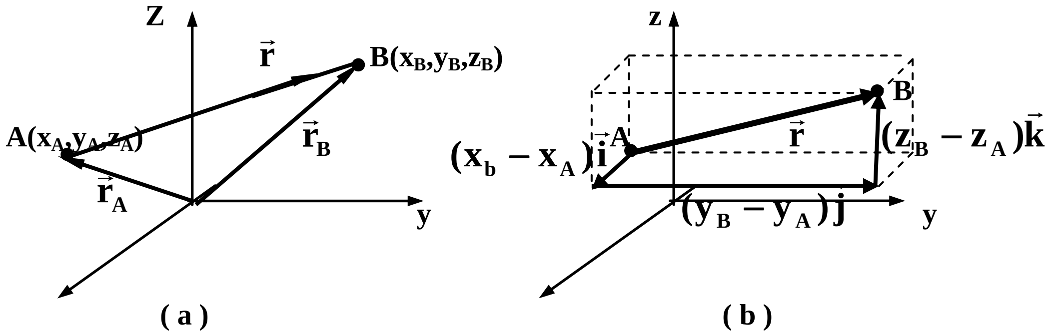


Fig. 2.14

$$\vec{r} = \vec{r}_{AB} = \vec{r}_B - \vec{r}_A = (x_B\vec{i} + y_B\vec{j} + z_B\vec{k}) - (x_A\vec{i} + y_A\vec{j} + z_A\vec{k})$$

or

$$\vec{r} = \vec{r}_{AB} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k} \qquad (2.18)$$

Also, note how the head-to-tail vector addition of the three components yields vector $\vec{\mathbf{r}}$, Fig. 2.14(b).

2.6.5 Force Vector Directed Along a Line

Quite often in three-dimensional statics problems, the direction of a force is specified by two points through which its line of action passes. Such a situation is shown in Fig. 2.15, where the force $\vec{F}$ is directed along the cord MN. We can formulate $\vec{F}$ as a Cartesian vector by realizing that it has the same direction and sense as the position vector $\overrightarrow{\mathbf{MN}}$ directed from point M with coordinates (x_1,y_1,z_1) to point N with coordinates (x_2,y_2,z_2).

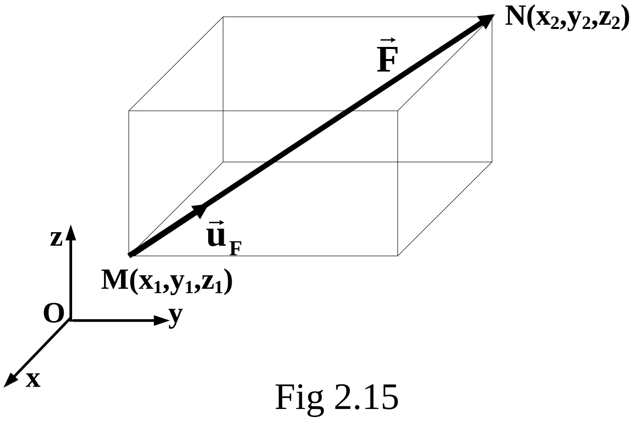


Fig 2.15

The vector $\overrightarrow{\mathbf{MN}}$ can be written as:

$$\overrightarrow{MN} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k} \qquad (2.19)$$

The unit vector $\vec{u}_F$ along the line of action of $\vec{F}$ (i.e. along the line MN) may be obtained by dividing the vector $\overrightarrow{\mathbf{MN}}$ by its magnitude MN. Where:

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \qquad (2.20)$$

Hence, we can write:

$$\vec{u}_F = \frac{\overrightarrow{MN}}{MN} \qquad (2.21)$$

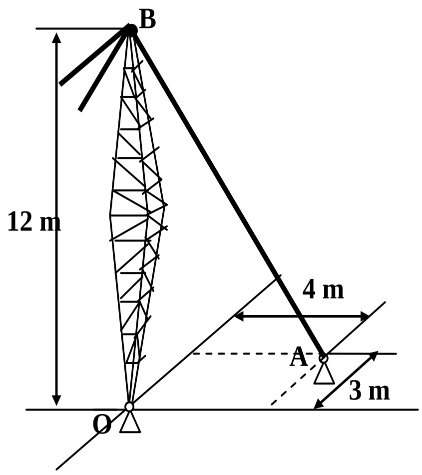
Recalling that $\vec{F}$ is equal to the product of its magnitude F and the unit vector $\vec{u}_F$ along $\vec{F}$, we can write:

$$\vec{F} = F\,\vec{u}_F = \frac{F}{MN}(\overrightarrow{MN}) \qquad (2.22)$$

Where the vector $\overrightarrow{\mathbf{MN}}$, and its magnitude MN are given by equations (2.19) and (2.20), respectively.

<u>Example 2.7</u>

A tower guy wire is anchored by means of a ring at A. The tension in the wire is 2600 N. Determine the components $\mathbf{F_x}$, $\mathbf{F_y}$ and $\mathbf{F_z}$ of the force acting on the ring A.



<u>Solution</u>

The unit vector along $\vec{\mathbf{F}}$ is directed along the line AB, hence:

A = (-3,4,0) m , B = (0,0,12) m

$$\vec{u}_F = \frac{\overrightarrow{AB}}{AB} = \frac{(0-(-3))\vec{i} + (0-4)\vec{j} + (12-0)\vec{k}}{\sqrt{(3)^2 + (-4)^2 + (12)^2}}$$

$$\vec{u}_F = \frac{1}{13}(3\vec{i} - 4\vec{j} + 12\vec{k})$$

Since, $\vec{F} = F\vec{u}_F$

Therefore, $\vec{F} = \frac{2600}{13}(3\vec{i} - 4\vec{j} + 12\vec{k})$

$$\vec{F} = (600\vec{i} - 800\vec{j} + 2400\vec{k}) \text{ N}$$

The components of the force acting on **A** are:

F_x = 600 N, F_y= -800 N, F_z= 2400 N

2.6.6 Resultant of Concurrent Forces

Let a system of forces $\vec{\mathbf{F}}_1$, $\vec{\mathbf{F}}_2$, $\vec{\mathbf{F}}_3$, ... acting at the point **P** with position vector $\vec{\mathbf{r}}$, Fig. 2.16. The resultant of this force system is obtained by summing their rectangular components.

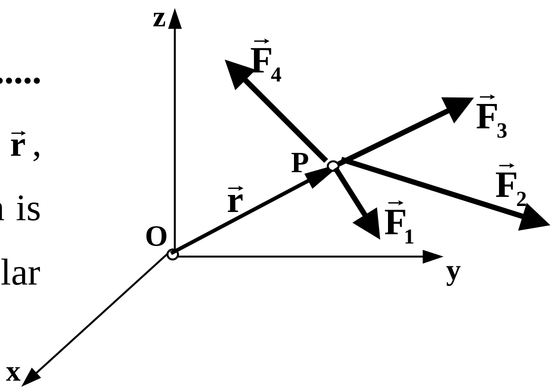


Fig 2.16

$$\vec{R} = \sum \vec{F} \qquad (2.23)$$

We resolve the resultant and all forces in their rectangular components and write:

$$R_x\vec{i} + R_y\vec{j} + R_z\vec{k} = (\sum F_x)\vec{i} + (\sum F_y)\vec{j} + (\sum F_z)\vec{k}$$

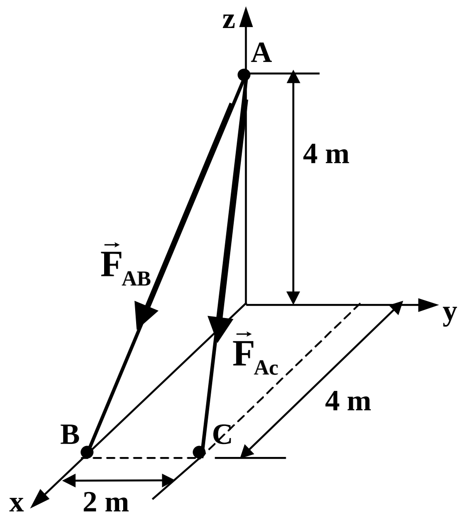
From which it follows that:

$$R_x = \sum F_x,\ R_y = \sum F_y,\ R_z = \sum F_z \qquad (2.24)$$

The magnitude of the resultant and its direction angles are obtained as:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} \qquad (2.25)$$

$$\cos \alpha_R = \frac{R_x}{R}, \quad \cos \beta_R = \frac{R_y}{R}, \quad \cos \gamma_R = \frac{R_z}{R} \qquad (2.26)$$



Example 2.8

The cables exert forces **F_{AB} = 100 N** and **F_{AC} = 120 N** as shown in figure. Determine the magnitude and the coordinate direction angles of the resultant force acting at A.

Solution

We can express the given forces as a Cartesian vectors, and then adding their components.

A = (0,0,4) m , B = (4,0,0) m , C = (4,2,0) m

With reference to figure, for $\vec{F}_{AB}$ we have:

$$\vec{u}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{(4-0))\vec{i} + (0-0)\vec{j} + (0-4)\vec{k}}{\sqrt{(4)^2 + (0)^2 + (-4)^2}}$$

$$= \frac{1}{5.66}(4\vec{i} + 0\vec{j} - 4\vec{k})$$

Since, $\vec{F}_{AB} = F_{AB}\vec{u}_{AB}$

Therefore, $\vec{F}_{AB} = \frac{100}{5.66}(4\vec{i} + 0\vec{j} - 4\vec{k})$

$$\vec{F}_{AB} = 70.67\vec{i} + 0\vec{j} - 70.67\vec{k} \quad N$$

For $\vec{F}_{AC}$ we have:

$$\vec{u}_{AC} = \frac{\overrightarrow{AC}}{AC} = \frac{(4-0))\vec{i} + (2-0)\vec{j} + (0-4)\vec{k}}{\sqrt{(4)^2 + (2)^2 + (-4)^2}}$$

$$\vec{u}_{AC} = \frac{1}{6}(4\vec{i} + 2\vec{j} - 4\vec{k})$$

Since, $\vec{F}_{AC} = F_{AC}\vec{u}_{AC}$

Therefore: $\vec{F}_{AC} = \frac{120}{6}(4\vec{i} + 2\vec{j} - 4\vec{k})$

$$\vec{F}_{AC} = 80\vec{i} + 40\vec{j} - 80\vec{k} \quad N$$

The resultant force $\vec{\mathbf{R}}$ is therefore:

$$\vec{R} = \sum \vec{F} = \vec{F}_{AB} + \vec{F}_{AC} = (70.67\vec{i} + 0\vec{j} - 70.67\vec{k}) + (80\vec{i} + 40\vec{j} - 80\vec{k})$$

$$= 150.67\vec{i} + 40\vec{j} - 150.67\vec{k} \quad N$$

The magnitude of $\vec{\mathbf{R}}$ is thus:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{(150.67)^2 + (40)^2 + (-150.67)^2} = 216.8 \quad N$$

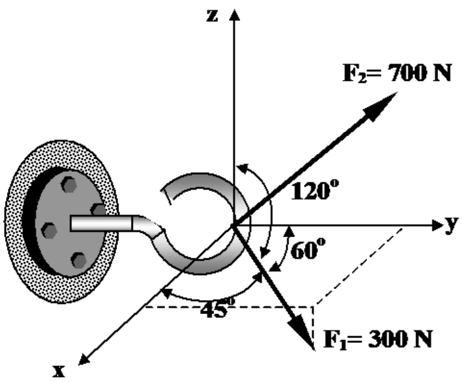
$$\cos\alpha_R = \frac{R_x}{R} = \frac{150.67}{216.8} \qquad \alpha_R = 46°$$

$$\cos\beta_R = \frac{R_y}{R} = \frac{40}{216.8} \qquad \beta_R = 79.4°$$

$$\cos\gamma_R = \frac{R_z}{R} = \frac{-150.67}{216.8} \qquad \gamma_R = 134°$$

Example 2.9

Two forces act on the hook shown in figure. Specify the coordinate direction angles of $\vec{\mathbf{F}}_2$ so that the resultant force acts along the positive **y** axis and has a magnitude of 800N.



Solution

We can express the given two forces as a Cartesian vectors, for $\vec{\mathbf{F}}_1$ we have:

$F_{1x} = F_1 \cos\alpha = 300 \cos 45° = 212.1$ N

$F_{1y} = F_1 \cos\beta = 300 \cos 60° = 150$ N

$F_{1z} = F_1 \cos\gamma = 300 \cos 120° = -150$ N

Then the expression of $\vec{\mathbf{F}}_1$ in Cartesian vector form is as follows:

$$\vec{F}_1 = F_{1x}\vec{i} + F_{1y}\vec{j} + F_{1z}\vec{k} = 212.1\vec{i} + 150\vec{j} - 150\vec{k} \quad \text{N}$$

For $\vec{\mathbf{F}}_2$ we have:

$$\vec{F}_2 = F_{2x}\vec{i} + F_{2y}\vec{j} + F_{2z}\vec{k} = 700\cos\alpha_2\vec{i} + 700\cos\beta_2\vec{j} + 700\cos\gamma_2\vec{k}$$

The resultant force $\vec{\mathbf{R}}$ has a magnitude of 800 N and acts in the $+\vec{\mathbf{j}}$ direction.

Hence, $\vec{R} = 800\vec{j}$ N

We require: $\vec{R} = \vec{F}_1 + \vec{F}_2$

$$800\vec{j} = (212.1\vec{i} + 150\vec{j} - 150\vec{k}) + (700\cos\alpha_2\vec{i} + 700\cos\beta_2\vec{j} + 700\cos\gamma_2\vec{k})$$

To satisfy this equation, the corresponding **x**, **y** and **z** components on the left and right sides must be equal. Hence:

$0 = 212.1 + 700\cos\alpha_2$ then, $\cos\alpha_2 = \frac{-212.1}{700}$, $\alpha_2 = 107.6°$,

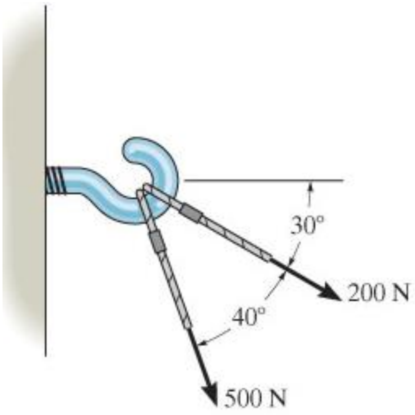
$800 = 150 + 700\cos\beta_2$ then, $\cos\beta_2 = \frac{650}{700}$, $\beta_2 = 21.8°$,

$0 = -150 + 700\cos\gamma_2$ then, $\cos\gamma_2 = \frac{150}{700}$, $\gamma_2 = 77.6°$.

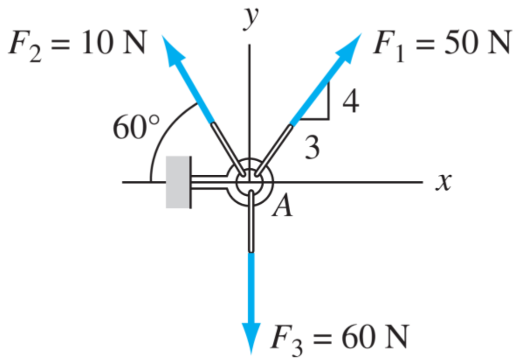
PROBLEMS

2.1 Two forces act on the hook. Determine the magnitude of the resultant force?

2.2 Determine the resultant of the three forces shown in Figure.



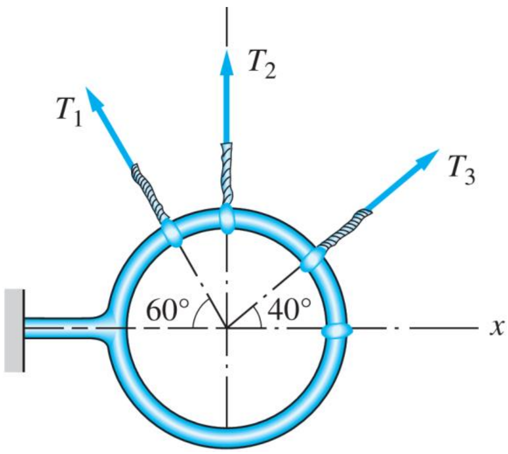
Problem 2.1



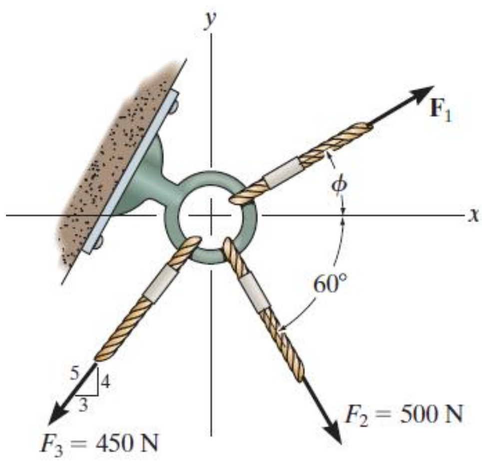
Problem 2.2

2.3 The magnitudes of the three forces applied to the eyebolt are $T_1 = 550\ \text{N}$, $T_2 = 200\ \text{N}$, and $T_2 = 750\ \text{N}$. Determine the resultant of the three forces.

2.4 If $F_1 = 600\ \text{N}$ and $\varphi = 30^0$, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.



Problem 2.3



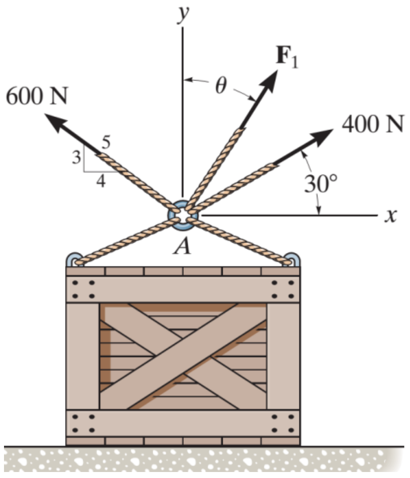
Problem 2.4

2.5 Determine the magnitude of F_1 and its direction so that the resultant force is directed vertically upward and has a magnitude of $800\ N$.

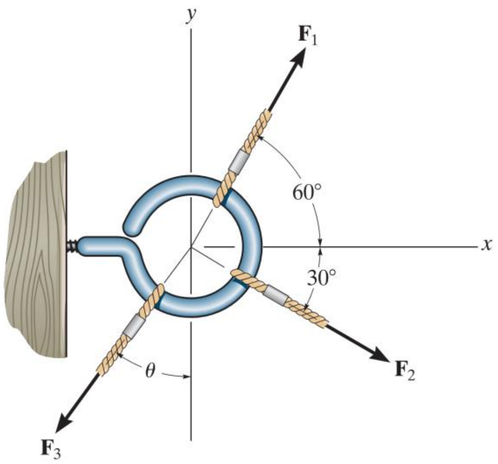
2.6 The three concurrent forces acting on the screw eye produce a resultant force $F_R = 0$. and $F_2 = \frac{2}{3}F_1$, determine:

(a) The angle θ.

(b) The magnitude of F_3 expressed in terms of F_1



Problem 2.5

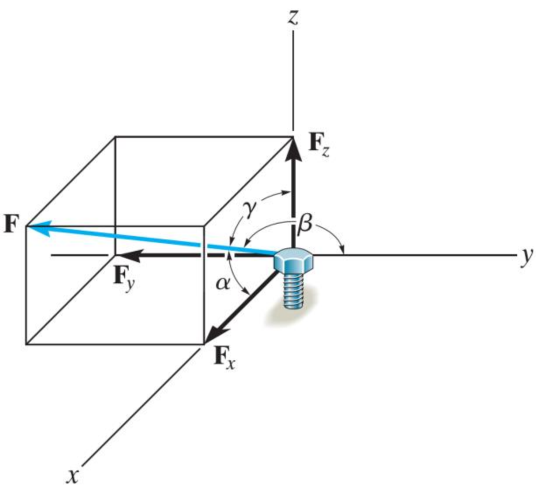


Problem 2.6

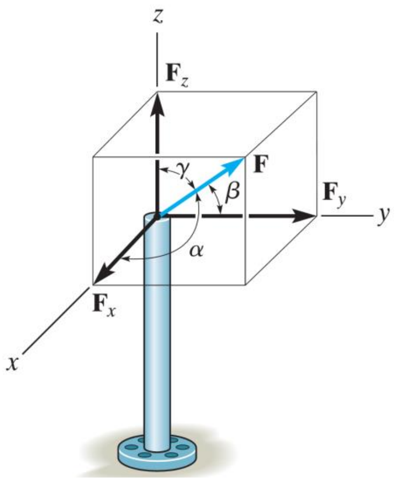
2.7 The bolt is subjected to the force F, which has components acting along the x, y, z axes as shown. If the magnitude of F is 80 N, and $\alpha = 60^\circ$ and $\beta = 45^\circ$, determine the magnitudes of its components.

2.8 The pole is subjected to the force F, which has components acting along the *x, y, z* axes as shown. If the magnitude of F is 3 kN, $\beta = 30^\circ$ and $\gamma = 75^\circ$, determine the magnitudes of its three components.

2.9 The pole is subjected to the force F which has components $F_x = 1.5$ kN and $F_z = 1.25$ kN. If $\beta = 75^\circ$, determine the magnitudes of F and F_y.



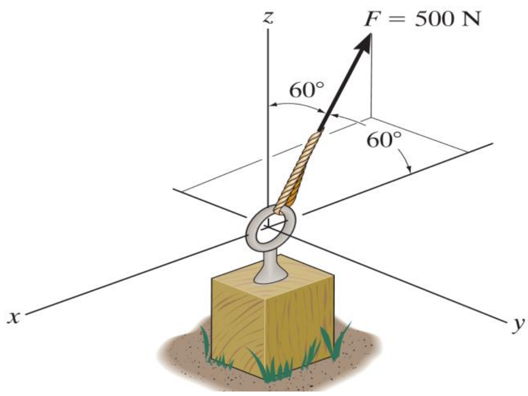
Problem 2.7



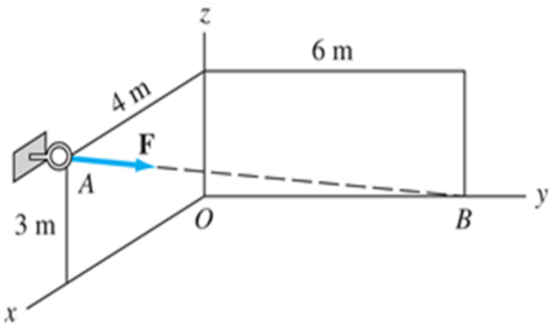
Problem 2.8&2.9

2.10 Express the force **F** as a Cartesian vector.

2.11 The cable attached to the eyebolt in Figure is pulled with the force **F** of magnitude 500 N. Determine the rectangular representation of this force.



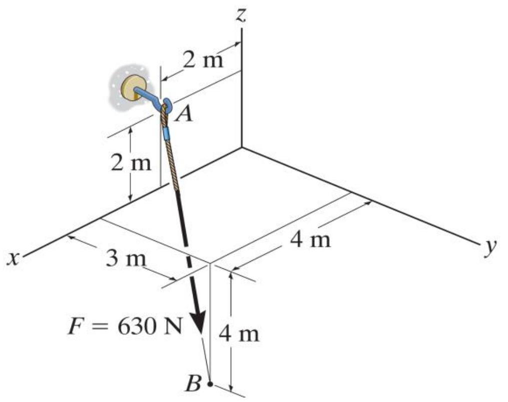
Problem 2.10



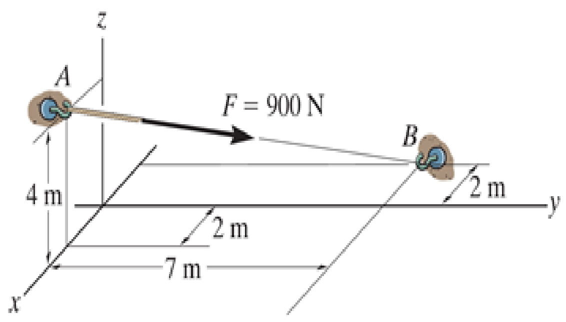
Problem 2.11

2.12 Express the force **F** as a Cartesian vector.

2.13 Express the force **F** as a Cartesian vector.



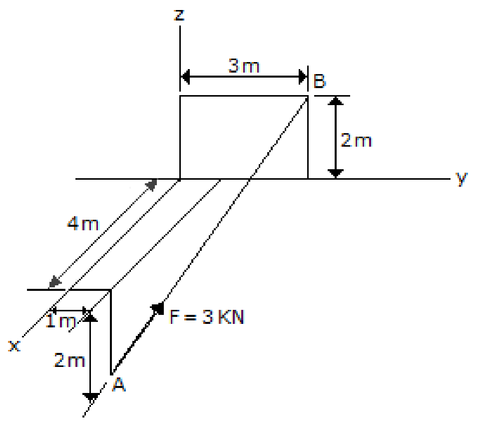
Problem 2.12



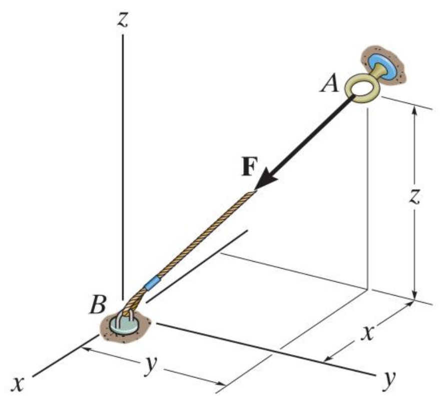
Problem 2.13

2.14 Express the force **F** as a Cartesian vector.

2.15 If $\vec{F} = 350\,\vec{i} - 250\,\vec{j} - 450\,\vec{k}$ and cable AB is 9 m long, determine the x, y, z coordinates of point A.



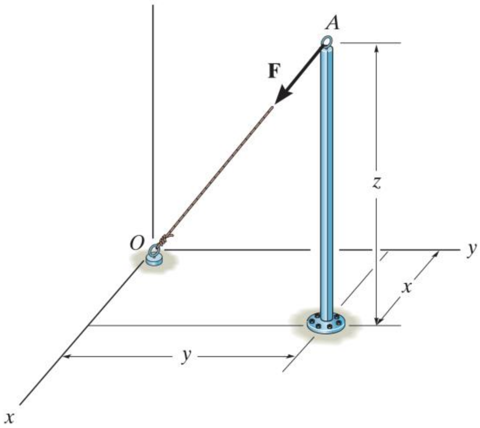
Problem 2.14



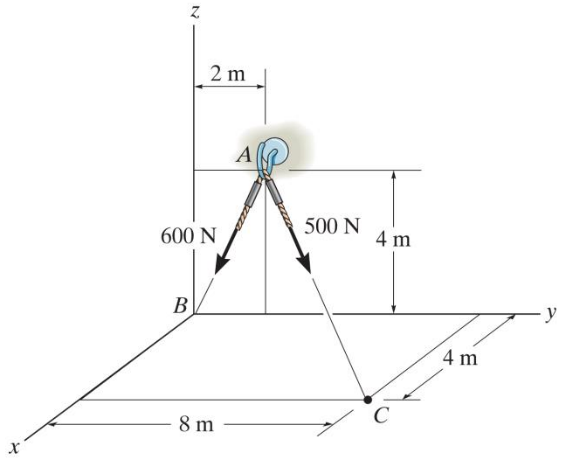
Problem 2.15

2.16 The cable AO exerts a force on the top of the pole of $\vec{F} = -120\,\vec{i} - 90\,\vec{j} - 80\,\vec{k}$ (N). If the cable has a length of 12 m. Determine the height z of the pole and the location (x,y) of the base.

2.17 Determine the magnitude and coordinate direction angles of the resultant force.



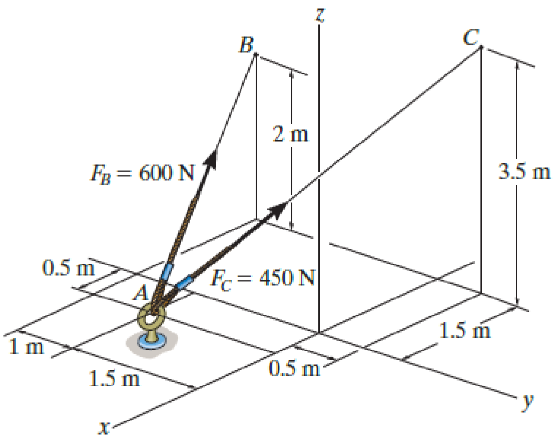
Problem 2.16



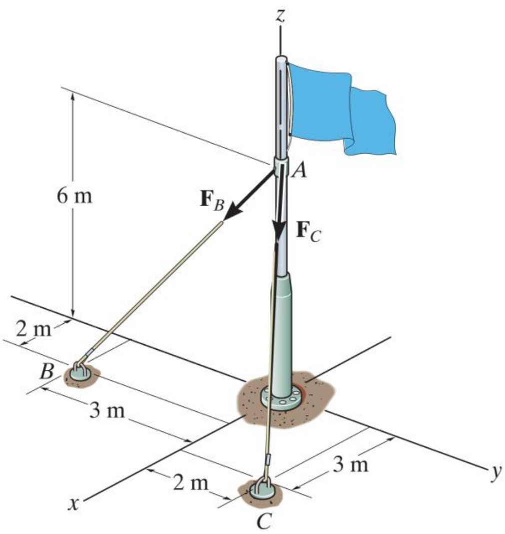
Problem 2.17

2.18 Determine the magnitude and coordinate direction angles of the resultant force.

2.19 If $F_B = 560$ N and $F_C = 700$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



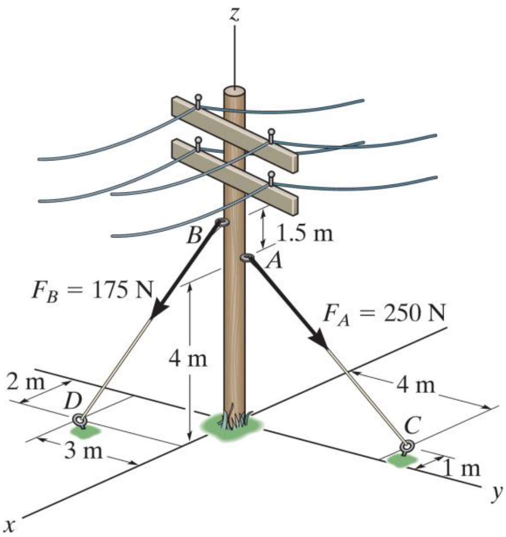
Problem 2.18



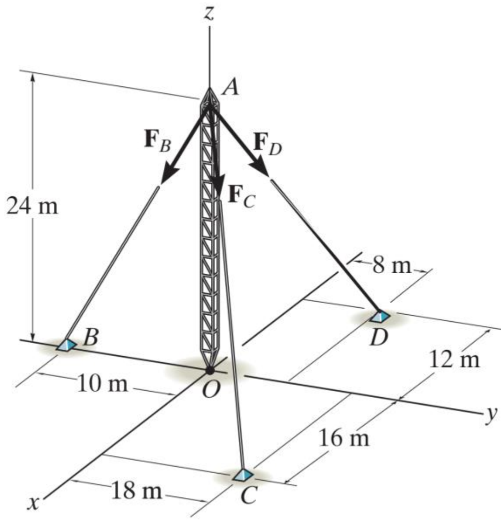
Problem 2.19

2.20 The guy wires are used to support the telephone pole. Determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.

2.21 The tower is supported by three cables. If the forces of these cables acting on the tower are $F_B = 520\ \text{N}$, $F_C = 680\ \text{N}$ and $F_D = 560\ \text{N}$. Determine the magnitude and coordinate direction angles of the resultant force.

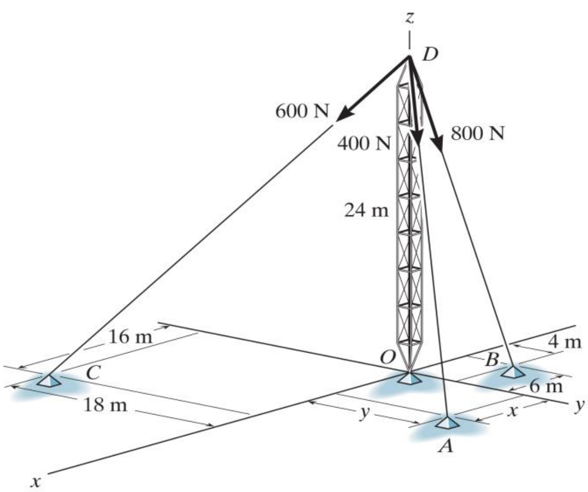


Problem 2.20

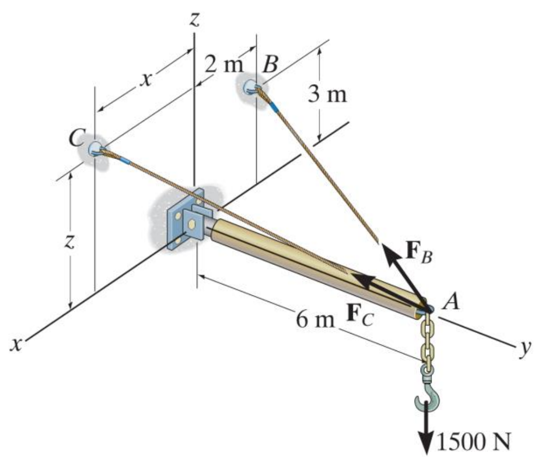


Problem 2.21

2.22 The tower is held in place by three cables. If the force of each cable on the tower is shown, determine the magnitude and coordinate direction angles of the resultant force. Take $x = 15\ \text{m}$ and $y = 20\ \text{m}$.



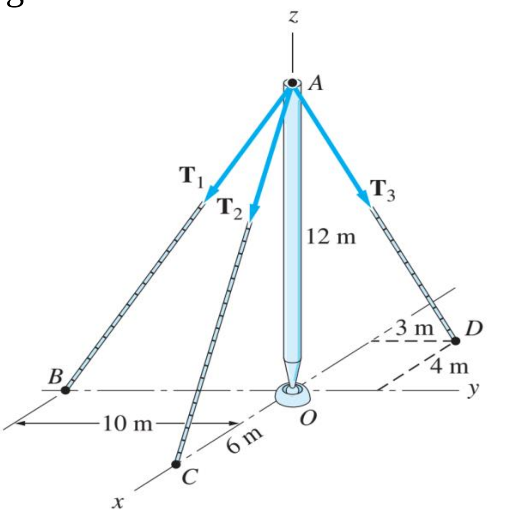
Problem 2.22



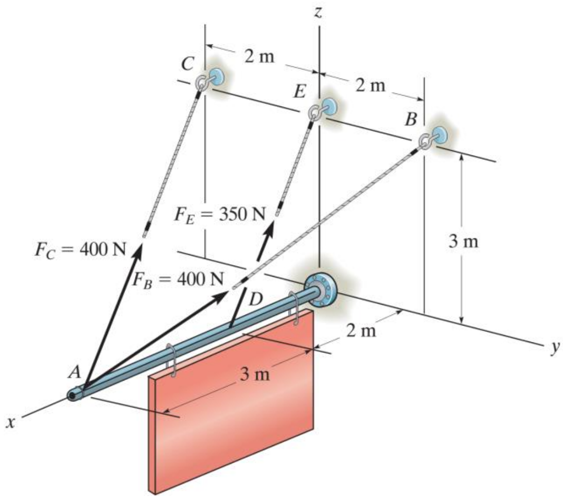
Problem 2.23

2.23 Two cables are used to secure the overhang boom in position and support the 1500 N load. If the resultant force is directed along the boom from point A towards O . determine the magnitudes of the resultant force and forces F_B and F_C. Set $x = 3$ m and $z = 2$ m.

2.24 Three cable tensions T_1, T_2 and T_3 act at the top of the flag pole. Given that the resultant force for the three tensions is $\vec{F}_R = -400\,\vec{K}$, find the magnitudes of the cable tensions.



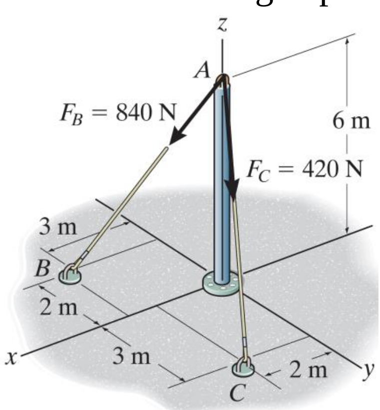
Problem 2.24



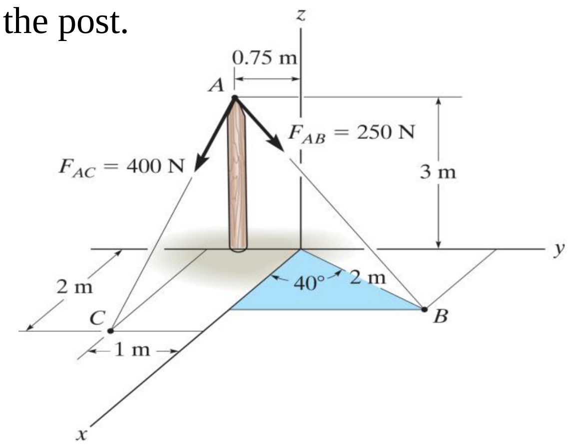
Problem 2.25

2.25 The three supporting cables exert the forces shown on the sign. Determine the magnitude and coordinate direction angles of the resultant force.

2.26 Determine the magnitude and coordinate direction angles of the resultant force acting at point A on the post.



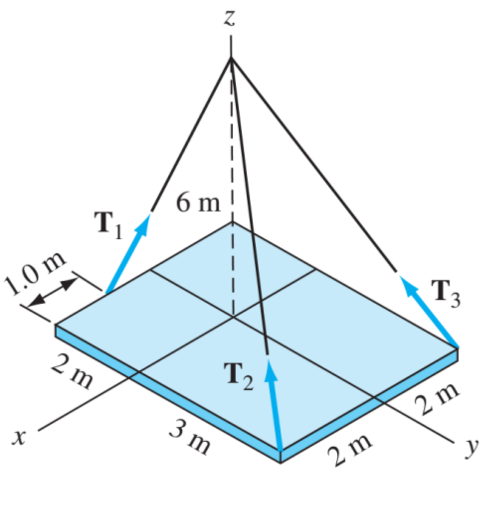
Problem 2.26

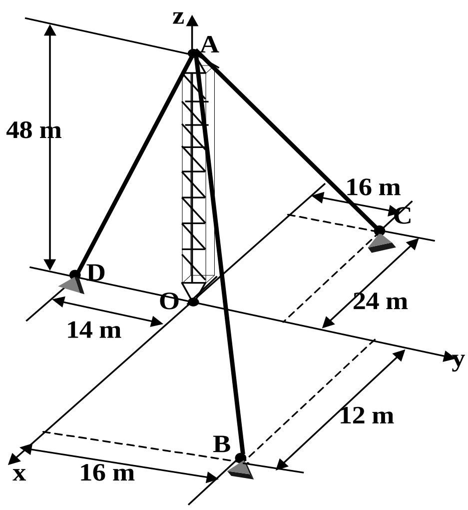


Problem 2.27

2.27 Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.

2.28 The magnitudes of the three forces acting on the plate are $T_1 = 100\,\text{kN}$, $T_2 = 80\,\text{kN}$ and $T_3 = 50\,\text{kN}$. Determine the magnitude and coordinate direction angles of the resultant force.



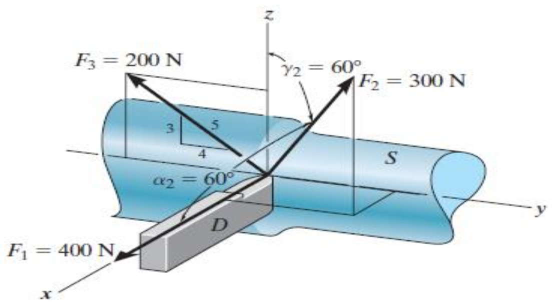


Problem 2.28 Problem 2.29&2.30

2.29 Knowing that the tension in the cable **AB** is 39 kN, determine the required values of tension in **AC** and **AD** so the resultant of the three forces applied at A is vertical.

2.30 Knowing that the tension in the cable **AC** is 28 kN, determine the required values of tension in **AB** and **AD** so the resultant of the three forces applied at A is vertical.

2.31 The shaft S exerts three force components on the die D. Find the magnitude and coordinate direction angles of the resultant force.



CHAPTER (3)

EQUILIBRIUM OF A PARTICLE

3.1 Condition for the Equilibrium of a Particle

A particle is said to be in equilibrium if it remains at rest if originally at rest, or has a constant velocity if originally in motion. Most often, however, the term “equilibrium” or, more specifically, “static equilibrium” is used to describe an object at rest. To maintain equilibrium, it is necessary to satisfy Newton’s first law of motion, which requires the resultant force acting on a particle to be equal to zero. This condition is stated by the equation of equilibrium,

$$\Sigma \vec{F} = 0 \qquad (3.1)$$

Where $\Sigma \vec{F}$ is the vector sum of all the forces acting on the particle. Not only Equation (3.1) is a necessary condition for equilibrium, it is also a sufficient condition.

To apply the equation of equilibrium, we must account for all the known and unknown forces **(**$\Sigma \vec{F}$**)** which act on the particle. The best way to do this is to think of the particle as isolated and “free” from its surroundings. A drawing that shows the particle with all the forces that act on it is called **free-body diagram** (FBD). Before presenting a formal procedure as to how to draw a free-body diagram, we will first consider three types of supports often encountered in particle equilibrium problems.

Springs. If a linearly elastic spring (or cord) of un-deformed length $\boldsymbol{l_0}$ is used to support a particle, the length of the spring will change in direct proportion to the force **F** acting on it, Fig. 3.1. A characteristic that defines the “elasticity” of a spring is the **spring constant** or **stiffness k**. The magnitude of force exerted on a linearly elastic spring which has a stiffness **k** and is deformed (elongated or compressed) a distance $\mathbf{s = \mathit{l} - \mathit{l}_0}$, measured from its unloaded position, is

$$F = k\ s \qquad (3.2)$$

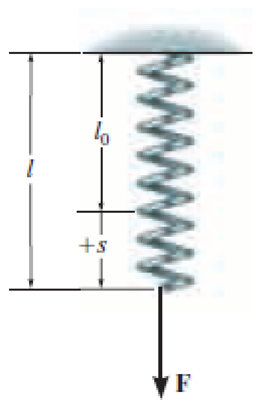


Fig. 3.1

Cables and Pulleys. Unless otherwise stated all cables (or cords) will be assumed to have negligible weight and they cannot stretch. Also, a cable can support only a tension or "pulling" force, and this force always acts in the direction of the cable. It is shown that the tension force developed in a continuous cable which passes over a frictionless pulley must have a constant magnitude to keep the cable in equilibrium. Hence, for any angle **θ**, shown in Fig. 3.2, the cable is subjected to a constant tension **T** throughout its length.

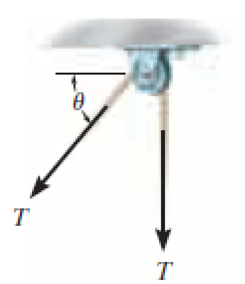


Fig. 3.2

Smooth Contact. If an object rests on a smooth surface, then the surface will exert a force on the object that is normal to the surface at the point of contact. An example of this is shown in Fig. 3.3(a). In addition to this normal force **N**, the cylinder is also subjected to its weight **W** and the force **T** of the cord, Fig. 3.3(b).

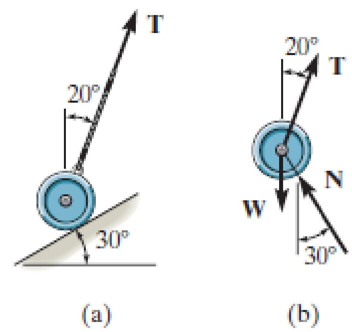


Fig. 3.3

3.2 Free-Body Diagram

Since we must account for all the forces acting on the particle when applying the equations of equilibrium, the importance of first drawing a free-body diagram cannot be overemphasized. To construct a free-body diagram, the following three steps are necessary.

Draw Outlined Shape.

Imagine the particle to be isolated or cut "free" from its surroundings. This requires removing all the supports and drawing the particle's outlined shape.

Show All Forces.

Indicate on this sketch all the forces that act on the particle. These forces can be **active forces**, which tend to set the particle in motion, or they can be **reactive forces** which are the result of the constraints or supports that tend to prevent motion. To account for all these forces, it may be helpful to trace around the particle's boundary, carefully noting each force acting on it.

Identify Each Force.

The forces that are known should be labeled with their proper magnitudes and directions. Letters are used to represent the magnitudes and directions of forces that are unknown.

3.3 Coplanar Force System

If a particle in equilibrium is subjected to a system of coplanar forces that lie in the x–y plane, then each force can be resolved into its $\vec{\mathbf{i}}$ and $\vec{\mathbf{j}}$ components. For equilibrium, these forces must sum to produce a zero force resultant, i.e.,

$$\Sigma \vec{F} = 0$$

$$\Sigma \vec{F} = \Sigma F_x \vec{i} + \Sigma F_y \vec{j}$$

For this vector equation to be satisfied, the resultant force's x and y components must both be equal to zero. Hence,

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad (3.3)$$

These two equations can be solved for at most two unknowns.

3.4 Force System in a Space

In the case of a three-dimensional force system, we can resolve the forces into their respective $\vec{\mathbf{i}}$, $\vec{\mathbf{j}}$ and $\vec{\mathbf{k}}$ components, so that

$$\Sigma \vec{F} = 0$$

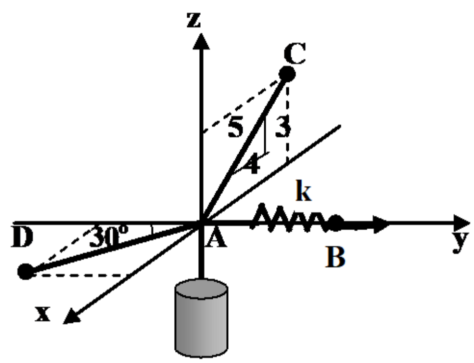
$$\Sigma \vec{F} = \Sigma F_x \vec{i} + \Sigma F_y \vec{j} + \Sigma F_z \vec{k}$$

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0 \qquad (3.4)$$

These three equations state that the algebraic sum of the components of all the forces acting on the particle along each of the coordinate axes must be zero. Using them we can solve for at most three unknowns.

<u>Example 3.1</u>

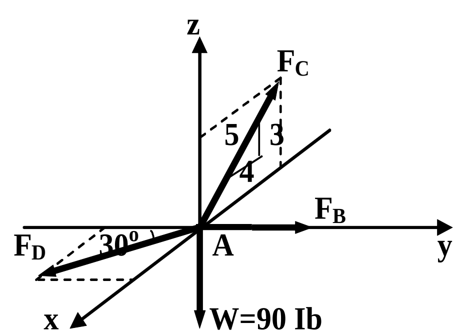
The 90 lb cylinder shown is supported by two cables and a spring having a stiffness k = 500 lb/ft. Determine the tension in each cable for equilibrium. Cable AD lies in the x-y plane and cable AC lies in the x-z plane.



Solution

The stretch of the spring can be determined once the force in the spring is determined.

Free-Body Diagram. The connection at A is chosen for the analysis since the cable forces are concurrent at this point.



Equations of Equilibrium. By inspection, each force can easily be resolved into its x, y, z components, and therefore the three scalar equations of equilibrium can be directly applied. Considering components directed along the positive axes as "positive", we have :

$$\Sigma F_x = 0; \qquad F_D \sin 30^o - \frac{4}{5} F_C = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad - F_D \cos 30^o + F_B = 0 \qquad (2)$$

$$\Sigma F_z = 0; \qquad \frac{3}{5} F_C - 90 = 0 \qquad (3)$$

Solving Eq.(3) for $\mathbf{F_C}$, then Eq. (1) for $\mathbf{F_D}$, and finally Eq.(2) for $\mathbf{F_B}$, we get :

F_C = 150 lb

F_D = 240 lb

F_B = 208 lb

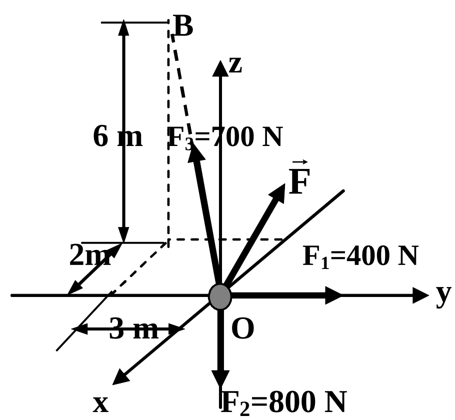
The tensile force $\mathbf{F_B}$ will cause stretching of the spring by an amount **s** such that:

F_B = k s

208 = 500 s Hence s = 0.416 ft

Example 3.2

Determine the magnitude and coordinate direction angles of force $\vec{\mathbf{F}}$ that are required for equilibrium of particle 0.



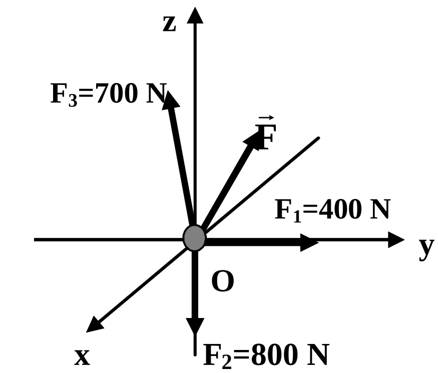
Solution

Free-Body Diagram. Four forces act on particle 0.

Equations of Equilibrium. A Cartesian vector analysis will be used for the solution in order to formulate the components of $\vec{\mathbf{F}}_3$. Hence:

$$\Sigma \vec{F} = 0 \qquad \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F} = 0 \qquad (1)$$

Expressing each of these forces in Cartesian vector form, noting that the coordinates of B are B (-2, -3, 6), we have



$\vec{F}_1 = (400\vec{j})$ N , $\vec{F}_2 = (-800\vec{k})$ N

$$\vec{F}_3 = F_3\vec{u}_B = F_3\frac{\vec{r}_B}{r_B} = 700\left[\frac{-2\vec{i} - 3\vec{j} + 6\vec{k}}{\sqrt{(-2)^2 + (-3)^2 + (6)^2}}\right]$$

$$= (-200\vec{i} - 300\vec{j} + 600\vec{k}) \quad \text{N}$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

Substituting into Eq. (1) yields:

$$400\vec{j} - 800\vec{k} - 200\vec{i} - 300\vec{j} + 600\vec{k} + F_x\vec{i} + F_y\vec{j} + F_z\vec{k} = 0$$

Equating the respective $\vec{i}, \vec{j}, \vec{k}$ components to zero, we have:

$\Sigma F_x = 0; \quad -200 + F_x = 0 \quad F_x = 200$ N

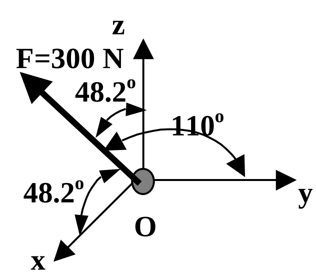
$\Sigma F_y = 0; \quad 400 - 300 + F_y = 0 \quad F_y = -100$ N

$\Sigma F_z = 0; \quad -800 + 600 + F_z = 0 \quad F_z = 200$ N

$$\vec{F} = (200\vec{i} - 100\vec{j} + 200\vec{k}) \quad \text{N}$$

$$F = \sqrt{(200)^2 + (-100)^2 + (200)^2} = 300 \quad \text{N}$$

$$\vec{u}_F = \frac{\vec{F}}{F} = \frac{200}{300}\vec{i} - \frac{100}{300}\vec{j} + \frac{200}{300}\vec{k}$$



$$\alpha = \cos^{-1}\left(\frac{200}{300}\right) = 48.2^{o} ,$$

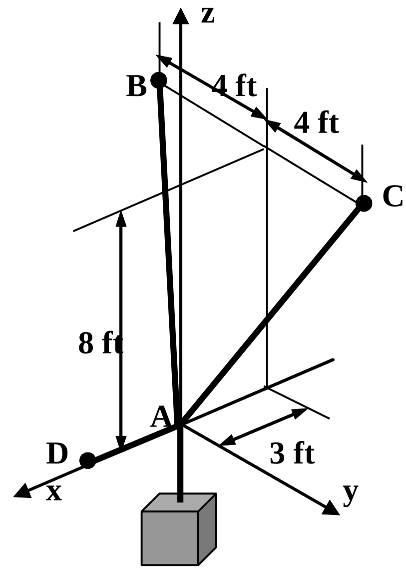
$$\beta = \cos^{-1}\left(\frac{-100}{300}\right) = 110^{o} ,$$

$$\gamma = \cos^{-1}\left(\frac{200}{300}\right) = 48.2^{o}$$

The magnitude and correct direction of $\vec{\mathbf{F}}$ are shown in last figure.

Example 3.3

Determine the force developed in each cable used to support the 40 lb crate shown



Solution

Free-Body Diagram. The free body diagram of the particle A is shown.

Equations of Equilibrium

$\Sigma\vec{F} = 0 \qquad \vec{F}_B + \vec{F}_C + \vec{F}_D + \vec{W} = 0 \qquad (1)$

Since the coordinates of points B and C are B (-3, -4, 8) and C(-3, 4, 8), we have:

$$\vec{F}_B = F_B \frac{-3\vec{i} - 4\vec{j} + 8\vec{k}}{\sqrt{(-3)^2 + (-4)^2 + (8)^2}}$$

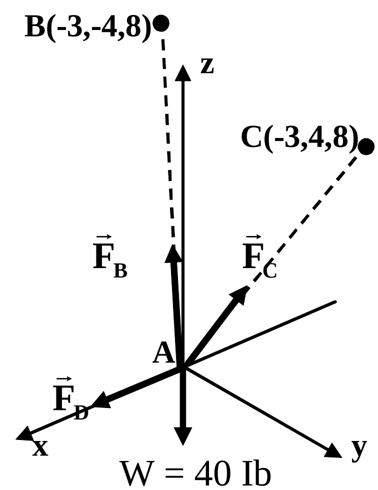
$$= -0.318F_B\vec{i} - 0.424F_B\vec{j} + 0.848F_B\vec{k}$$

$$\vec{F}_C = F_C \frac{-3\vec{i} + 4\vec{j} + 8\vec{k}}{\sqrt{(-3)^2 + (4)^2 + (8)^2}}$$

$$= -0.318F_C\vec{i} + 0.424F_C\vec{j} + 0.848F_C\vec{k}$$

$$\vec{F}_D = F_D\vec{i}$$

$$\vec{W} = -40\vec{k}$$



Substituting these forces into Eq. (1) gives:

$$-0.318F_B\vec{i} - 0.424F_B\vec{j} + 0.848F_B\vec{k}$$
$$-0.318F_C\vec{i} + 0.424F_C\vec{j} + 0.848F_B\vec{k}$$
$$+F_D\vec{i} - 40\vec{k} = 0$$

Equating the respective $\vec{\mathbf{i}}, \vec{\mathbf{j}}, \vec{\mathbf{k}}$ components to zero, yields:

$$\Sigma F_x = 0; \qquad -0.318F_B - 0.318F_C + F_D = 0 \qquad (1)$$

$$\Sigma F_y = 0; \qquad -0.424F_B + 0.424F_C = 0 \qquad (2)$$

$$\Sigma F_z = 0; \qquad 0.848F_B + 0.848F_C - 40 = 0 \qquad (3)$$

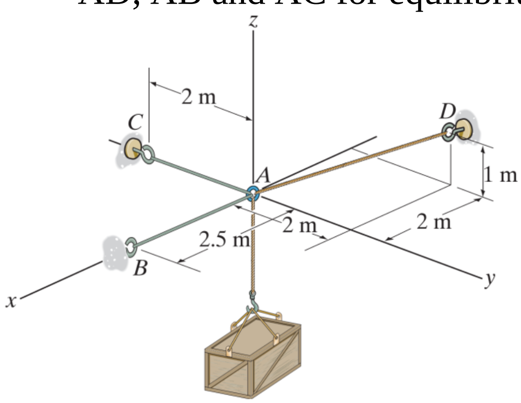
Equation (2) indicates that $F_B = F_C$. Thus, solving Eq.(3) for F_B and F_C and substituting the result into Eq.(1) to obtain F_D, we have:

$F_B = F_C = 23.6$ Ib and $F_D = 15.0$ Ib

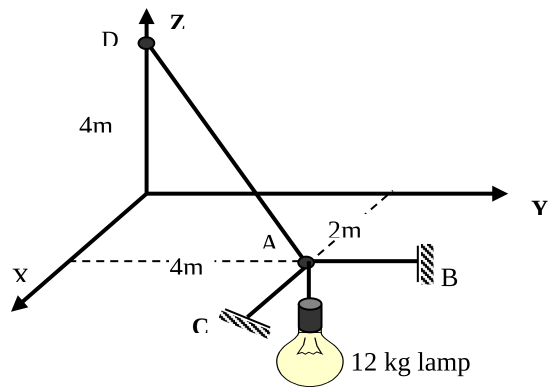
PROBLEMS

3.1 Determine the tension in the cables in order to support the 100 kg crate in the equilibrium position shown.

3.2 The three cables are used to support 12 kg lamp. Determine the Tension in AD, AB and AC for equilibrium.



Problem 3.1

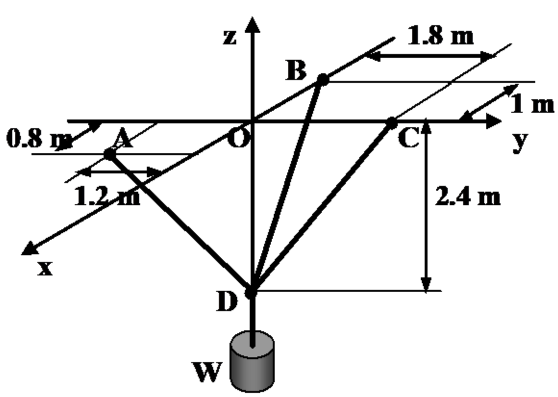


Problem 3.2

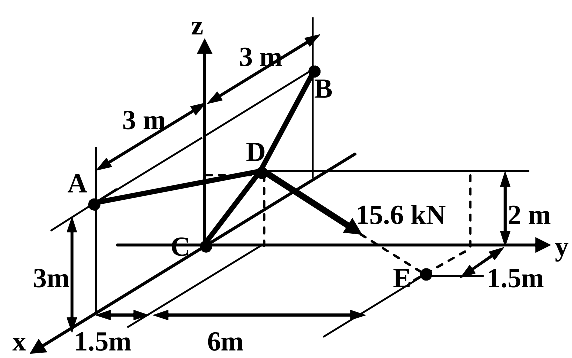
3.3 Three cables as shown support a cylinder. Determine the weight W of the cylinder, knowing that the tension in cable BD is 975 N.

3.4 Three cables as shown support a cylinder. Determine the weight W of the cylinder, knowing that the tension in cable CD is 350 N.

3.5 A cylinder of weight W = 110 N is supported by three cables as shown. Determine the tension in each cable.



Problem 3.3 – 3.5

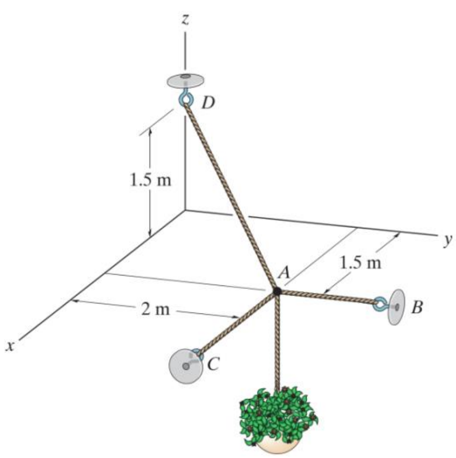


Problem 3.6

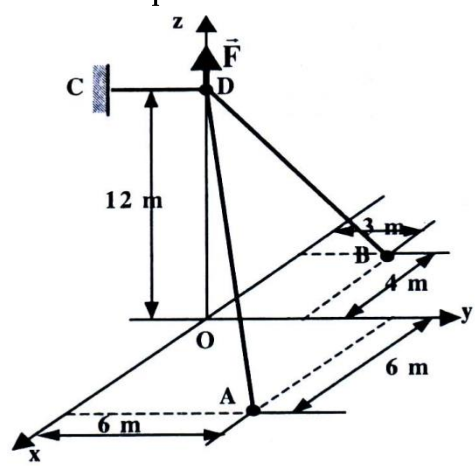
3.6 Three cables are connected at D , where a 15.60 kN force applied as shown. Determine the tension in each cable.

3.7 The three cables are used to support the 40 kg flowerpot. Determine the force developed in each cable for equilibrium.

3.8 Three cables are jointed at point D where an upward force F of magnitude 6 kN is applied. Find the tension in each cable for equilibrium.



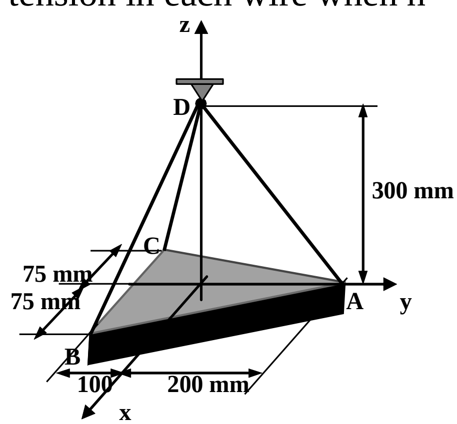
Problem 3.7



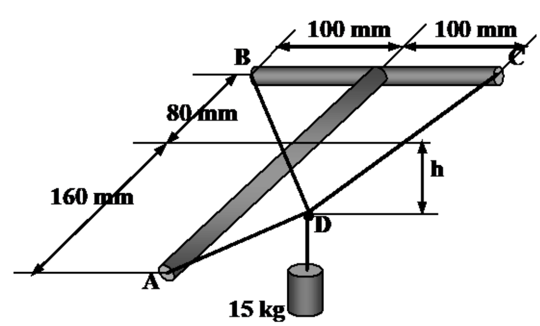
Problem 3.8

3.9 A 10 kg triangular plate is supported by three wires shown. determine the tension in each wire.

3.10 A 15 kg cylinder is supported by three wires as shown. Determine the tension in each wire when h = 60 mm.



Problem 3.9

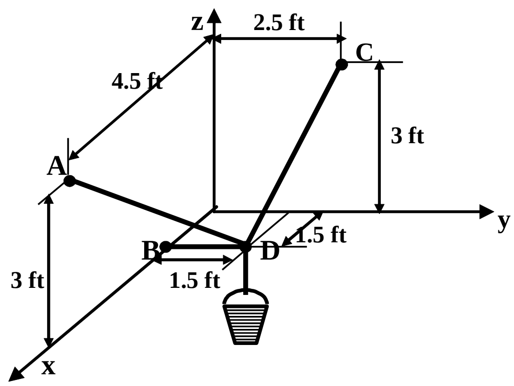


Problem 3.10 & 3.11

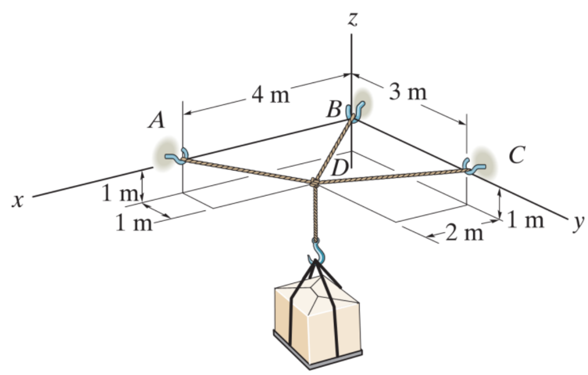
3.11 Solve Prob. 3.10 for the case when h = 20 mm why this arrangement results in higher tensions.

3.12 If the bucket and its contents have a total weight of 20 lb, determine the force in the supporting cables DA, DB and DC.

3.13 The crate has a mass of 130 kg. Determine the tension developed in each cable for equilibrium.



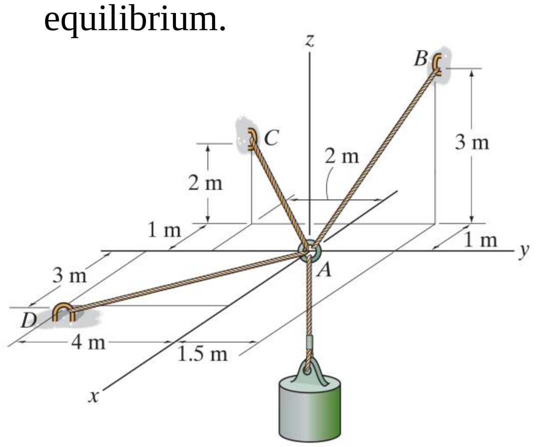
Problem 3.12



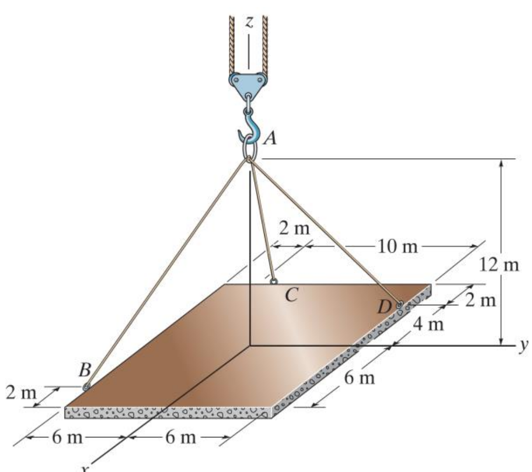
Problem 3.13

3.14 Determine the tension developed in cables **AB**, **AC** and **AD** required for equilibrium of the 75 kg cylinder.

3.15 The ends of the three cables are attached to a ring at *A* and to the edge of a uniform 150 kg plate. Determine the tension in each of the cables for equilibrium.



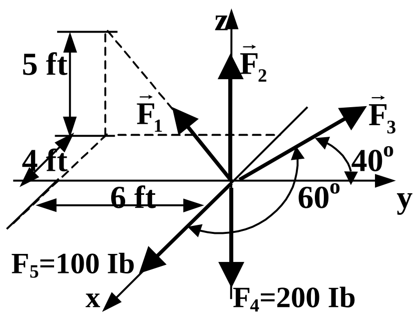
Problem 3.14



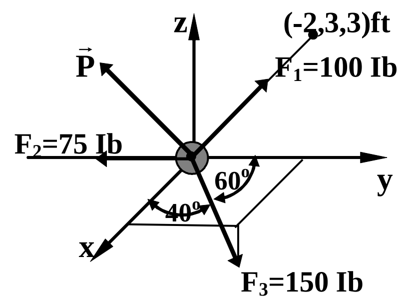
Problem 3.15

3.16 Determine the magnitude of forces $\vec{\mathbf{F}}_1, \vec{\mathbf{F}}_2$ and $\vec{\mathbf{F}}_3$ for equilibrium of the particle. Force $\vec{\mathbf{F}}_3$ is located in the positive x, y, z octant.

3.17 Determine the magnitude and coordinate direction angles of the force $\vec{\mathbf{P}}$ required to keep the concurrent force system in equilibrium.



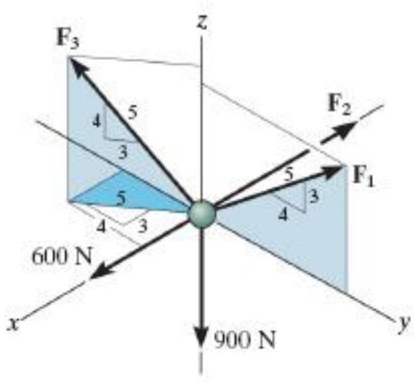
Problem 3.16



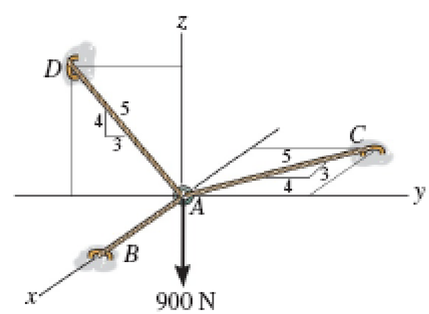
Problem 3.17

3.18 Determine the magnitude of forces **F**1, **F**2, **F**3, so that the particle is held in equilibrium.

3.19 Determine the tension developed in cables AB, AC and AD.



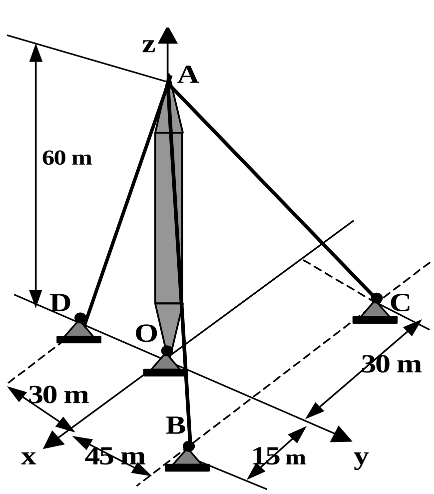
Problem 3.18



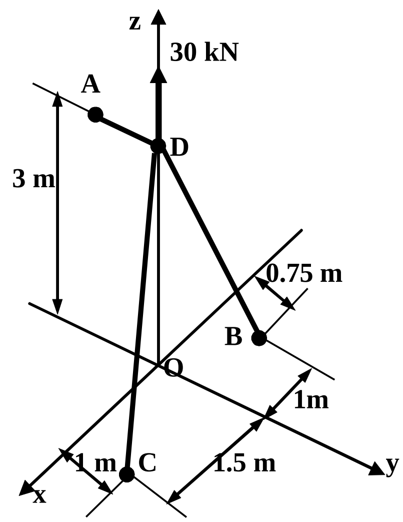
Problem 3.19

3.20 Three wires are attached to the top of the tower at A as shown in Figure. The tension in the wire AC is found to be 70 kN. Determine the required values of the tensions in AB and AD so that the resultant of the three forces applied at A is vertical.

3.21 Three cables are connected at D , where an upward force of 30 kN is applied. Determine the tension in each cable.



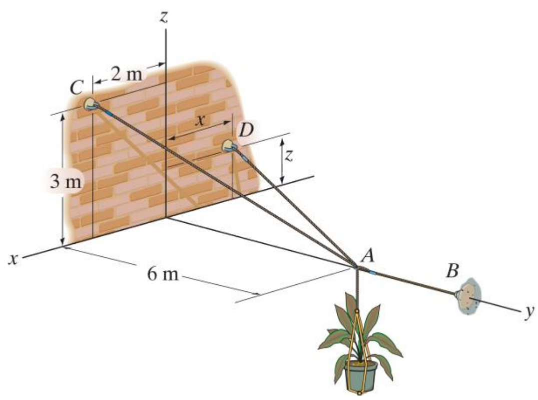
Problem 3.20



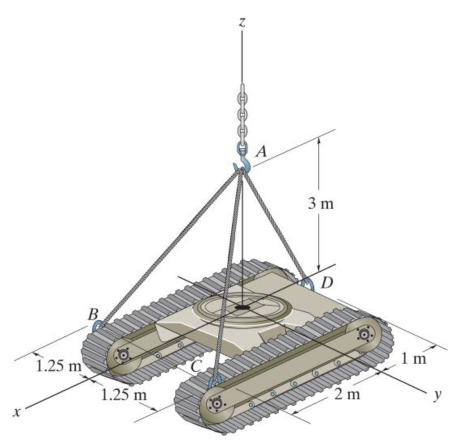
Problem 3.21

3.22 If the mass of the flowerpot is $50\,\text{kg}$, determine the tension developed in each wire for equilibrium. Set $x = 1.5\,\text{m}$ and $z = 2\,\text{m}$.

3.23 Determine the force in each of the three cables needed to lift the tractor which has a mass of $8\,\text{Mg}$.



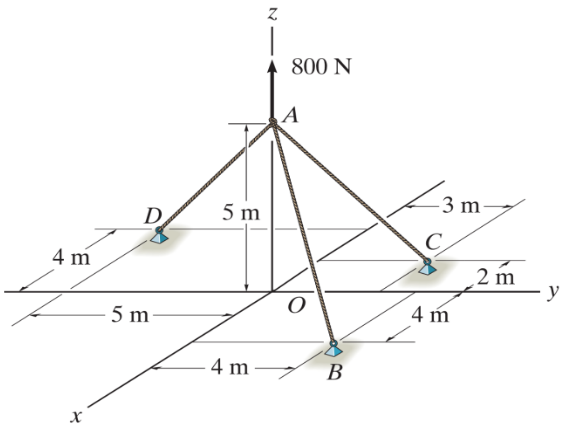
Problem 3.22



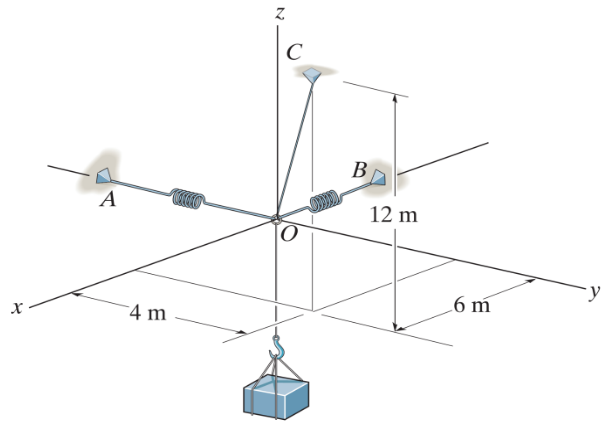
Problem 3.23

3.24 Determine the tension in each cable for equilibrium.

3.25 Determine the stretch in each of the two springs required to hold the 20 kg crate in the equilibrium position shown. Each spring has a stiffness of $k = 300\ \text{N/m}$.



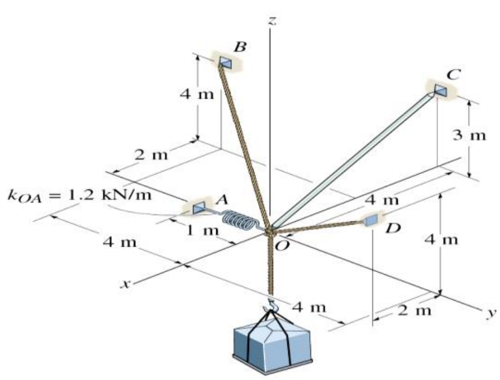
Problem 3.24



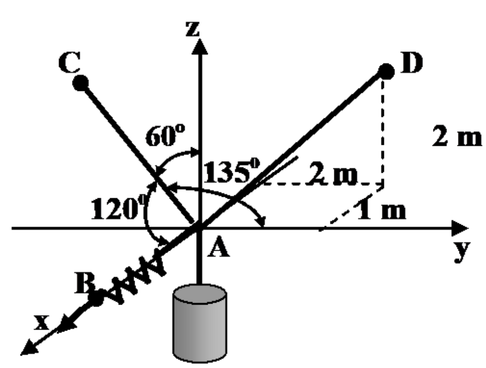
Problem 3.25

3.26 The 100 kg cylinder shown in Figure is supported by three cords, one of which is connected to a spring. Determine the tension in each cord and the stretch of the spring. Take the spring stiffness $k = 300\ \text{N/m}$.

3.27 If the maximum allowable tension in cable OB is 120 N, determine the tension developed in cables OC and OD, required to support the 490.5 N crate. The spring OA has an unstretched length of 0.8 m and a stiffness kOA = 1200 N/m. The force in the strut acts along the axis of the strut.



Problem 3.27



Problem 3.26

CHAPTER (4)

FORCE SYSTEM RESULTANTS

In Chapter (3), it was shown that the condition of equilibrium of a particle or a concurrent force system requires that the resultant of the force system must be equal to zero. In the next chapter, it will be shown that such a condition is necessary but not sufficient for the equilibrium of a rigid body. A further restriction must be made with regard to the non-concurrency of the applied force system, giving rise to the concept of moment. In this chapter, a formal definition of a moment will be presented. We will also present methods for determining the resultant of non-concurrent force systems.

4.1 Moment of a Force – Scalar Formulation

When a force is applied to a body it will produce a tendency for the body to rotate about a point that is not on the line of action of the force. This tendency to rotate is sometimes called the moment of a force or simply the **moment**. For example, consider a wrench used to unscrew the bolt in Fig. 4.1(a). If a force is applied to the handle of the wrench it will tend to turn the bolt about point O (or the z axis). The magnitude of the moment is directly proportional to the magnitude of **F** and the perpendicular distance or **moment arm d.** The larger the force or the longer the moment arm, the greater the moment or turning effect. Note that if the force **F** is applied at an angle $\theta \neq 90$, Fig. 4.1(b), then it will be more difficult to turn the bolt since the moment arm $d'' = d \sin \theta$ will be smaller than d. If **F** is applied along the wrench, Fig. 4.1(c), its moment arm will be zero since the line of action of **F** will intersect point O (the z axis). As a result, the moment of **F** about O is also zero and no turning can occur.

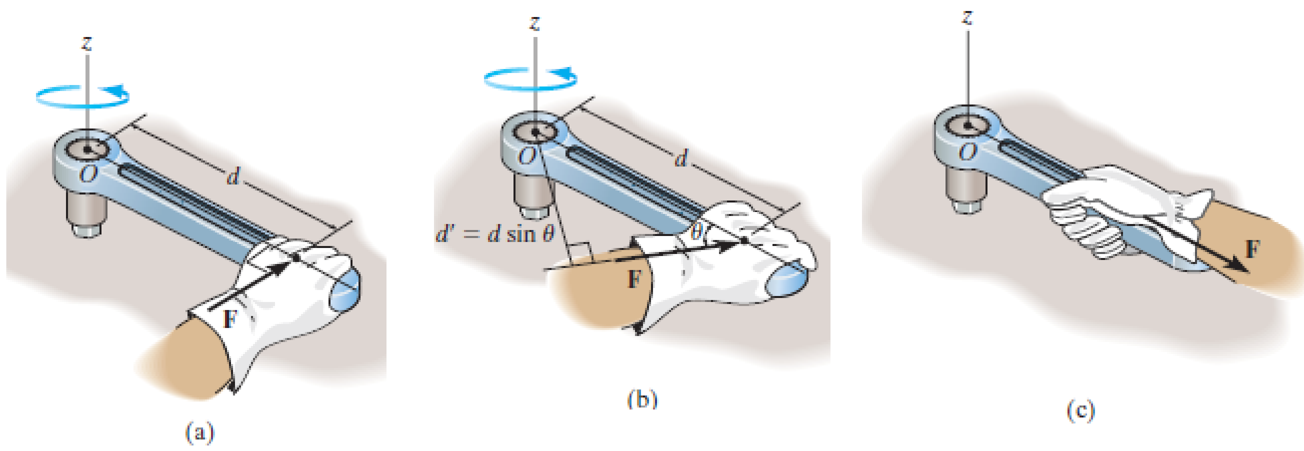


Fig. 4.1

We can generalize the above discussion and consider the force **F** and point O which lie in the shaded plane as shown in Fig. 4.2(a). The moment $\mathbf{M}_O$ about point O, or about an axis passing through O and perpendicular to the plane, is a vector quantity since it has a specified magnitude and direction.

Magnitude. The magnitude of $\mathbf{M}_O$ is

$$M_O = F\,d \qquad (4.1)$$

where d is the **moment arm** or perpendicular distance from the axis at point O to the line of action of the force. Units of moment magnitude consist of force times distance, e.g., N. m or lb. ft.

Direction. The direction of $\mathbf{M}_O$ is defined by its **moment axis**, which is perpendicular to the plane that contains the force **F** and its moment arm d. The right-hand rule is used to establish the sense of direction of $\mathbf{M}_O$. According to this rule, the natural curl of the fingers of the right hand, as

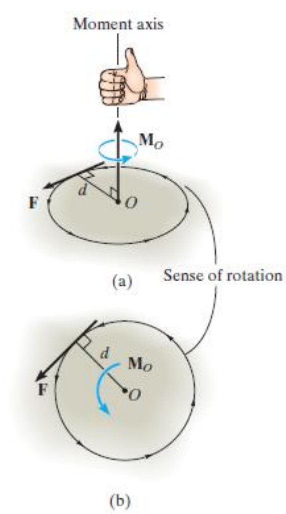


Fig. 4.2

they are drawn towards the palm, represent the rotation, or if no movement is possible, there is a tendency for rotation caused by the moment. As this action is performed, the thumb of the right hand will give the directional sense of $\mathbf{M}_O$, Fig. 4.2(a). Notice that the moment vector is represented three-dimensionally by a curl around an arrow. In two dimensions this vector is represented only by the curl as in Fig. 4.2(b). Since in this case the moment will tend to cause a counterclockwise rotation, the moment vector is actually directed out of the page.

Resultant Moment. For two-dimensional problems, where all the forces lie within the x–y plane, Fig. 4.3, the resultant moment $\mathbf{(M_R)_O}$ about point O (the z axis) can be determined by finding the algebraic sum of the moments caused by all the forces in the system. As a convention, we will generally consider positive moments as counterclockwise since they are directed along the positive z axis (out of the page). Clockwise moments will be negative. Doing this, the directional sense of each moment can be represented by a plus or minus sign. Using this sign convention, with a symbolic curl to define the positive direction, the resultant moment in Fig. 4.3 is therefore

$$\circlearrowleft + (M_R)_O = \sum Fd; \qquad (M_R)_O = F_1d_1 - F_2d_2 + F_3d_3$$

If the numerical result of this sum is a positive scalar, $\mathbf{(M_R)_O}$ will be a counterclockwise moment (out of the page); and if the result is negative, $\mathbf{(M_R)}_O$ will be a clockwise moment (into the page).

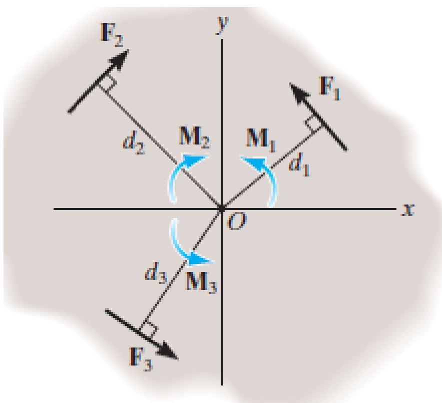


Fig. 4.3

4.2 Vector Product

The moment of a force will be formulated using Cartesian vectors in the next section. Before doing this, however it is first necessary to introduce the vector product (or cross product) of two vectors.

The vector product of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ yields the vector $\vec{\mathbf{C}}$, Fig.4.4, which is written:

$$\vec{C} = \vec{A} x \vec{B} \qquad (4.2)$$

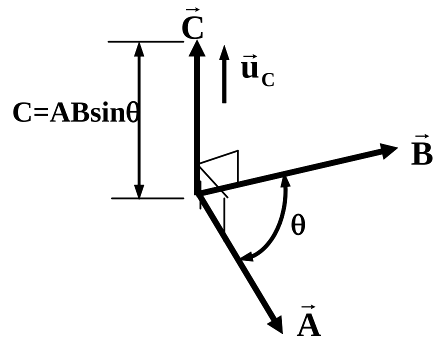


Fig. 4.4

and is read "$\vec{\mathbf{C}}$ equals $\vec{\mathbf{A}}$ cross $\vec{\mathbf{B}}$". **The magnitude** of $\vec{\mathbf{C}}$ is defined as the product of the magnitudes of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ and the sine of the angle θ between their tails.

Thus, $\mathbf{C = A\, B \sin \theta}$

The direction of the vector $\vec{\mathbf{C}}$ is perpendicular to the plane containing $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ such that the direction of $\vec{\mathbf{C}}$ is specified by the right hand rule, Fig.4.5. Knowing both the magnitude and direction of $\vec{\mathbf{C}}$, we can write :

$$\vec{C} = \vec{A} x \vec{B} = (AB \sin\theta)\vec{u}_C \qquad (4.3)$$

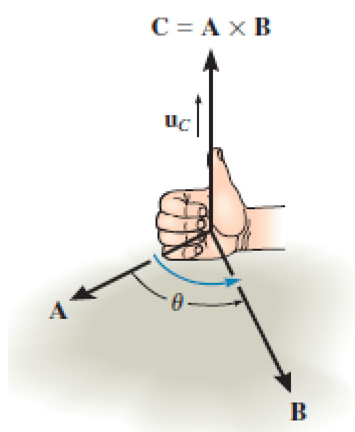


Fig. 4.5

where the scalar **AB sinθ** defines the **magnitude** of $\vec{\mathbf{C}}$ and the unit vector $\vec{\mathbf{u}}_C$ defines the **direction** of $\vec{\mathbf{C}}$.

Laws of Operation.

• The commutative law is not valid; i.e., $\vec{A} x \vec{B} \neq \vec{B} x \vec{A}$ Rather, $\vec{A} x \vec{B} = -\vec{B} x \vec{A}$

This is shown in Fig. 4.6 by using the right-hand rule. The cross product $\vec{B} x \vec{A}$ yields a vector that has the same magnitude but acts in the opposite direction to $\vec{C}$; i.e., $\vec{B} x \vec{A} = -\vec{C}$

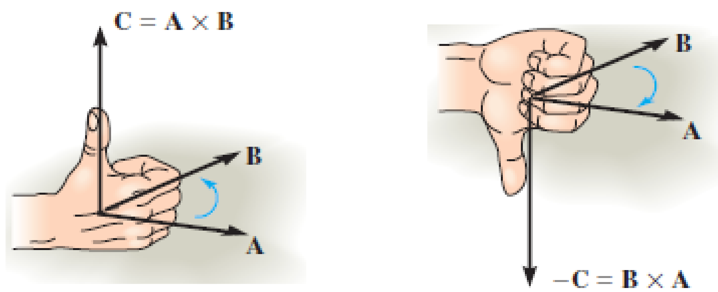


Fig. 4.6

• The vector cross product also obeys the distributive law of addition,

$\vec{A} \text{ x } (\vec{B} + \vec{D}) = (\vec{A} \text{ x } \vec{B}) + (\vec{A} \text{ x } \vec{D})$

Cartesian Vector Formulation

Equation (4.3) may be used to find the vector product of a pair of fundamental unit vectors. For example to find $\vec{\mathbf{i}} \mathbf{x} \vec{\mathbf{j}}$, the magnitude of the resultant vector is **(1) (1) (sin 90º) = (1) (1) (1) = 1**, and its direction is determined, using the right hand rule, Fig.4.7, as $+\vec{\mathbf{k}}$ direction.

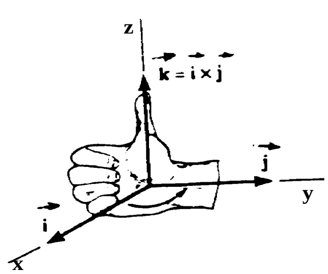


Fig. 4.7

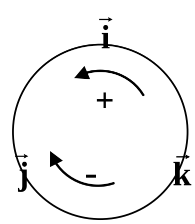


Fig. 4.8

Thus,

In a similar manner,

$\vec{i} \times \vec{j} = \vec{k}$ $\vec{i} \times \vec{k} = -\vec{j}$ $\vec{j} \times \vec{k} = \vec{i}$

$\vec{j} \times \vec{i} = -\vec{k}$ $\vec{k} \times \vec{i} = \vec{j}$ $\vec{k} \times \vec{j} = -\vec{i}$

$\vec{i} \times \vec{i} = 0$ $\vec{j} \times \vec{j} = 0$ $\vec{k} \times \vec{k} = 0$ (4.4)

These results should not be memorized; rather, it should be clearly understood how each is obtained by using the right-hand rule and the definition of the cross product. A simple scheme shown in Fig. 4.8 is helpful for obtaining the same results when the need arises. If the circle is constructed as shown, then "crossing" two unit vectors in a counterclockwise fashion around the circle yields the positive third unit vector; e.g., $\vec{k} \times \vec{i} = \vec{j}$. "Crossing" clockwise, a negative unit vector is obtained; e.g., $\vec{i} \times \vec{k} = -\vec{j}$.

Vector Product in Terms of Rectangular Components

Consider the vector product of two general vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ which are expressed in Cartesian vector form.

$$\vec{A} \times \vec{B} = (A_x \vec{i} + A_y \vec{j} + A_z \vec{k}) \times (B_x \vec{i} + B_y \vec{j} + B_z \vec{k})$$

Carrying up the vector product operations and considering equation (4.4), the above equation can be written in a compact determinant form as:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - A_z B_y)\vec{i} - (A_x B_z - A_z B_x)\vec{j} + (A_x B_y - A_y B_x)\vec{k}$$

(4.5)

4.3 Force Transmissibility

The principle of transmissibility states that the condition of equilibrium or motion of a rigid body will remain unchanged if a force $\vec{\mathbf{F}}$ acting at a given point A of the rigid body is transmitted to act at point B or at any other point on the force line of action Fig. 4.9. This principle, which states that the action of a force may be transmitted along its line of action, is based on experimental evidence. The force $\vec{\mathbf{F}}$, acting on a rigid body, has the properties of sliding vector and therefore act at any point along its line of action without changing effect on the rigid body.

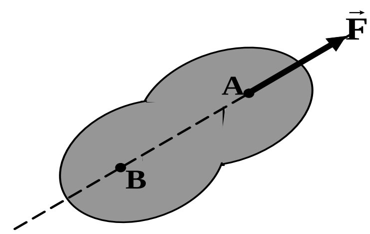


Fig. 4.9

4.4 Moment of a Force – Vector Formulation

The moment of a force $\vec{\mathbf{F}}$ about point O, or actually about the moment axis passing through O and perpendicular to the plane containing O and $\vec{\mathbf{F}}$, Fig. 4.10, can be expressed using the vector cross product, namely,

$$\vec{M}_O = \vec{r} \times \vec{F} \qquad (4.6)$$

Here $\vec{r}$ represents a position vector directed from O to any point P on the line of action of $\vec{\mathbf{F}}$. The position vector $\vec{\mathbf{r}}$ and the force $\vec{\mathbf{F}}$ define the plane shown in the figure.

According to the definition of the vector product,

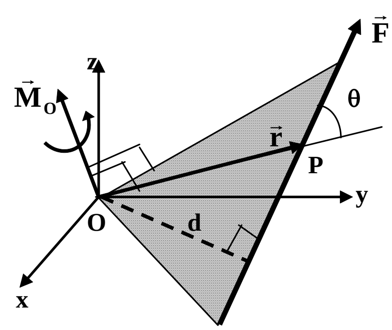


Fig. 4.10

given in the previous section, the moment vector $\vec{\mathbf{M}}_o$ must be perpendicular to the plane containing $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$. The sense of $\vec{\mathbf{M}}_o$ is defined by the sense of the rotation which would bring the vector $\vec{\mathbf{r}}$ in line with the vector $\vec{\mathbf{F}}$. The magnitude of the moment of $\vec{\mathbf{F}}$ about point o is:

$$M_O = F\,r\sin\theta = F\,d$$

Cartesian Vector Formulation.

If the components of the position vector are **x, y** and **z**, then the vector $\vec{\mathbf{r}}$ can be expressed as:

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

Knowing the force $\vec{\mathbf{F}}$ in terms of its rectangular components:

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

The moment vector $\vec{\mathbf{M}}_o$ of the force $\vec{\mathbf{F}}$ about the point o can be written in the form:

$$\vec{M}_o = \vec{r} x \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= (yF_z - ZF_y)\vec{i} - (xF_z - zF_x)\vec{j} + (xF_y - yF_x)\vec{k} \qquad (4.7)$$

In the SI system of units, where a force is measured in Newton (N) and a distance in meters (m), the moment of a force will be expressed in Newton meters (N.m).

Rectangular Components of the Moment Vector:

The obtained moment vector $\vec{\mathbf{M}}_{\mathbf{O}}$ in Eq. (4.7) may be written in the form:

$$\vec{M}_o = M_{ox}\vec{i} + M_{oy}\vec{j} + M_{oz}\vec{k} \qquad (4.8)$$

where the components $\mathbf{M_{Ox}}$, $\mathbf{M_{Oy}}$ and $\mathbf{M_{Oz}}$, Fig. 4.11, are expressed as:

$$M_{ox} = yF_z - zF_y$$
$$M_{oy} = zF_x - xFz$$
$$M_{oz} = xF_y - yF_x$$

These components measure the ability of the force $\vec{\mathbf{F}}$ to cause rotation about **x**, **y** and **z** axes, respectively.

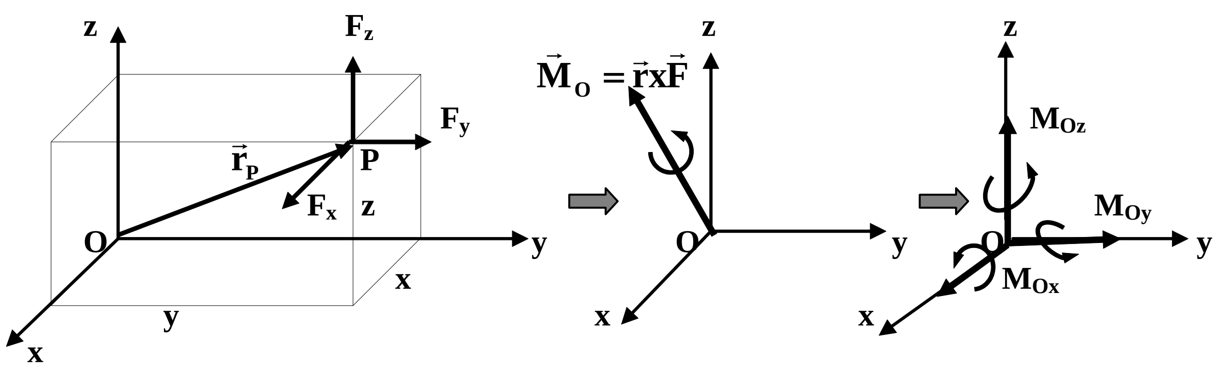


Fig. 4.11

Resultant Moment of a System of Forces

If a body is acted upon by a system of forces, Fig. 4.12, the resultant moment of the forces about point O can be determined by vector addition of the moment of each force. This resultant can be written symbolically as:

$$(\vec{M}_R)_o = \Sigma(\vec{r} \times \vec{F}) \qquad (4.9)$$

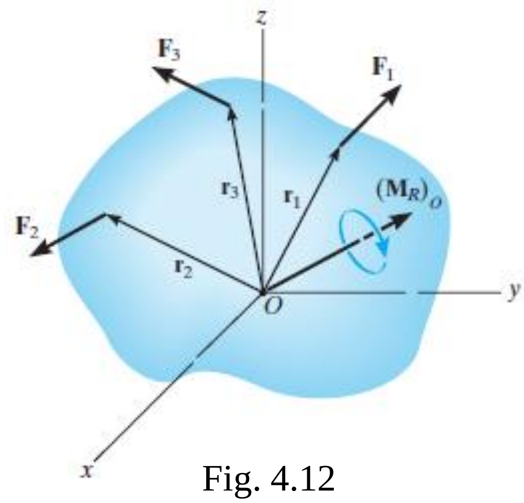


Fig. 4.12

Moment of force about a General Point

The moment of $\vec{\mathbf{F}}$ about a general point Q is given by:

$$\vec{M}_Q = \vec{r}_{QP} x \vec{F}$$

but $\qquad \vec{r}_{QP} = \vec{r}_P - \vec{r}_Q$

$$\vec{M}_Q = (\vec{r}_P - \vec{r}_Q) x \vec{F}$$

$$= \vec{r}_P x \vec{F} - \vec{r}_Q x \vec{F}$$

$$\vec{M}_Q = \vec{M}_O - \vec{r}_Q x \vec{F} \qquad (4.10)$$

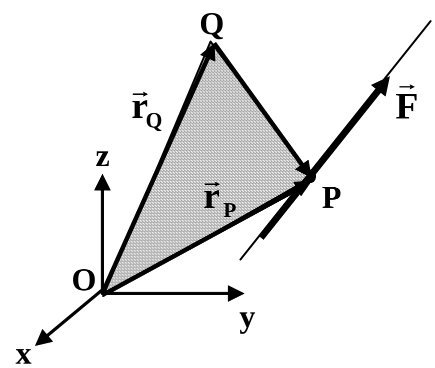


Fig. 4.13

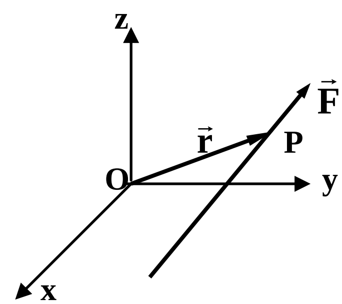
This equation is used when $\vec{\mathbf{M}}_o$ as well as the coordinates of a general point Q are known.

Example 4.1

Determine the moment of the force $\vec{F} = 4\vec{i} - 8\vec{j} + 3\vec{k}$ [N], which acts at the point P with coordinates (-3, 8, 2) m, about the origin o.

Solution

The position vector of the point P is:



$$\vec{r} = \overrightarrow{OP} = -3\vec{i} + 8\vec{j} + 2\vec{k} \qquad [m]$$

The moment of the force $\vec{F}$ about o is obtained as:

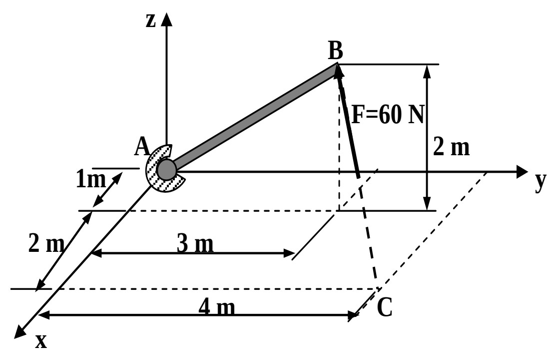
$$\vec{M}_O = \vec{r}x\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 8 & 2 \\ 4 & -8 & 3 \end{vmatrix}$$

Getting the determinant value we obtain:

$$\vec{M}_O = (24+16)\vec{i} - (-9-8)\vec{j} + (24-32)\vec{k}$$

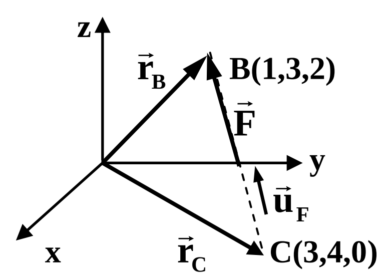
or $\quad \vec{M}_O = 40\vec{i} + 17\vec{j} - 8\vec{k} \qquad [N.m]$

Example 4.2



The pole is subjected to a 60 N force that is directed from C to B. Determine the magnitude of the moment created by this force about point A.

Solution



As shown, either one of two position vectors can be used for the solution, since $\vec{M}_A = \vec{r}_B x\vec{F}$ or $\vec{M}_A = \vec{r}_C x\vec{F}$ the position vectors are represented as:

$$\vec{\mathbf{r}}_B = \vec{\mathbf{i}} + 3\vec{\mathbf{j}} + 2\vec{\mathbf{k}} \qquad \mathbf{m}$$

$$\vec{\mathbf{r}}_C = 3\vec{\mathbf{i}} + 4\vec{\mathbf{j}} \qquad \mathbf{m}$$

The force has a magnitude of 60 N and a direction specified by the unit vector $\vec{\mathbf{u}}_F$, directed from C to B. Thus,

$$\vec{F} = 60\vec{u}_F = 60\left[\frac{(1-3)\vec{i} + (3-4)\vec{j} + (2-0)\vec{k}}{\sqrt{(-2)^2 + (-1)^2 + (2)^2}}\right]$$

$$= (-40\vec{i} - 20\vec{j} + 40\vec{k}) \qquad \text{N}$$

Substituting into the determinant formulation, equation (4.7) we have:

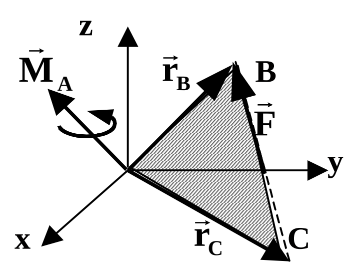
$$\vec{M}_A = \vec{r}_B x\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & 2 \\ -40 & -20 & 40 \end{vmatrix}$$

$$= [3(40) - 2(-20)]\vec{i} - [1(40) - 2(-40)]\vec{j} + [1(-20) - 3(-40)]\vec{k}$$

or

$$\vec{M}_A = \vec{r}_C x\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 4 & 0 \\ -40 & -20 & 40 \end{vmatrix}$$

$$= [4(40) - 0(-20)]\vec{i} - [3(40) - 0(-40)]\vec{j} + [3(-20) - 4(-40)]\vec{k}$$



In both cases,

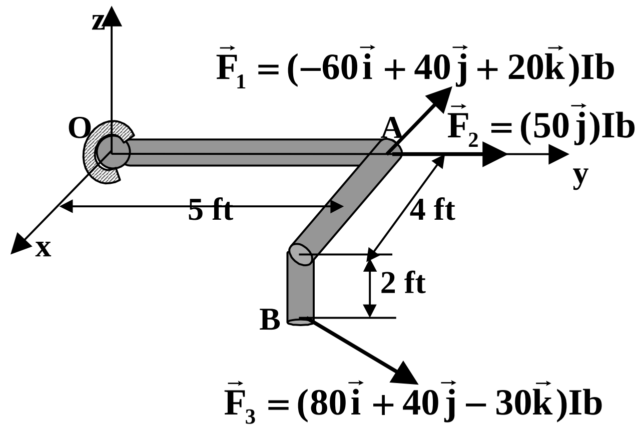
$$\vec{M}_A = 160\vec{i} - 120\vec{j} + 100\vec{k} \qquad \text{N.m}$$

The magnitude of $\vec{\mathbf{M}}_\mathbf{A}$ is therefore:

$$M_A = \sqrt{(160)^2 + (-120)^2 + (100)^2} = 224 \qquad \text{N.m}$$

Example 4.3

Three forces act on the rod shown. Determine the resultant moment they create about the flange at o, and determine the direction of this moment axis.

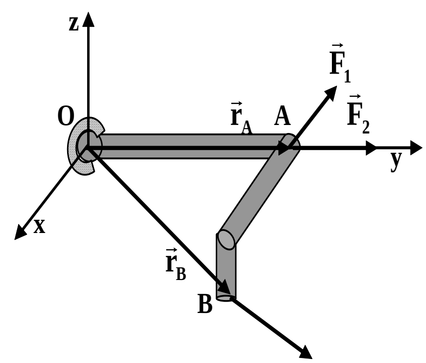


Solution

Position vectors are directed from point o to each force. These vectors are:

$\vec{r}_A = 5\vec{j}$ ft

$\vec{r}_B = 4\vec{i} + 5\vec{j} - 2\vec{k}$ ft



Since $\vec{F}_2 = 50\vec{j}$ Ib, and the Cartesian components of the other forces are given, the resultant moment about O is given by, equation (4.9).

$$\vec{M}_o = \Sigma\vec{r}x\vec{F} = \vec{r}_A x\vec{F}_1 + \vec{r}_A x\vec{F}_2 + \vec{r}_B x\vec{F}_3$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 5 & 0 \\ 0 & 50 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix}$$

$$= [5(20) - 40(0)]\vec{i} - 0\,\vec{j} + [0(40) - (-60)(5)]\vec{k}$$
$$+ [0\vec{i} + 0\vec{j} + 0\vec{k}] + [5(-30) - 40(-2)]\vec{i} - [4(-30) - 80(-2)]\vec{j} + [4(40) - 80(5)]\vec{k}$$

$$= 30\,\vec{i} - 40\,\vec{j} + 60\,\vec{k} \qquad \text{Ib.ft}$$

The moment axis is directed along the line of action of $\mathbf{M_o}$. Since the magnitude of this moment is:

$$M_O = \sqrt{(30)^2 + (-40)^2 + (60)^2} = 78.1 \qquad \text{Ib.ft}$$

The unit vector which defines the direction of the moment axis is:

$$\vec{u} = \frac{\vec{M}_o}{M_o} = \frac{30\vec{i} - 40\vec{j} + 60\vec{k}}{78.1}$$

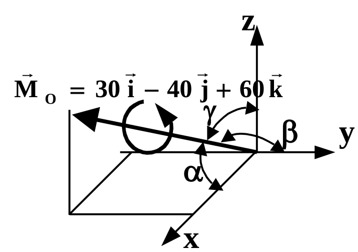
$$= 0.384\vec{i} - 0.512\vec{j} + 0.768\vec{k}$$

Therefore, the coordinate direction angles of the axis are:

$\cos\alpha = 0.384 \qquad \alpha = 67.4^o$

$\cos\beta = -0.512 \qquad \beta = 121^o$

$\cos\gamma = 0.768 \qquad \gamma = 39.8^o$



4.5 Scalar Product and Scalar Triple Product

Scalar Product

The scalar (or dot) product of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is defined as the product of the magnitude of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ and the cosine of the angle **θ** between their tails, Fig.4.14. Expressed in equation form as:

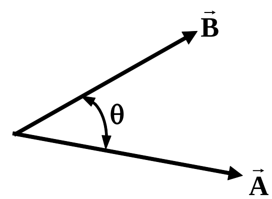


Fig. 4.14

$$\vec{A}.\vec{B} = AB\cos\theta \qquad (4.11)$$

Laws of Operation.

1. Commutative law: $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

2. Distributive law: $\vec{A}.(\vec{B} + \vec{D}) = \vec{A}.\vec{B} + \vec{A}.\vec{D}$

Cartesian Vector Formulation

This definition may be used to find the scalar product of the fundamental-unit vectors. For example; $\vec{i}.\vec{i} = (1)(1)\cos 0^o = 1$ and $\vec{i}.\vec{j} = (1)(1)\cos 90^o = 0$.

In a similar manner we can get:

$$\begin{aligned} \vec{i}.\vec{i} &= 1 \quad , \quad \vec{i}.\vec{j} = 0 \\ \vec{j}.\vec{j} &= 1 \quad , \quad \vec{i}.\vec{k} = 0 \\ \vec{k}.\vec{k} &= 1 \quad , \quad \vec{k}.\vec{j} = 0 \end{aligned} \qquad (4.12)$$

If the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are expressed in Cartesian vector form, application of equations (4.11) and (4.12) leads to:

$$\begin{aligned} \vec{A}.\vec{B} &= (A_x\vec{i} + A_y\vec{j} + A_z\vec{k}).(B_x\vec{i} + B_y\vec{j} + B_z\vec{k}) \\ &= A_xB_x + A_yB_y + A_zB_z \end{aligned} \qquad (4.13)$$

Thus, to determine the dot product of two Cartesian vectors, multiply their corresponding x, y, z components and sum these products algebraically. Note that the result will be either a positive or negative scalar, or it could be zero.

Applications

The dot product has two important applications in mechanics.

• **The angle formed between two vectors or intersecting lines.**

The angle θ between the tails of vectors $\vec{A}$ and $\vec{B}$ in Fig. 2.14 can be determined from equation (4.11) and written as

$$\theta = \cos^{-1}\left(\frac{\vec{A}.\vec{B}}{AB}\right) \qquad 0^o \le \theta \le 180^o$$

Here $\vec{A}.\vec{B}$ is found from equation (4.13). In particular, notice that if $\vec{A}.\vec{B} = 0$, $\theta = \cos^{-1} 0 = 90^o$ so that $\vec{A}$ will be perpendicular to $\vec{B}$.

• **The components of a vector parallel to a line**

The component of vector $\vec{A}$ parallel to or collinear with the line aa in Fig. 4.15 is defined by A_a where A_a = A cos θ. This component is sometimes referred to as the **projection** of $\vec{A}$ onto the line, since a right angle is formed in the construction. If the direction of the line is specified by the unit vector $\vec{u}_a$, then since u_a = 1, we can determine the magnitude of A_a directly from the dot product (Eq. 4.11); i.e.,

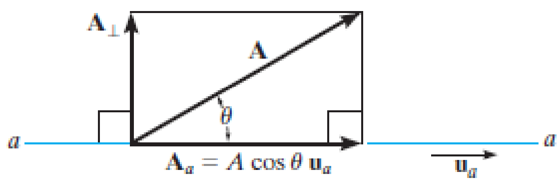


Fig. 4.15

$$A_a = A\cos\theta = \vec{A}.\vec{u}_a$$

Hence, the scalar projection of $\vec{A}$ along a line is determined from the dot product of $\vec{A}$ and the unit vector $\vec{u}_a$ which defines the direction of the line.

<u>**The scalar triple product**</u>

The scalar triple product of three vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$ is defined as:

$$\vec{A}.(\vec{B}x\vec{C})$$

obtained by forming the scalar product of $\vec{\mathbf{A}}$ with the vector product of $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$, Fig.4.16.

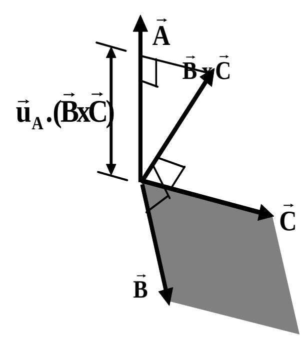


Fig. 4.16

The scalar triple product of vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$ may be written in terms of their rectangular components. Denoting $\vec{\mathbf{B}}\mathbf{x}\vec{\mathbf{C}}$ by $\vec{\mathbf{D}}$, and using formula (4.13) to express the scalar product of $\vec{\mathbf{A}}$ and $\vec{\mathbf{D}}$, we write

$$\vec{A}.(\vec{B}x\vec{C}) = \vec{A}.\vec{D} = A_x D_x + A_y D_y + A_z D_z$$

Substituting the components of $\vec{\mathbf{D}}$ and using vector product, the above triple scalar product can be written in the following determinant:

$$\vec{A}.(\vec{B}x\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} \qquad (4.14)$$

4.6 Moment of a Force about a Given Axis

Consider a force $\vec{\mathbf{F}}$ acting at point A as shown in Fig.4.17. The moment of this force about the origin o is given by equation (4.6).

$$\vec{M}_O = \vec{r}x\vec{F}$$

Let OL be an axis through 0 and $\vec{\mathbf{u}}_L$ is a unit vector along OL. The moment of $\vec{\mathbf{F}}$ about OL is denoted by $\mathbf{M_L}$ and is given by the projection of the moment $\vec{\mathbf{M}}_0$ on the axis OL. Thus:

$$M_L = M_O \cos\theta$$

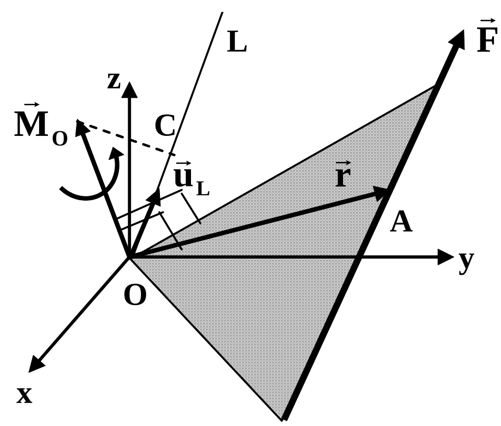


Fig. 4.17

where $\boldsymbol{\theta}$ is the angle between the tails of the two vectors $\vec{\mathbf{M}}_O$ and $\vec{\mathbf{u}}_L$.

Utilizing the properties of the scalar product, and noting that the magnitude of $\vec{\mathbf{u}}_L$ is 1, we can write:

$$M_L = \vec{u}_L.\vec{M}_O = \vec{u}_L.(\vec{r}x\vec{F}) \qquad (4.15)$$

The unit vector $\vec{\mathbf{u}}_L$ may be expressed in Cartesian vector form as:

$$\vec{u}_L = \ell\vec{i} + m\vec{j} + n\vec{k} \qquad (4.16)$$

where $\ell = \cos\alpha_L$, $m = \cos\beta_L$, $n = \cos\gamma_L$

Equations (4.15) and (4.16) show that the moment $\mathbf{M_L}$ of $\vec{\mathbf{F}}$ about the axis OL is the scalar value obtained by forming the scalar triple product of $\vec{\mathbf{u}}_L$, $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$.

In terms of the rectangular components of the three vectors $\vec{\mathbf{u}}_L$, $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$, we may write:

$$M_L = \begin{vmatrix} \ell & m & n \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \qquad (4.17)$$

where: ℓ, **m** and **n** are the components of unit vector $\vec{\mathbf{u}}_L$ along the axis OL.

x, y and **z** are the components of the position vector $\vec{\mathbf{r}}$ drawn from a point on line L to a point on the line of action of the force $\vec{\mathbf{F}}$.

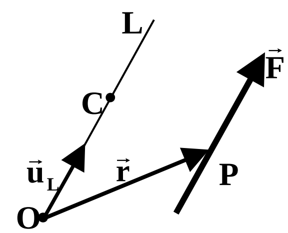
$\mathbf{F_x}$, $\mathbf{F_y}$ and $\mathbf{F_z}$ are the components of the force $\vec{\mathbf{F}}$.

Example 4.4

Find the moment of the force $\vec{F} = -2\vec{i} + 3\vec{j}$ [N] which passes through the point P with coordinates (1, 4,-2) m about the axis OL containing the point C having coordinates (2, 2, 1) m, where o is the origin of the coordinate system of axes.

Solution

The unit vector along line OL can be obtained in terms of coordinates of the two points o and C as:



$$\vec{u}_L = \frac{\overrightarrow{OC}}{OC} = \frac{2\vec{i} + 2\vec{j} + \vec{k}}{\sqrt{(2)^2 + (2)^2 + (1)^2}}$$

$$= \frac{1}{3}(2\vec{i} + 2\vec{j} + \vec{k}) \qquad \text{(a)}$$

The position vector $\vec{r} = \overrightarrow{OP}$ can be written as:

$$\vec{r} = \vec{i} + 4\vec{j} - 2\vec{k} \qquad \text{(b)}$$

The force $\vec{\mathbf{F}}$ is written as:

$$\vec{F} = -2\vec{i} + 3\vec{j} \qquad [N] \qquad \text{(c)}$$

Substituting the components of $\vec{\mathbf{u}}_L$, $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$ from (a), (b) and (c) into expression (4.17), the moment of the force $\vec{\mathbf{F}}$ about the line OL is obtained as:

$$M_L = \begin{vmatrix} \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ 1 & 4 & -2 \\ -2 & 3 & 0 \end{vmatrix}$$

or

$$M_L = \frac{2}{3}(0+6) - \frac{2}{3}(0-4) + \frac{1}{3}(3+8)$$

$$M_L = 4 + \frac{8}{3} + \frac{11}{3} = \frac{31}{3} \qquad [N.m]$$

4.7 Moment of a Couple

A **couple** is defined as two parallel forces that have the same magnitude, but opposite directions. They are separated by a perpendicular distance d, Fig. 4.18. Since the resultant force is zero, the only effect of a couple is to produce or make tendency of rotation in a specified direction.

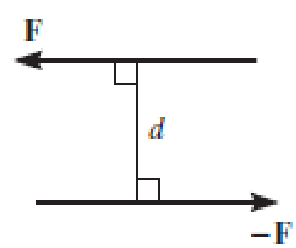


Fig. 4.18

The moment produced by a couple is called a **couple moment**. We can determine its value by finding the sum of the moments of both couple forces about any arbitrary point O.

For example, in Fig. 4.19, position vectors $\vec{r}_A$ and $\vec{r}_B$ are directed from point O to points A and B lying on the line of action of $-\vec{\mathbf{F}}$ and $\vec{\mathbf{F}}$.

The couple moment determined about O is therefore

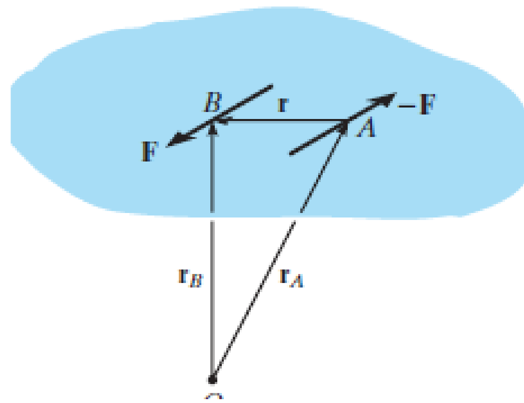


Fig. 4.19

$$\vec{M} = \vec{r}_B \times \vec{F} + \vec{r}_A \times (-\vec{F}) = (\vec{r}_B - \vec{r}_A) \times \vec{F}$$

$$\vec{M} = \vec{r} \times \vec{F} \qquad (4.18)$$

This result indicates that a couple moment is a vector perpendicular to the plane containing the two forces. It is called a **free vector**, i.e., it can act at any point since $\vec{M}$ depends only upon the position vector $\vec{r}$ directed between the forces and not the position vectors $\vec{r}_A$ and $\vec{r}_B$, directed from the arbitrary point O to the forces.

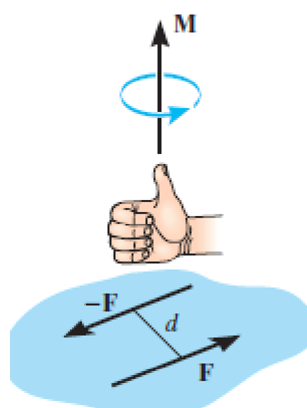


Fig. 4.20

The magnitude of M is:

$$M = F.\, d \qquad (4.19)$$

where **d** is the perpendicular distance between the two forces. The sense of $\vec{\mathbf{M}}$ is defined by the right hand rule.

Example 4.5

$\vec{F}_1 = 2\vec{i} - 3\vec{j}$ [N] and $\vec{F}_2 = -2\vec{i} + 3\vec{j}$ [N], are two forces. P and Q are two points on their lines of action with coordinates (0, -5, 2) m and (1, -2, 0) m, respectively.

a) Find the moment of the two forces about Q.

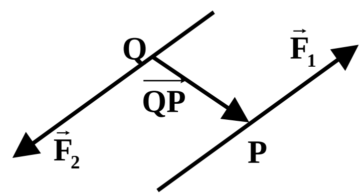
b) Prove that the two forces form a couple and find its magnitude and direction.

Solution

$$\vec{M}_Q = \overrightarrow{QP} x \vec{F}_1 + 0 \qquad (a)$$

where $$\overrightarrow{QP} = -\vec{i} - 3\vec{j} + 2\vec{k} \qquad (b)$$

$$\vec{F}_1 = 2\vec{i} - 3\vec{j} \qquad (c)$$



Substituting (b), (c) into (a) we have:

$$\vec{M}_Q = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -3 & 2 \\ 2 & -3 & 0 \end{vmatrix}$$

$$\vec{M}_Q = 6\vec{i} + 4\vec{j} + 9\vec{k} \quad [N.m]$$

To prove that the system forms a couple, we find the moment of the two forces about any other point such as P.

$$\vec{M}_P = 0 + \overrightarrow{PQ} x \vec{F}_2 \qquad (d)$$

$$\overrightarrow{PQ} = \vec{i} + 3\vec{j} - 2\vec{k} \qquad (e)$$

$$\vec{F}_2 = -2\vec{i} + 3\vec{j} \qquad (f)$$

Substituting (e), (f) into (d) we have:

$$\vec{M}_P = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & -2 \\ -2 & 3 & 0 \end{vmatrix}$$

$$\vec{M}_P = 6\vec{i} + 4\vec{j} + 9\vec{k} \qquad [\text{N.m}]$$

$$\vec{M}_P = \vec{M}_Q = \vec{M}$$

So $\vec{\mathbf{M}}$ is a "free" vector, hence $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ form a couple, having the magnitude:

$$M = \sqrt{(6)^2 + (4)^2 + (9)^2} = 11.53 \qquad [\text{N.m}]$$

The direction of the couple is given by its direction angles **α, β** and **γ** such as:

$$\cos\alpha = \frac{M_x}{M} = \frac{6}{11.53} = 0.52 \quad , \quad \alpha = 58.64^o$$

$$\cos\beta = \frac{M_y}{M} = \frac{4}{11.53} = 0.35 \quad , \quad \beta = 69.7^o$$

$$\cos\gamma = \frac{M_z}{M} = \frac{9}{11.53} = 0.78 \quad , \quad \gamma = 38.69^o$$

4.8 Resolution of a Force at A into a Force at Point o and a Couple

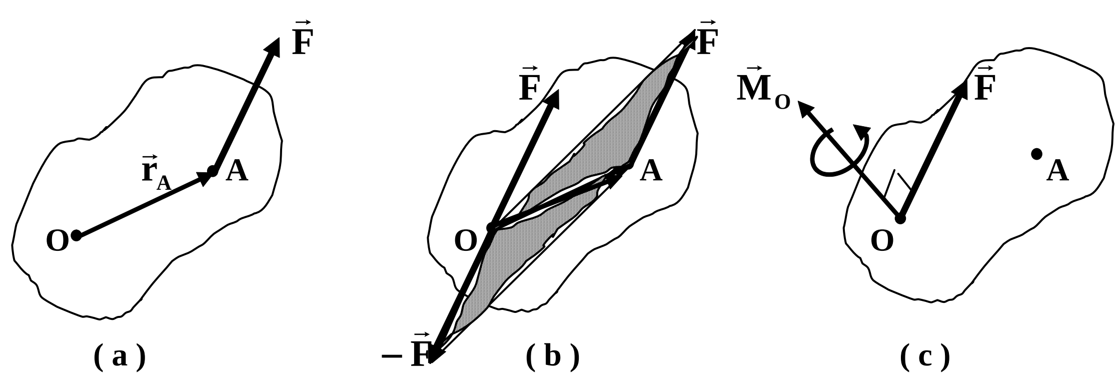


Fig. 4.21

Suppose that a force $\vec{\mathbf{F}}$ acts at the point A as shown in Fig.4.21(a). We may attach two forces ($\vec{\mathbf{F}}$ and $-\vec{\mathbf{F}}$) at o without modifying the action of the original force on the rigid body, Fig.4.21(b). So, as a result, we have at the point o a force $\vec{\mathbf{F}}$ and a couple of moment $\vec{\mathbf{M}}_{\mathbf{O}} = \vec{\mathbf{r}}_{\mathbf{A}} \mathbf{x} \vec{\mathbf{F}}$, as shown in Fig.4.9(c). Thus, any force $\vec{\mathbf{F}}$ may be moved to an arbitrary point o, provided that a couple is added, of moment equal to the moment of $\vec{\mathbf{F}}$ about o. One can note that $\vec{\mathbf{F}}$ and $\vec{\mathbf{M}}_{\mathbf{O}}$ vectors are perpendicular to each other.

4.9 Reduction of a System of Forces to One Force and One Couple

Consider a system of n forces $\vec{\mathbf{F}}_1$, $\vec{\mathbf{F}}_2$, ..., $\vec{\mathbf{F}}_i$, ..., $\vec{\mathbf{F}}_n$ acting on a rigid body at points **A_1 , A_2, ..., A_i, ..., A_n**, respectively. The position vectors of these points are labeled $\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2, ..., \vec{\mathbf{r}}_i, ..., \vec{\mathbf{r}}_n$ as shown in Fig. 4.22(a).

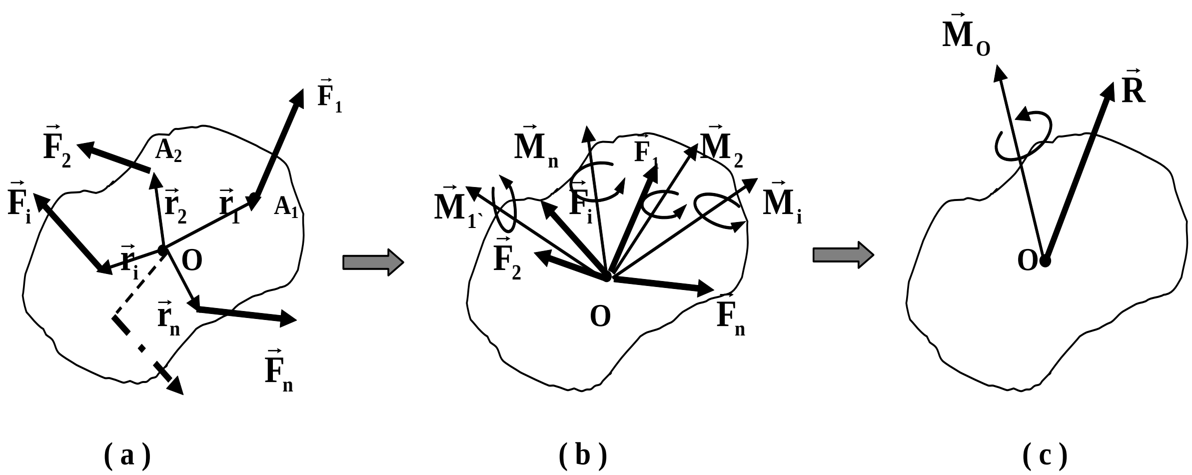


Fig. 4.22

Each force $\vec{\mathbf{F}}$ can be replaced at O by a force $\vec{\mathbf{F}}$ plus a couple of moment $\vec{\mathbf{M}} = \vec{\mathbf{r}} \mathbf{x} \vec{\mathbf{F}}$. Thus, at O we have $\vec{\mathbf{F}}_1$, $\vec{\mathbf{F}}_2$, ..., $\vec{\mathbf{F}}_i$, ..., $\vec{\mathbf{F}}_n$ plus $\vec{\mathbf{M}}_1, \vec{\mathbf{M}}_2, ..., \vec{\mathbf{M}}_i, ..., \vec{\mathbf{M}}_n$ Fig.4.22(b). So, we say that the given system of space forces may be reduced to a force $\vec{\mathbf{R}}$ and a couple $\vec{\mathbf{M}}_o$ acting at O, Fig.4.22(c), such that the resultant force $\vec{\mathbf{R}}$ is equal to the vectorial sum of all forces.

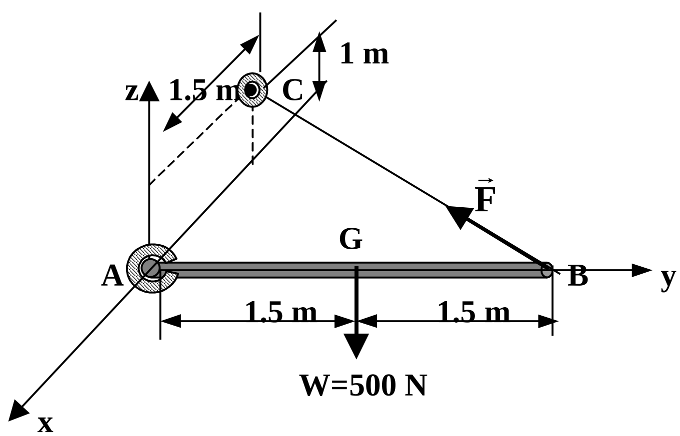
$$\vec{R} = \sum \vec{F}_i \tag{4.20}$$

and the resultant couple $\vec{\mathbf{M}}_o$ is equal to the vectorial sum of moments of all forces about O.

$$\vec{M}_O = \sum \vec{r}_i x \vec{F}_i \qquad \text{(for i = 1, 2,…,n:)} \tag{4.21}$$

<u>Example 4.6</u>

The 3 m boom AB has a fixed end A and is subjected to its 500 N weight and to 700 N force exerted by cable BC. Determine the force and couple at A equivalent to the two given forces.



<u>Solution</u>

The magnitude of the weight is W= 500 N. The magnitude of the applied force is F =700 N.

The resultant force $\vec{\mathbf{R}}$ is obtained as:

$$\vec{R} = \sum \vec{F}_i = \vec{W} + \vec{F}$$

where the coordinates of important point:

A (0,0,0) , B (0,3,0) , C (-1.5, 0, 1) , G (0,1.5,0)   m

$\vec{W} = -W\vec{k}$ acts at point G (0, 1.5, 0) m

$\vec{F} = F\vec{u}_{BC}$ acts at point B (0, 3, 0) m

$$\vec{u}_{BC} = \frac{\overrightarrow{BC}}{BC} = \frac{-1.5\vec{i} - 3\vec{j} + \vec{k}}{\sqrt{(-1.5)^2 + (-3)^2 + (1)^2}} = \frac{1}{3.5}(-1.5\vec{i} - 3\vec{j} + \vec{k})$$

Hence $\vec{W} = -500\vec{k}$

$$\vec{F} = \frac{700}{3.5}(-1.5\vec{i} - 3\vec{j} + \vec{k}) = -300\vec{i} - 600\vec{j} + 200\vec{k}$$

$$\vec{R} = \vec{W} + \vec{F} = -300\vec{i} - 600\vec{j} - 300\vec{k} \qquad [N]$$

The moment of the two forces about o, is

$$\vec{M}_O = \sum \vec{r}_i x \vec{F}_i = \vec{r}_{AB} x \vec{F} + \vec{r}_{AG} x \vec{W}$$

but $\vec{r}_{AB} = 3\vec{j}$ $\vec{r}_{AG} = 1.5\vec{j}$

$$\vec{M}_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 3 & 0 \\ -300 & -600 & 200 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1.5 & 0 \\ 0 & 0 & -500 \end{vmatrix}$$

Hence: $\vec{M}_O = (600 - 750)\vec{i} - (0 + 0)\vec{j} + (900 - 0)\vec{k}$

$$= -150\vec{i} \qquad + 900\vec{k} \qquad [N.m]$$

The equivalent force couple system at A is:

$$\vec{R} = -300\vec{i} - 600\vec{j} - 300\vec{k} \qquad [N]$$

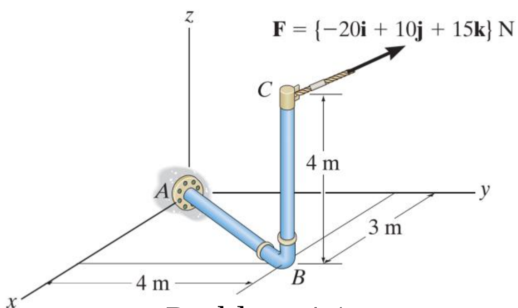
and

$$\vec{M}_O = -150\vec{i} \qquad + 900\vec{k} \qquad [N.m]$$

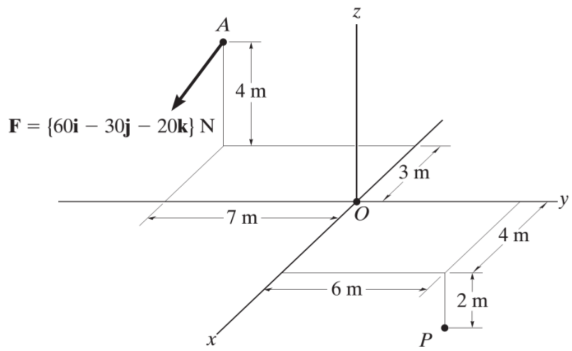
PROBLEMS

4.1 Determine the moment produced by force $\vec{F} = -20\,\vec{i} + 10\,\vec{j} + 15\,\vec{k}$ about point **A**. Express the result as a Cartesian vector.

4.2 Determine the moment of the force **F** at **A** about point **P**. Express the result as a Cartesian vector.



Problem 4.1



Problem 4.2

4.3 Determine the moment about the origin o of the force $\vec{F} = -2\vec{i} - 3\vec{j} + 5\vec{k}$ which acts at a point A. Assume that the position vector of A is:

(a) $\vec{r} = \vec{i} + \vec{j} + \vec{k}$

(b) $\vec{r} = 4\vec{i} + 6\vec{j} - 10\vec{k}$

(c) $\vec{r} = 4\vec{i} + 3\vec{j} - 5\vec{k}$

4.4 Determine the moment about the origin o of the force $\vec{F} = 4\vec{i} + 10\vec{j} + 6\vec{k}$ which acts at a point A. Assume that the position vector of A is:

(a) $\vec{r} = 2\vec{i} + 3\vec{j} + 4\vec{k}$

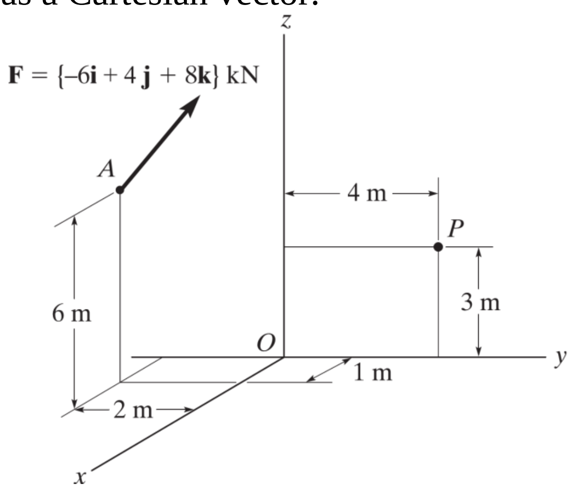
(b) $\vec{r} = 2\vec{i} + 6\vec{j} - 3\vec{k}$

(c) $\vec{r} = 2\vec{i} + 5\vec{j} - 6\vec{k}$

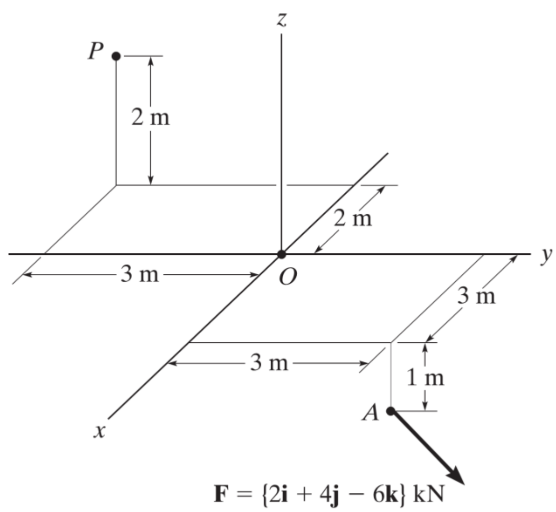
4.5 A single force $\vec{\mathbf{F}}$ acts at a point A of coordinates $\mathbf{x} = \mathbf{y} = \mathbf{z} = \mathbf{a}$ show that the algebraic sum of the rectangular components of the moment of $\vec{\mathbf{F}}$ about o is zero, (i.e. $\mathbf{M_x} + \mathbf{M_y} + \mathbf{M_Z} = \mathbf{0}$).

4.6 Determine the moment of the force **F** at **A** about point **P**. Express the result as a Cartesian vector.

4.7 Determine the moment of the force **F** at **A** about point **P**. Express the result as a Cartesian vector.



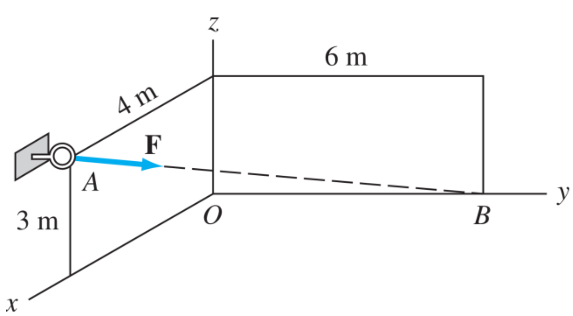
Problem 4.6



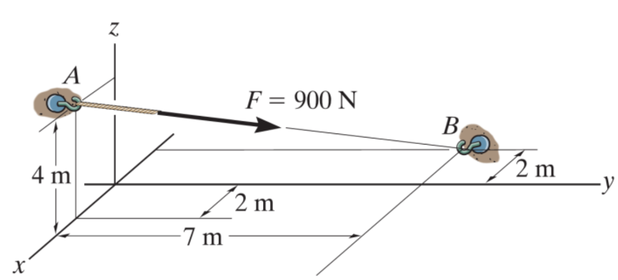
Problem 4.7

4.8 The cable attached to the eyebolt in Figure is pulled with the force **F** of magnitude 500 N . Determine the moment of this force **F** about point o.

4.9 Determine the moment of this force **F** about point o.



Problem 4.8

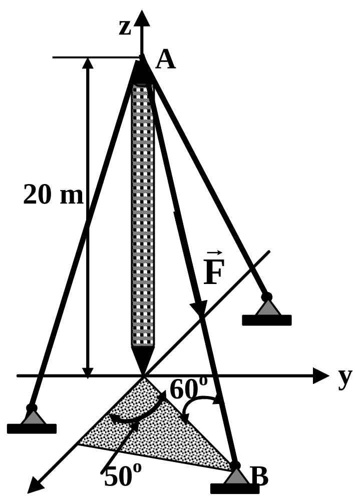


Problem 4.9

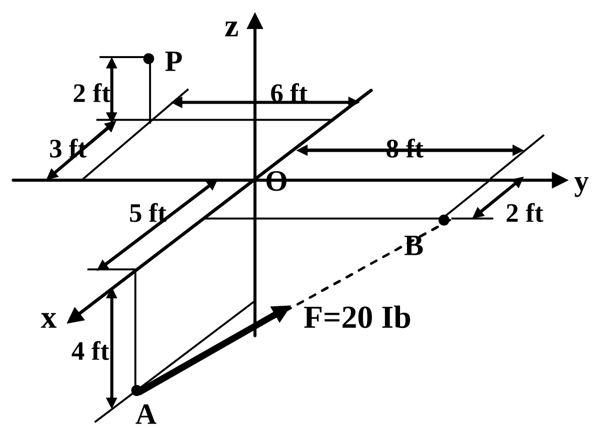
4.10 The line of action of the force $\vec{\mathbf{F}}$ of magnitude 2100 N passes through the two points A and B as shown. Determine the moment of $\vec{\mathbf{F}}$ about o using the position vector: (a) of point A, (b) of point B.

4.11 Determine the moment of the force $\vec{\mathbf{F}}$ at A about point o. Express the result as a Cartesian vector.

4.12 Determine the moment of the force $\vec{\mathbf{F}}$ at A about point P. Express the result as a Cartesian vector.



Problem 4.10



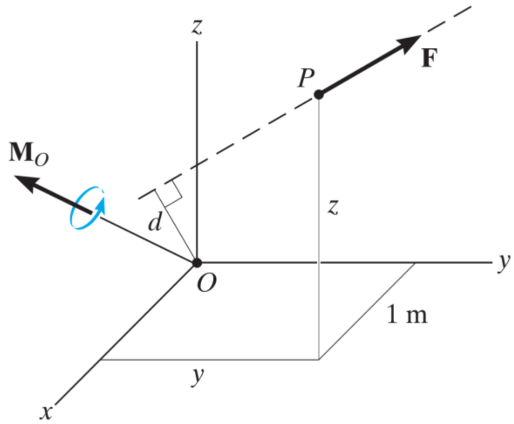
Problem 4.11 & 4.12

4.13 The force $\vec{F} = 6\,\vec{i} - 2\,\vec{j} + 1\,\vec{k}$ (kN) Creates a moment about the point o of $\vec{M}_O = 4\,\vec{i} + 5\,\vec{j} - 14\,\vec{k}$ (kN.m) If the force acts at a point having x coordinate of $x = 1\,m$, Determine:

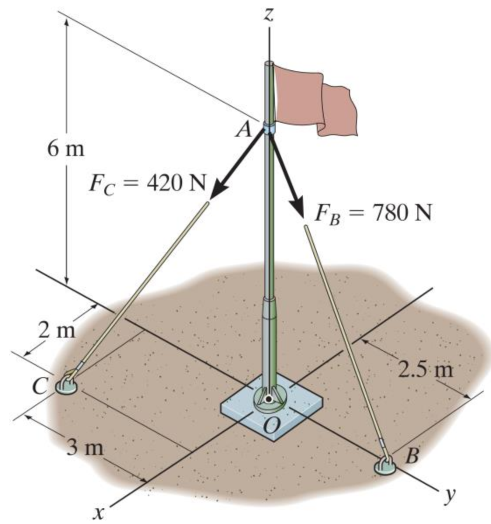
(a) The coordinates **y** and **z**.

(b) The perpendicular distance **d** from the point **o** to the line of action of force F.

4.14 Determine the resultant moment produced by forces and about point o. Express the result as a Cartesian vector.



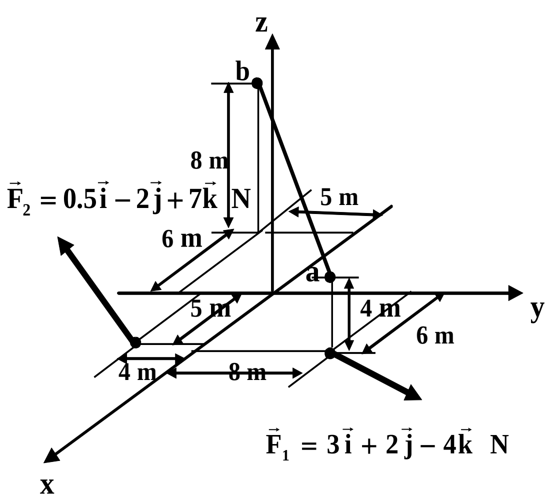
Problem 4.13



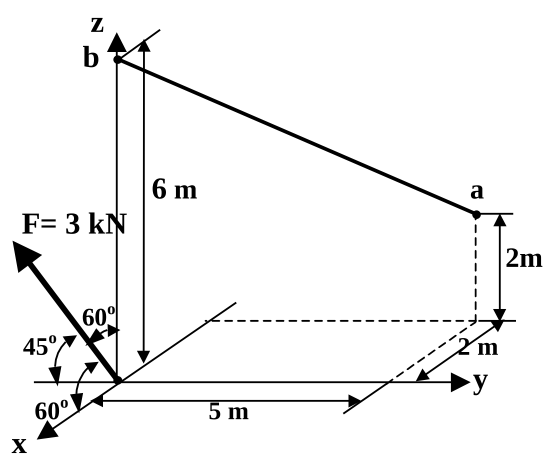
Problem 4.14

4.15 Determine the resultant moment of two forces about the line ab. Express the result as a Cartesian vector.

4.16 Determine the moment of the force $\vec{\mathbf{F}}$ about the line ab. Express the result as a Cartesian vector.



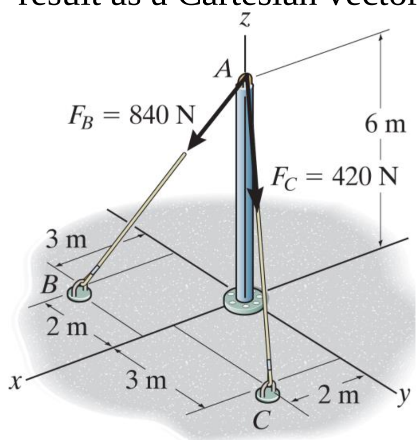
Problem 4.15



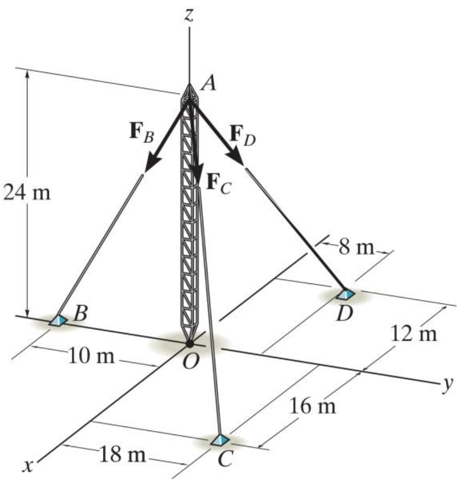
Problem 4.16

4.17 Determine the resultant moment produced by forces and about point o. Express the result as a Cartesian vector.

4.18 The tower is supported by three cables. If the forces of these cables acting on the tower are $F_B = 520\ \text{N}$, $F_C = 680\ \text{N}$ and $F_D = 560\ \text{N}$. Determine the resultant moment produced by forces and about point o. Express the result as a Cartesian vector.



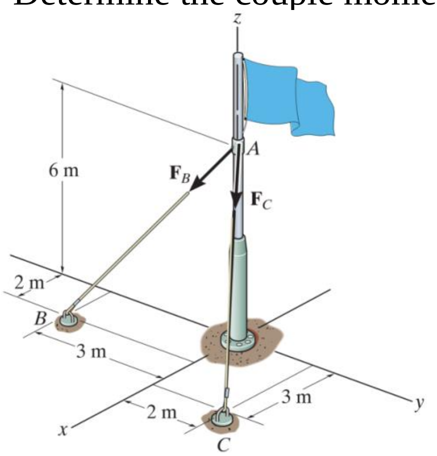
Problem 4.17



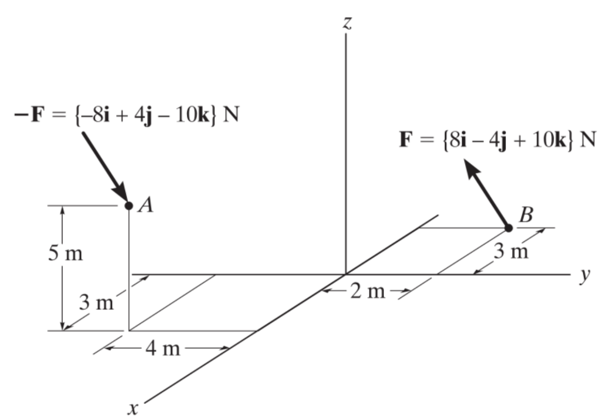
Problem 4.18

4.19 If $F_B = 560\ \text{N}$ and $F_C = 700\ \text{N}$, Determine the resultant moment produced by forces and about point o. Express the result as a Cartesian vector.

4.20 Determine the couple moment. Express the result as a Cartesian vector.



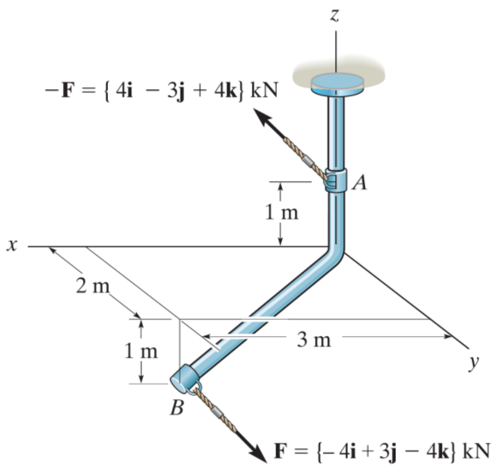
Problem 4.19



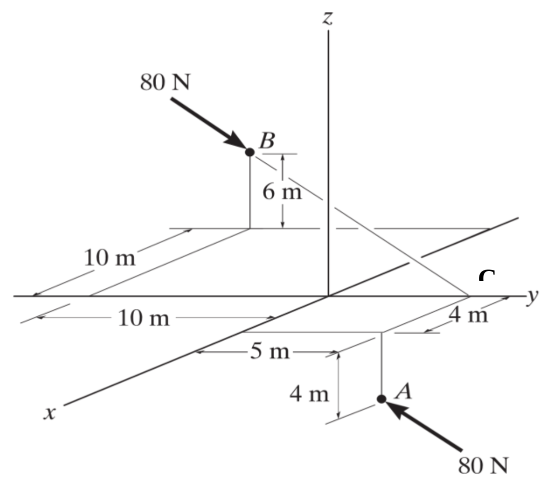
Problem 4.20

4.21 Express the moment of the couple acting on the rod in Cartesian vector form. What is the magnitude of the couple moment?

4.22 Determine the couple moment. Express the result as a Cartesian vector.



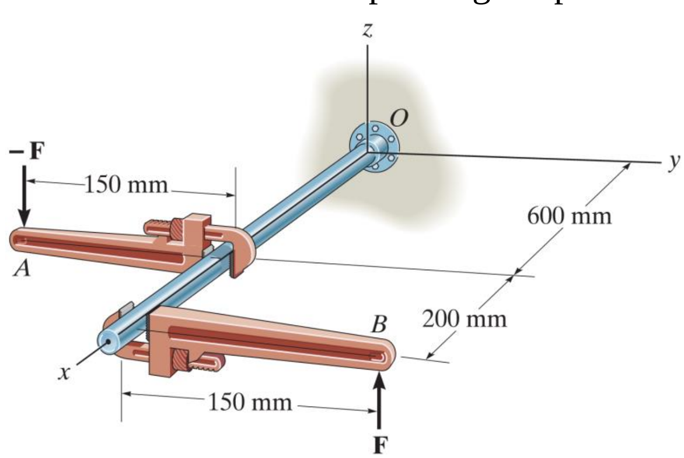
Problem 4.21



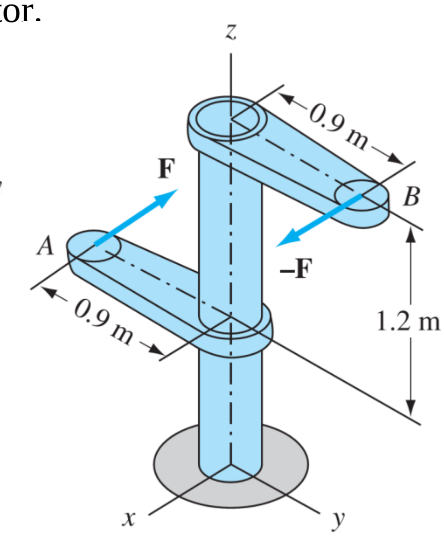
Problem 4.22

4.23 Express the moment of the couple acting on the pipe in Cartesian vector form. What is the magnitude of the couple moment? Take $F = 125\ \text{N}$.

4.24 The two forces of magnitude $F = 24\ \text{kN}$ parallel to x–axis form a couple. Determine the corresponding couple–vector.



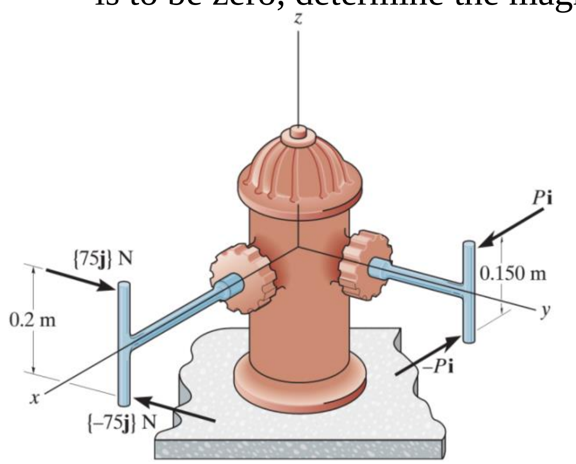
Problem 4.23



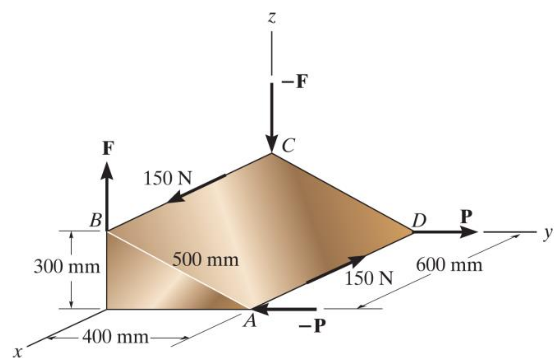
Problem 4.24

4.25 If the resultant couple of the two couples acting on the fire hydrant is $\vec{M}_{CR} = -15\,\vec{i} + 30\,\vec{j} + 0\,\vec{k}$ (N.m), determine the force magnitude **P**.

4.26 If the resultant couple of the three couples acting on the triangular block is to be zero, determine the magnitude of forces **F** and **P**.



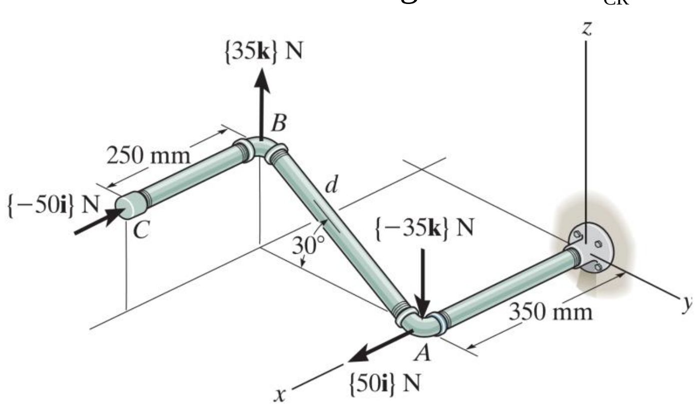
Problem 4.25



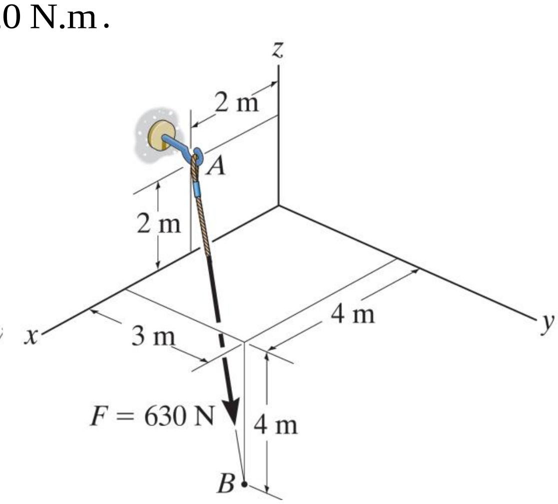
Problem 4.26

4.27 Determine the resultant couple moment of the two couples that act on the pipe assembly. The distance from A to B is $d = 400$ mm. Express the result as a Cartesian vector.

4.28 Determine the distance d between A and B so that the resultant couple moment has a magnitude of $M_{CR} = 20$ N.m.



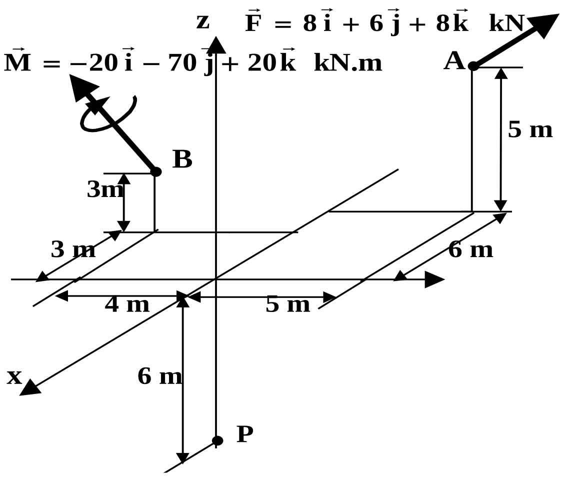
Problem 4.27 & 28



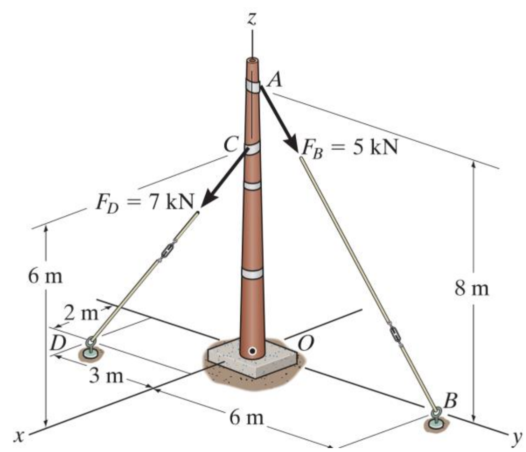
Problem 4.29

4.29 Replace the force by an equivalent force–couple system at point at o.

4.30 Replace the force and couple-moment system by an equivalent resultant force and couple moment at point **P**. express the results in Cartesian vector form.



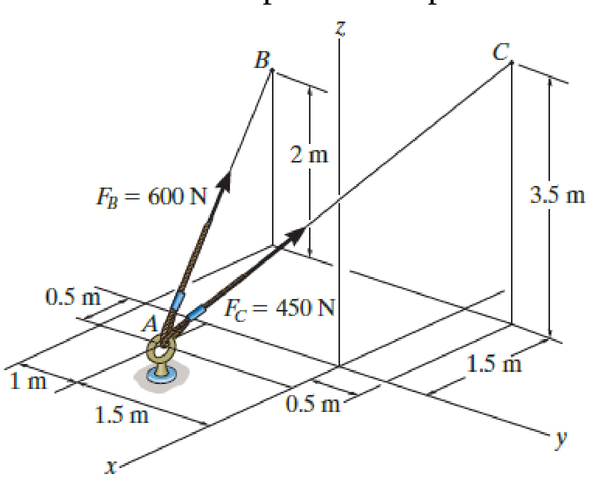
Problem 4.30



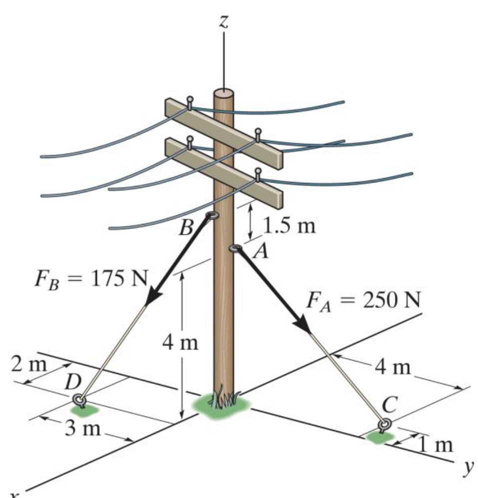
Problem 4.31

4.31 Replace the two forces acting on the post by equivalent resultant force and couple moment at point **o**. Express the results in Cartesian vector form.

4.32 Replace the two forces acting on the post by a resultant force and couple moment at point **o**. Express the results in Cartesian vector form.



Problem 4.32

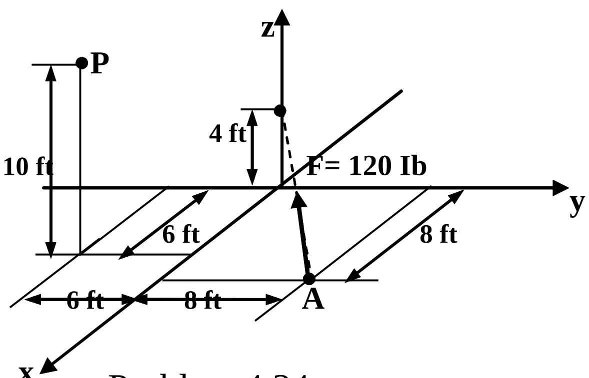


Problem 4.33

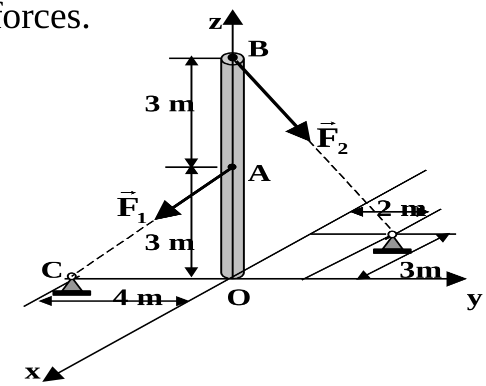
4.33 Replace the two forces acting on the post by a resultant force and couple moment at point o. Express the results in Cartesian vector form.

4.34 Replace the force at A by an equivalent resultant force and couple moment at point P. Express the results in Cartesian vector form.

4.35 Two forces $\vec{F}_1$ and $\vec{F}_2$, of magnitudes $\mathbf{F_1 = 5\ kN}$ and $\mathbf{F_2 = 7\ kN}$, are acting on the vertical pole shown. Determine the components of the force and couple at o equivalent to the given two forces.

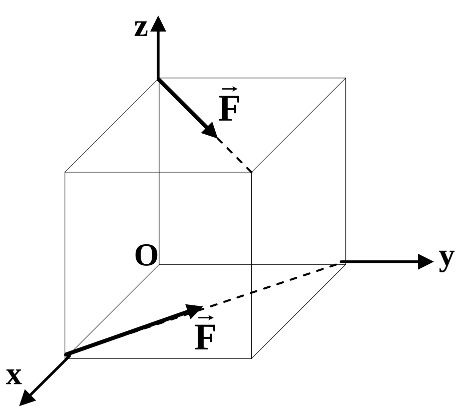


Problem 4.34

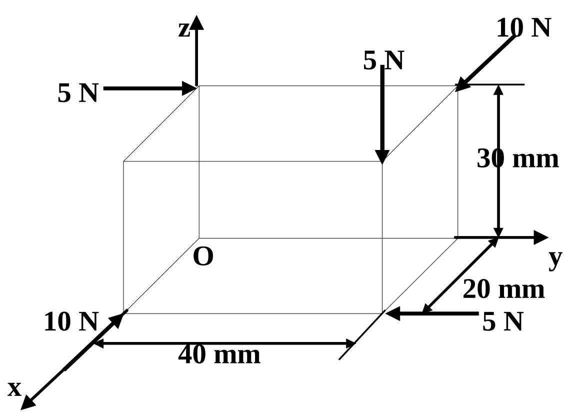


Problem 4.35

4.36 Two forces of magnitude F act along the diagonals of two opposite faces of a cube of side **a** as shown. Replace the two forces by force and a couple at o,



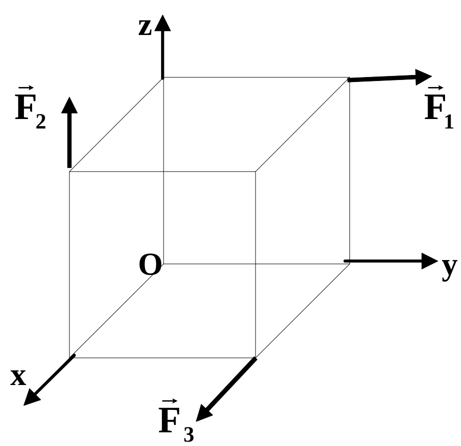
Problem 4.36



Problem 4.37

4.37 A rectangular block is acted upon by the five forces shown, which are directed along the edges. Reduce the system of forces to a force and a couple at o.

4.38 Three forces act on a cube of side a, as shown. Knowing that $\mathbf{F_1} = \mathbf{F_2} = \mathbf{F}$, determine the magnitude of $\vec{\mathbf{F}}_3$ so that the system may be reduced to a single force $\vec{\mathbf{R}}$. Determine, also, the magnitude of $\vec{\mathbf{R}}$ and its line of action.



Problem 4.38

CHAPTER (5)

EQUILIBRIUM OF A RIGID BODY

5.1 Conditions of Equilibrium of a Rigid Body

The rigid body, as defined in chapter (1), consists of a large number of particles. The size of the body must be taken into consideration, since the applied forces will act at different particles on the body. A body is said to be "rigid" if it does not deform under application of external forces. In this section, we will develop both the necessary and sufficient conditions for the equilibrium of the rigid body, Fig. 5.1(a). As shown, this body is subjected to an external force and couple moment system that is the result of different effects such as of gravitational, electrical, magnetic, or contact forces caused by adjacent bodies. The internal forces caused by interactions between particles within the body are not shown in this figure because these forces occur in equal but opposite collinear pairs and hence will cancel out.

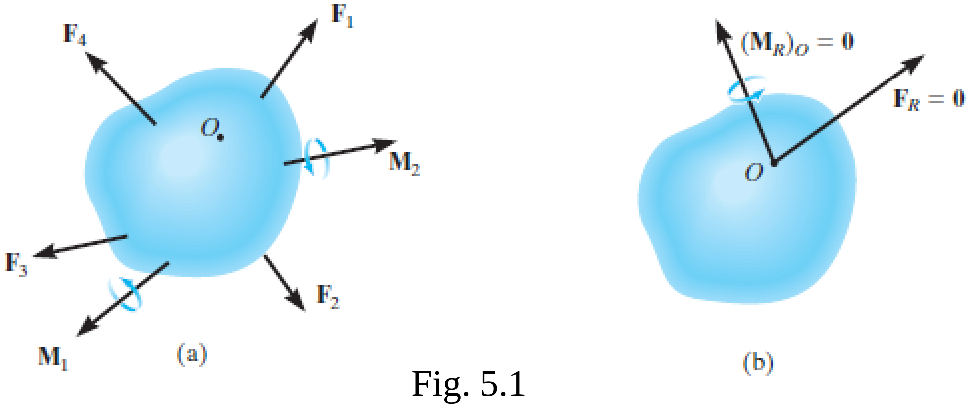


Fig. 5.1

Using the methods of the previous chapter, the force and couple moment system acting on a body can be reduced to an equivalent resultant force and resultant couple moment at any arbitrary point O on or off the body, Fig. 5.1(b). If this resultant force and couple moment are both equal to zero, then the body is said to be in **equilibrium**. Mathematically, the equilibrium of a body is expressed as

$$\vec{F}_R = \sum \vec{F} = 0 \qquad (5.1)$$

$$(\vec{M}_R)_O = 0 \qquad (5.2)$$

5.2 Equilibrium Equations in Two dimensions

When the body is subjected to a system of forces, which all lie in the x–y plane, then the forces can be resolved into their x and y components. Consequently, the conditions for equilibrium in two dimensions are:

$$\sum F_x = 0, \qquad \sum F_y = 0 \qquad \sum M_O = 0 \qquad (5.3)$$

Here $\sum F_x$ and $\sum F_y$ represent, respectively, the algebraic sum of the x and y components of all the forces acting on the body, and $\sum M_O$ represents the algebraic sum of the couple moments and the moments of all the force components about the z axis, which is perpendicular to the x–y plane and passes through the arbitrary point O.

5.3 Two-and Three-Force Members

The solution of some problems involving equilibrium can be simplified if one is able to recognise members that are subjected to only two or three forces.

Two-Force Members

When a member is subjected to no couples, and forces are applied at only two points on a member, the member is called a two-force member, Fig. 5.2. The forces at A and B are first summed to obtain their respective resultants $\vec{\mathbf{F}}_{\mathbf{A}}$ and $\vec{\mathbf{F}}_{\mathbf{B}}$. These two resultant forces will maintain force equilibrium ($\sum \vec{\mathbf{F}} = \mathbf{0}$) provided $\vec{\mathbf{F}}_{\mathbf{A}}$ is of equal magnitude and opposite direction to $\vec{\mathbf{F}}_{\mathbf{B}}$.

Furthermore, moment equilibrium ($\vec{\mathbf{M}}_{\mathbf{A}} = \mathbf{0}$) is satisfied if $\vec{\mathbf{F}}_{\mathbf{A}}$ is collinear with $\vec{\mathbf{F}}_{\mathbf{B}}$. As a result, the line of action of both forces is known, since it always passes through A and B. Hence, only the magnitude must be determined or stated. Other example of two-force members held in equilibrium is shown in Fig.5.3.

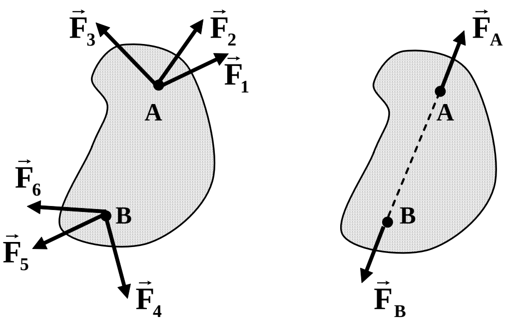


Fig. 5.2

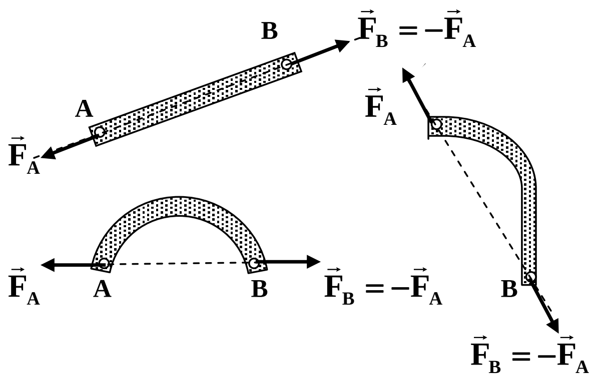


Fig. 5.3

Three-Force Members

If a member is subjected to three coplanar forces, then it is necessary that the forces be either concurrent or parallel, if the member is to be in equilibrium. Consider the three forces acting on a body, Fig.5.4, and suppose that any two of the three forces have lines of action that intersect at point O.

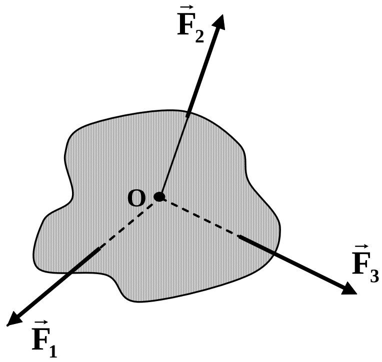


Fig. 5.4

To satisfy moment equilibrium about O($\vec{\mathbf{M}}_{\mathbf{O}} = \mathbf{0}$), the third force must also pass through o, which then makes the force system concurrent. If two of the three forces are parallel, the point of concurring, O is considered to be at "infinity" and the third force must be parallel to the other two forces to intersect at this "point".

5.4 Equilibrium Equations in Three dimensions

Resolving each of equations (5.1) and (5.2) into its three rectangular components, we find that the necessary and sufficient conditions for the equilibrium of a rigid body may also be expressed by six scalar equations:

$$\sum F_x = 0, \qquad \sum F_y = 0, \qquad \sum F_z = 0 \tag{5.4}$$

$$\sum M_x = 0, \qquad \sum M_y = 0, \qquad \sum M_z = 0 \tag{5.5}$$

Equations (5.4) express the fact that the components of the external forces in the x, y and z directions are balanced. Equations (5.5) express the fact that the moments about the x, y and z axes are balanced. The system of the external forces, therefore, will cause no motion of translation or rotation to the rigid body considered. These six scalar equilibrium equations may be used to solve for at most six unknowns shown on the free-body diagram.

5.5 Free-Body Diagram

The first step in solving problems of equilibrium of rigid body, is to draw a free-body diagram. Before we can do this, however, it is first necessary to discuss the types of reactions that can occur at the supports.

Reactions at Supports and Connections

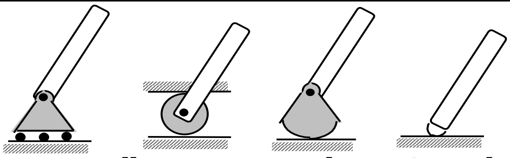
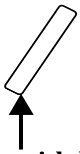
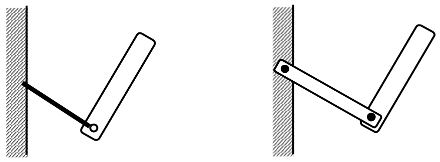
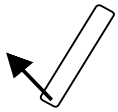
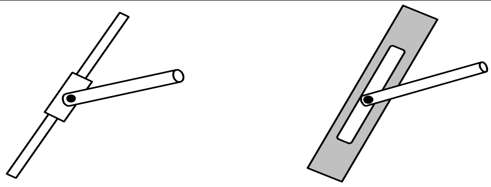
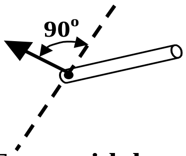
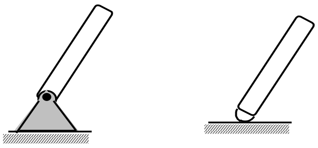
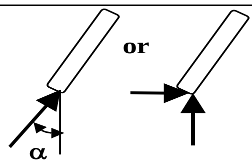
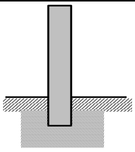
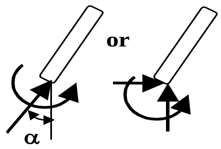
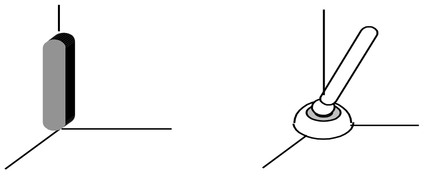
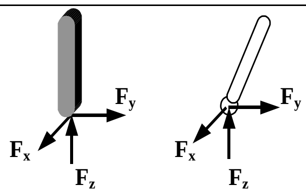
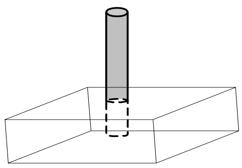
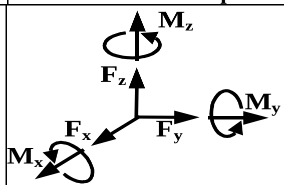
As a general rule:

- A support prevents the translation of a body in a given direction by exerting a force on the body in the opposite direction.
- A support prevents the rotation of a body in a given direction by exerting a couple moment on the body in the opposite direction.

According to the above rule and the type of support or connection, the reactions on a body in space range from a single force with known direction to a force-couple system preventing translation and rotation of the body in space. Hence, the number of unknowns associated with the reaction at a support or connection may vary from one to six, in problems involving equilibrium of a rigid body in space. Various types of supports and connections with corresponding reaction are shown in the following table.

Rollers, smooth surfaces and cables prevent translation in one direction only, perpendicular to the possible direction of motion, and thus exert a single force of known line of action with unknown magnitude. Hinges and rough surfaces prevent translation in two directions, hence the corresponding reactions consist of two unknown force components. Ball and socket supports prevent translation in three directions, hence, these supports have three unknown force components.

Some supports and connections may prevent rotation as well as translation, hence the corresponding reactions include couples as well as forces. For example, the reactions at a fixed support in a plane, which prevent rotation as well as translation in that plane, consist of unknown reacting couple magnitude and two unknown force reacting components.

Support or Connection	Reaction	Number of Unknowns
 Roller, Rocker, Smooth Surface	 Force with known line of action	1
 Short cable, Short link	 Force with known line of action	1
 Collar on frictionless rod, Frictionless pin in slot	 Force with known line of acion	1
 Fixed pin or hinge, Rough surface, 2 dimensions	 Force of unknown direction	2
 Fixed support, 2 dimensions	 Force of unknown direction and couple	3
 Rough surface, Ball and socket	 Three force components	3
 Fixed support	 Three forces components and three couples	6

<u>Procedure for Analysis</u>

To construct a free-body diagram for a rigid body, the following steps should be performed :

Step 1. Imagine the body to be isolated or cut "free" from its constraints and connections, and draw (sketch) its outlined shape.

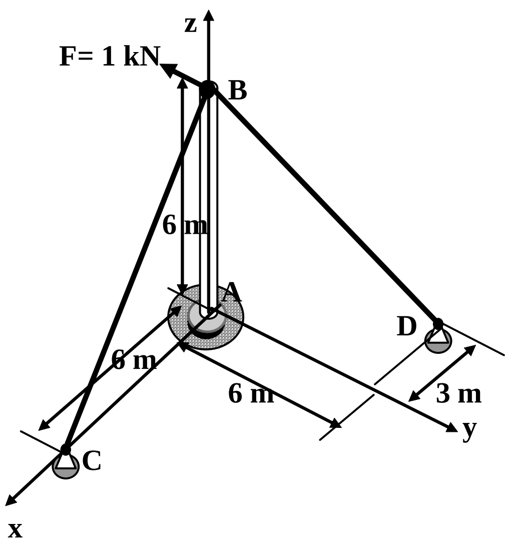
Step 2. Identify all the external forces and couples that act on the body. Those generally are due to applied loading, reactions at supports and the weight of the body.

Step 3. Indicate the dimensions of the body necessary for computing the moments of forces. The forces and couple moments that are known should be labelled with their proper magnitudes and directions. Letters are used to represent the magnitudes and directions of unknown forces and couple moments.

<u>Example 5.1</u>

Determine the tension in cables BC and BD and the reactions at the ball-and-socket support A for the shown mast.

<u>Solution</u>



Free-body-diagram of the mast AB indicates that the mast is acted upon by 4 force vectors which are:

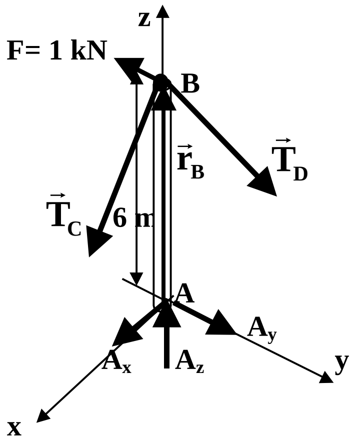
The applied force $\vec{\mathbf{F}}$, the reaction force vector at ball-and-socket A the tension in cables $\vec{\mathbf{T}}_C$ and $\vec{\mathbf{T}}_D$.

Applying the force equation of equilibrium, Eq.(5.1) gives:

$$\vec{R} = 0; \qquad \vec{F} + \vec{A} + \vec{T}_C + \vec{T}_D = 0$$

Expressing each force in terms of its rectangular components,

we have:



$$\vec{F} = -1\vec{j}$$

$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$

$$\vec{T}_C = T_C\frac{\overrightarrow{BC}}{BC} = \frac{1}{\sqrt{2}}T_C\vec{i} - \frac{1}{\sqrt{2}}T_C\vec{k}$$

$$\vec{T}_D = T_D\frac{\overrightarrow{BD}}{BD} = -\frac{1}{3}T_D\vec{i} + \frac{2}{3}T_D\vec{j} - \frac{2}{3}T_D\vec{k}$$

Applying the above force equation of equilibrium; we have:

$$(A_x + \frac{1}{\sqrt{2}}T_C - \frac{1}{3}T_D)\vec{i} + (-1 + A_y + \frac{2}{3}T_D)\vec{j} + (A_z - \frac{1}{\sqrt{2}}T_C - \frac{2}{3}T_D)\vec{k} = 0$$

Hence

$$A_x + \frac{1}{\sqrt{2}}T_C - \frac{1}{3}T_D = 0 \qquad (1)$$

$$A_y + \frac{2}{3}T_D - 1 = 0 \qquad (2)$$

$$A_z - \frac{1}{\sqrt{2}}T_C - \frac{2}{3}T_D = 0 \qquad (3)$$

Expressing moments about A and applying equation of equilibrium (5.2),

we have:

$$\sum\vec{M}_O = 0; \qquad \vec{r}_{AB}x\vec{F} + \vec{r}_{AB}x\vec{T}_C + \vec{r}_{AB}x\vec{T}_D = 0$$

$$\text{or} \qquad \vec{r}_{AB}x(\vec{F} + \vec{T}_C + \vec{T}_D) = 0$$

Expressing this equation in a determinant form, we have:

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 6 \\ \frac{1}{\sqrt{2}}T_C - \frac{1}{3}T_D & \frac{2}{3}T_D - 1 & -\frac{1}{\sqrt{2}}T_C - \frac{2}{3}T_D \end{vmatrix} = 0$$

Expanding the obtained determinant yields:

$$(0-6(\frac{2}{3}T_D-1))\vec{i}-(0-6(\frac{1}{\sqrt{2}}T_C-\frac{1}{3}T_D))\ \vec{j}+(0-0)\ \vec{k}=0$$

or $\quad \frac{2}{3}T_D-1=0 \qquad (4)$

$\frac{1}{\sqrt{2}}T_C-\frac{1}{3}T_D \qquad (5)$

Therefore, we have 5 equations in terms of 5 unknowns (**A_x, A_y ,A_z ,T_D ,T_C).**
Solution of the above 5 equations, yields:

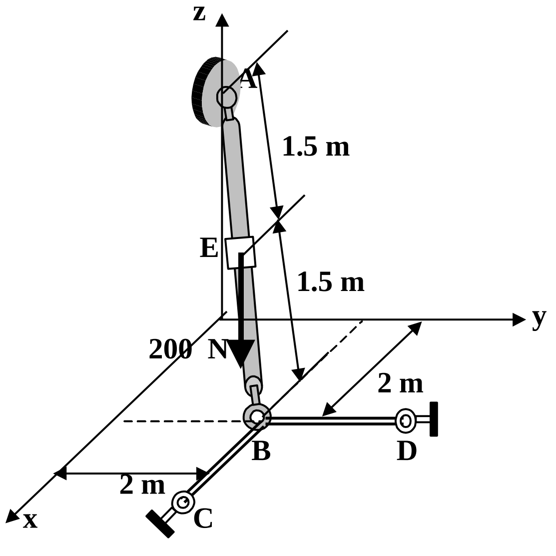
$T_D = 1.5$ kN $\qquad$ $T_C = 0.707$ kN $\qquad$ $A_x = 0$,

$A_y = 0$, $\qquad$ $A_z = 1.5$ kN

Note that the forces acting on the mast act at two points A and B, hence, the mast is a two force member and the reacting force at A must act along the line AB, so that the value $A_x = A_y = 0$ could have been determined by inspection.

Example 5.2

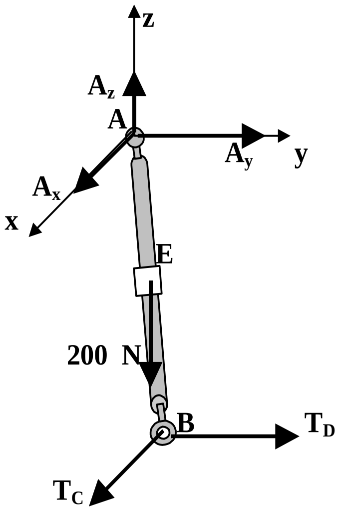
The shown rod AB is subjected to 200 N vertical force. Determine the reactions at the ball-and-socket support A and the tension in cables BD and BC.



Solution

Free-body-diagram of the rod AB indicates that the rod is acted upon by 4 force vectors which are:

The applied force $\vec{\mathbf{F}}$, the reacting force at ball-and-socket $\vec{\mathbf{A}}$, and the tension in cables $\vec{\mathbf{T}}_C$ and $\vec{\mathbf{T}}_D$.



Applying the force equilibrium equation (5. 1) gives.

$\vec{R} = 0; \qquad \vec{F} + \vec{A} + \vec{T}_C + \vec{T}_D = 0$

Representing each force in terms of its rectangular components, we have:

$$\vec{F} = \qquad -200\vec{k}$$
$$\vec{A} = A_x\vec{i} + A_y\vec{j} + A_z\vec{k}$$
$$\vec{T}_C = T_C\vec{i}$$
$$\vec{T}_D = \qquad T_D\vec{j}$$

Applying the above force equation of equilibrium; we have:

$$(A_x + T_C)\vec{i} + (A_y + T_D)\vec{j} + (A_z - 200)\vec{k} = 0$$

Hence

$$A_x + T_C = 0 \qquad (1)$$
$$A_y + T_D = 0 \qquad (2)$$
$$A_z - 200 = 0 \qquad (3)$$

Summing moments about A yields,

$\sum \vec{M}_A = 0; \qquad \vec{r}_{AC} x\vec{F} + \vec{r}_{AB} x(\vec{T}_C + \vec{T}_D) = 0$

Since: $\qquad \vec{r}_{AC} = \frac{1}{2}\vec{r}_{AB}, \qquad \vec{r}_{AB} = 2\vec{i} + 2\vec{j} - \vec{k} \qquad$ [m]

we have

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 0.5 \\ 0 & 0 & -200 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 2 & 1 \\ T_C & T_D & \end{vmatrix} = 0$$

Expanding these two determinants, yields

$$(-200 - T_D)\vec{i} - (-200 + T_C)\vec{j} + (2T_D - 2T_C)\vec{k} = 0$$

$$-200 - T_D = 0 \qquad (4)$$

$$-200 + T_C = 0 \qquad (5)$$

$$2T_D - 2T_C = 0 \qquad (6)$$

Solving Eqs.(1) through (6), we get:

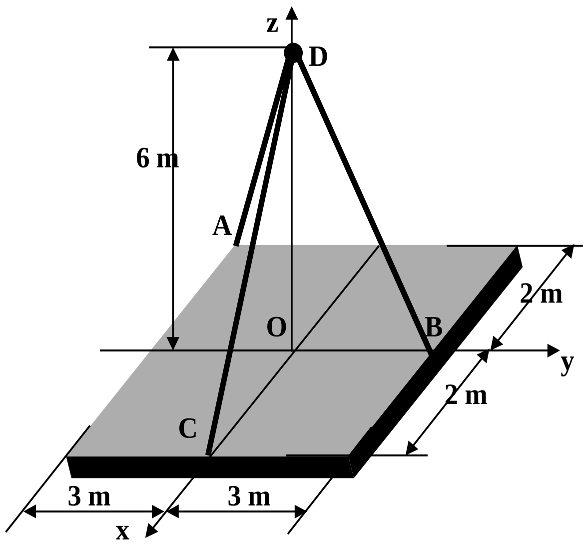
$$A_x = A_y = -200 \qquad N$$

$$A_z = T_C = T_D = 200 \qquad N$$

The negative sign indicates that $\mathbf{A_x}$ and $\mathbf{A_y}$ have a sense, which is opposite to that shown on the free-body diagram.

<u>Example 5.3</u>

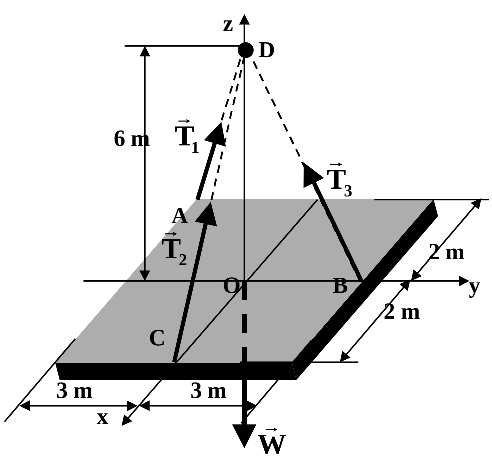
A 4 x 6 meters plate of weight W = 9 kN is kept in a horizontal plane using three cables intersecting at one point D. Determine the tension in each cable.



<u>Solution</u>

The free- body- diagram indicates that the applied forces on the plate are:

- the weight of the plate $\vec{\mathbf{W}}$ acting at its center.
- the tension in the three cables $\vec{\mathbf{T}}_1$, $\vec{\mathbf{T}}_2$ and $\vec{\mathbf{T}}_3$.



Since the three forces $\vec{\mathbf{T}}_1$, $\vec{\mathbf{T}}_2$ and $\vec{\mathbf{T}}_3$ intersect at point D, the line of action of the weight $\vec{\mathbf{W}}$ must pass through the point D, and only the force equation of equilibrium (5. 1) is sufficient to determine the unknown tensions.

$$\vec{R} = 0, \qquad \vec{W} + \vec{T}_1 + \vec{T}_2 + \vec{T}_3 = 0$$

Expressing each force in terms of its rectangular form, we have:

$$\vec{W} = W(-\vec{k}) = -9\vec{k} \quad \text{kN}$$

$$\vec{T}_1 = T_1 \frac{\overrightarrow{AD}}{AD} = \frac{2}{7}T_1\vec{i} + \frac{3}{7}T_1\vec{j} + \frac{6}{7}T_1\vec{k}$$

$$\vec{T}_2 = T_2 \frac{\overrightarrow{CD}}{CD} = -\frac{2}{\sqrt{40}}T_2\vec{i} + \frac{6}{\sqrt{40}}T_2\vec{k}$$

$$\vec{T}_3 = T_3 \frac{\overrightarrow{BD}}{BD} = -\frac{3}{\sqrt{45}}T_3\vec{j} + \frac{6}{\sqrt{45}}T_3\vec{k}$$

Summing up these forces, the above force equilibrium equation yields :

$$(\frac{2}{7}T_1 - \frac{2}{\sqrt{40}}T_2)\vec{i} + (\frac{3}{7}T_1 - \frac{3}{\sqrt{45}}T_3)\vec{j} + (-9 + \frac{6}{7}T_1 + \frac{6}{\sqrt{40}}T_2 + \frac{6}{\sqrt{45}}T_3)\vec{k} = 0$$

Hence $\frac{2}{7}T_1 - \frac{2}{\sqrt{40}}T_2 = 0$ (1)

$$\frac{3}{7}T_1 - \frac{3}{\sqrt{45}}T_3 = 0 \qquad (2)$$

$$-9 + \frac{6}{7}T_1 + \frac{6}{\sqrt{40}}T_2 + \frac{6}{\sqrt{45}}T_3 = 0 \qquad (3)$$

Solving these three equations, we get:

$T_1 = 3.5$ kN

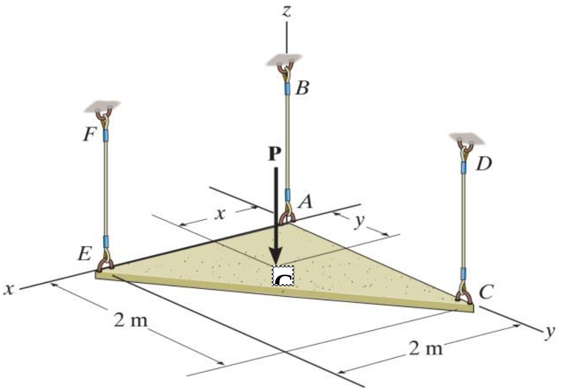
$T_2 = 3.16$ kN

$T_3 = 3.35$ kN

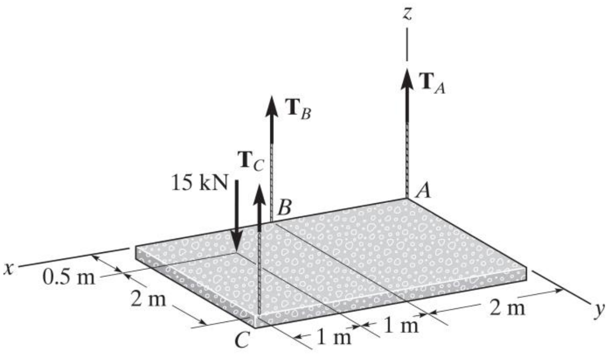
PROBLEMS

5.1 If $P = 6\,\text{kN}$, $x = 0.75\,\text{m}$ and $y = 1\,\text{m}$, determine the tension developed in cables AB, CD, and EF.

5.2 The uniform concrete slab has a mass of $2400\,\text{kg}$. Determine the tension in each of the three parallel supporting cables when the slab is held in the horizontal plane as shown.



Problem 5.1

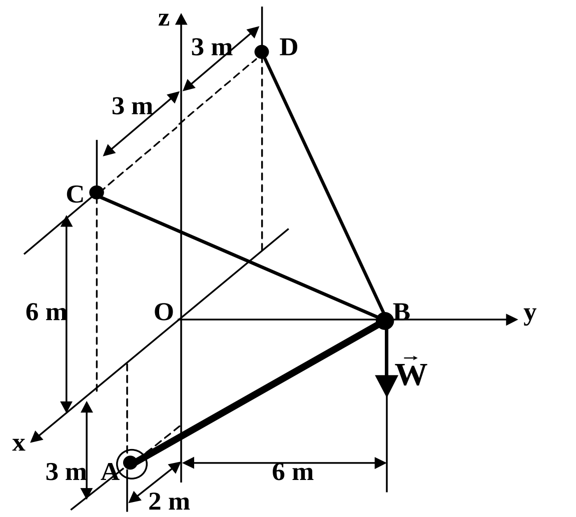


Problem 5.2

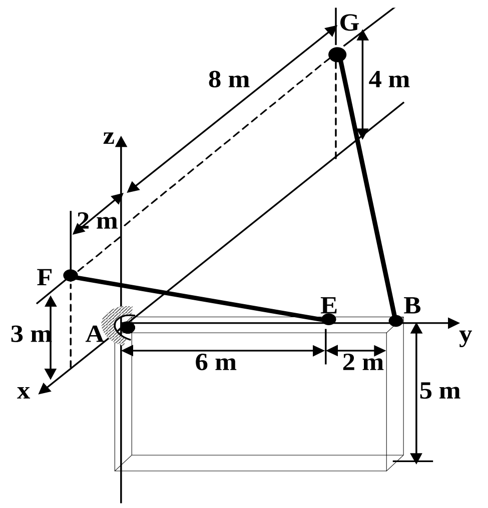
5.3 A 7 m boom supports a load W = 1.8 kN as shown. The boom is held by a ball and socket at A and by two cables BC and BD. Neglecting the weight of the boom, determine the tension in each cable and the reaction components at A.

5.4 A 5 x 8 meters plate of uniform density weighs 2.7 kN. It is supported by a ball and socket at A and by two cables as shown. Determine the reaction at A and tension in each cable.

5.5 The weight of uniform rod AB is W = 240 N. It is supported by a ball and socket at B and the wire CD which is attached to the midpoint of the rod (C). The rod leans against a smooth vertical wall at A. Determine the tension in the wire and the reactions at A and B.

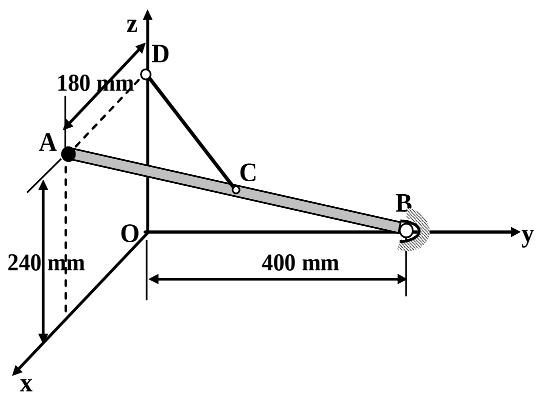


Problem 5.3

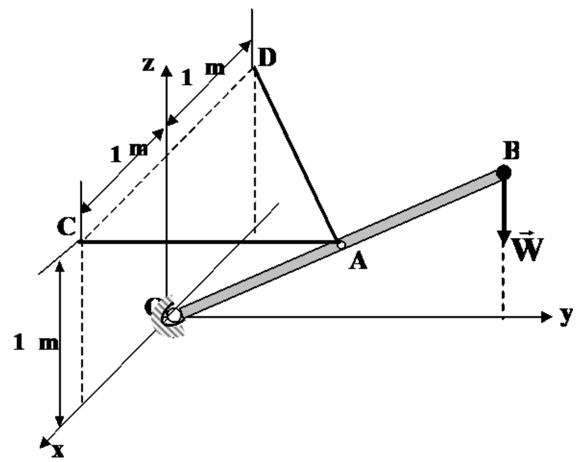


Problem 5.4

5.6 A link OB of length 3 m forms an angle of 60° with the vertical axis. It is held by a ball and socket at O and by two rods AD and AC. The distance OA is 1 m. Determine the components of the reaction at O and the force in each rod when the link carries a load W = 1 kN at the top B.

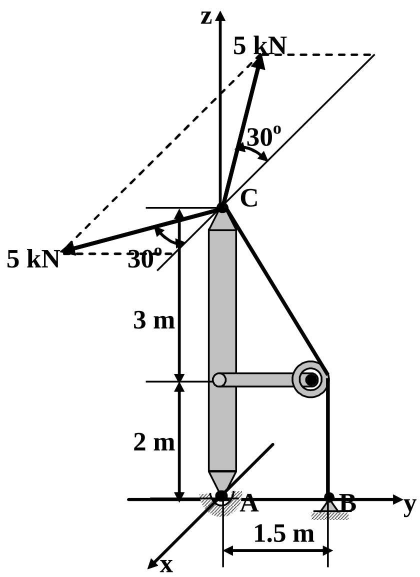


Problem 5.5

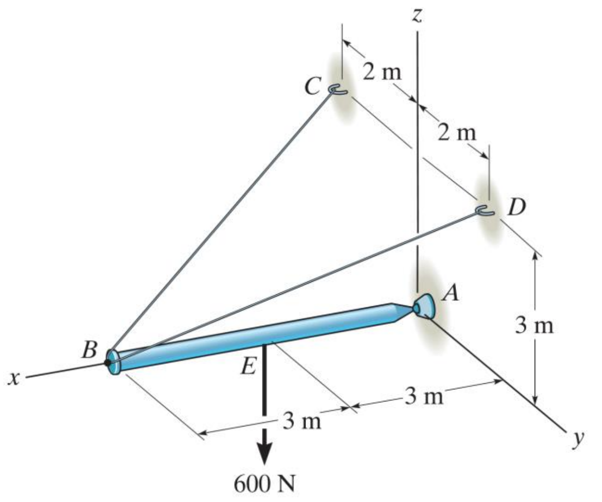


Problem 5.6

5.7 The boom is supported by a ball-and-socket joint at A and a guy wire at B. If the 5 kN loads lie in a plane which is parallel to the x–y plane, determine the x, y, z components of reaction at A and the tension in the cable at B.



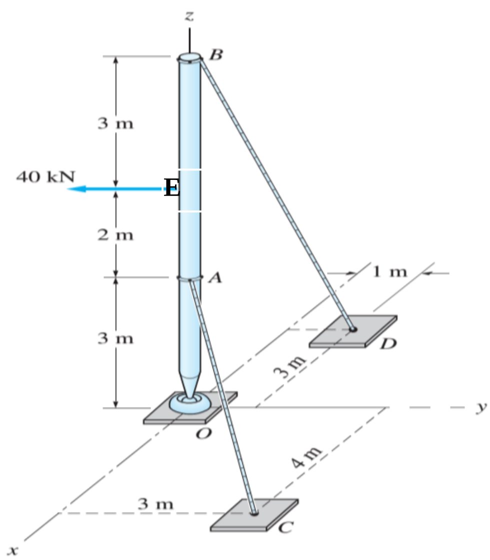
Problem 5.7



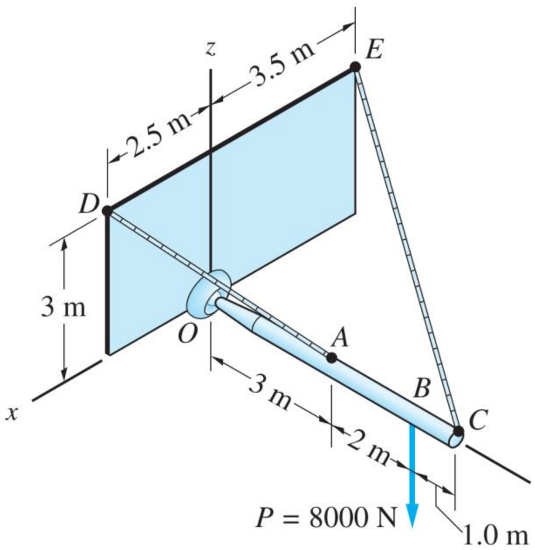
Problem 5.8

5.8 Determine the components of reaction at the ball-and-socket joint A and the tension in each cable necessary for equilibrium of the rod.

5.9 A pole weight 19.62 kN is supported by a ball-and-socket joint at O and two cables. Determine the components of reaction at the ball-and-socket joint O and the tension in each cable necessary for equilibrium of the pole.



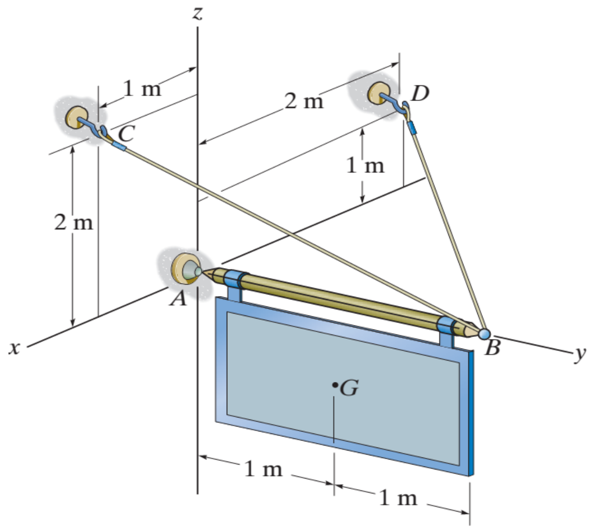
Problem 5.9



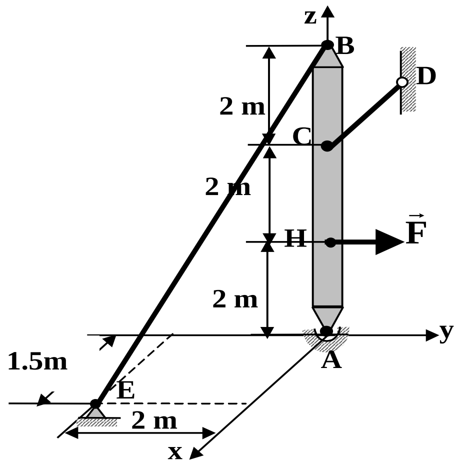
Problem 5.10

5.10 The horizontal boom OC, which is supported by a ball-and-socket joint and two cables, carries the vertical force $P = 8000\,N$. Calculate T_{AD} and T_{CE}, the tensions in the cables, and the components of the force exerted on the boom by the joint at O.

5.11 The sign has a mass of $100\,kg$ with center of mass at G. Determine the x, y, z components of reaction at the ball–and–socket joint A and the tension in wires BC and BD.



Problem 5.11

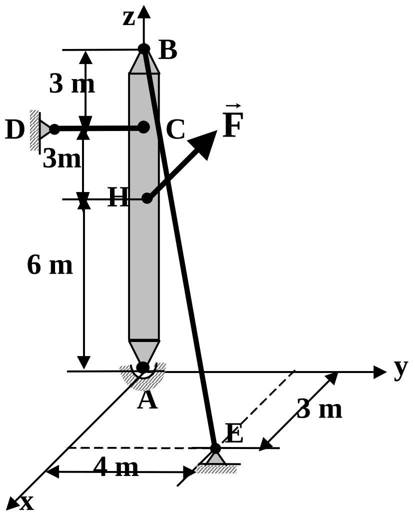


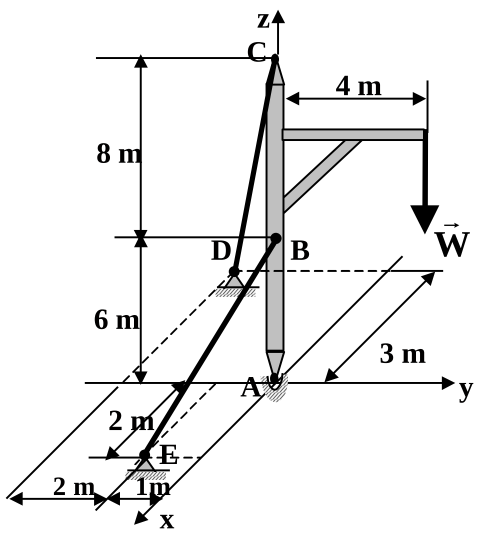
Problem 5.12

5.12 A tower AB is held in vertical position by a ball-and-socket at A and two cables CD and BE. An external force of magnitude F = 600 N is applied as shown. Neglecting the weight of the tower, determine the reaction at A and the tension in each cable.

5.13 A mast of height 12 m is held at A by a ball and socket and two cables CD and BE as shown. The weight of the mast can be neglected with respect to the force F = 20 kN applied at H and is parallel to x axis. Find the reaction at A and the tension in the cables.

5.14 The shown crane is used to lift a load W = 250 N. Determine the tensions in the cables CD and BE.



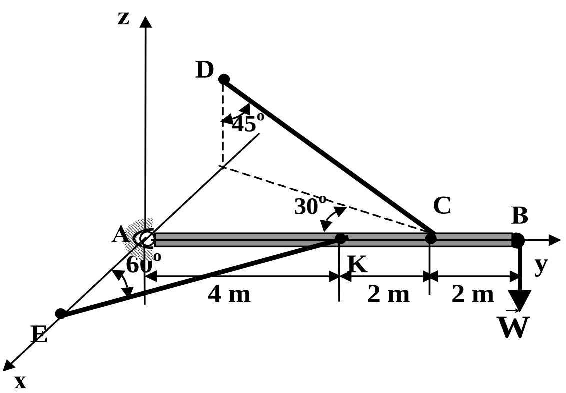


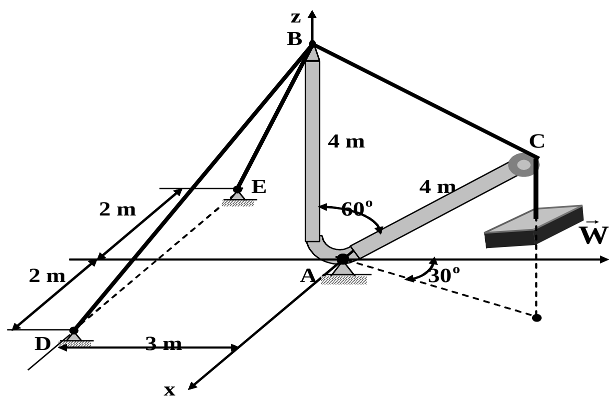
Problem 5.13

Problem 5.14

5.15 The rod AB is kept in the shown direction by a ball and socket at A and two cables CD and KE. A weight W = 40 kN is hanged at B. Neglecting the weight of the rod, determine the reaction at A and the tension in the cables.

5.16 The shown crane supports a load W = 2 kN. At A we have a ball and socket. Determine the tension in the cables BD and BE and the reaction at A.



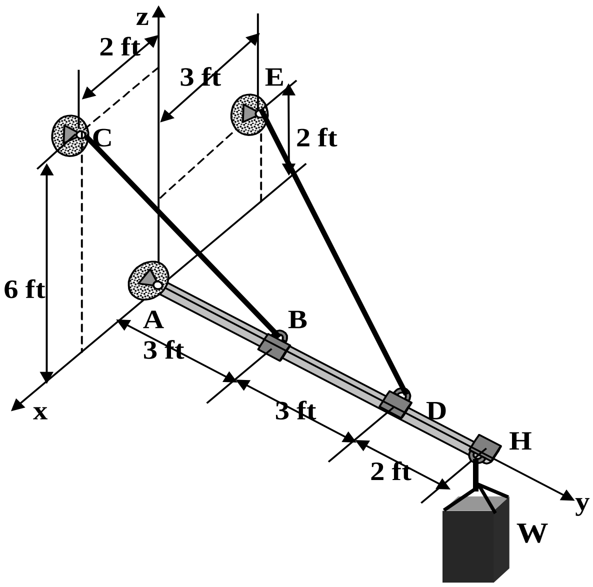


Problem 5.15

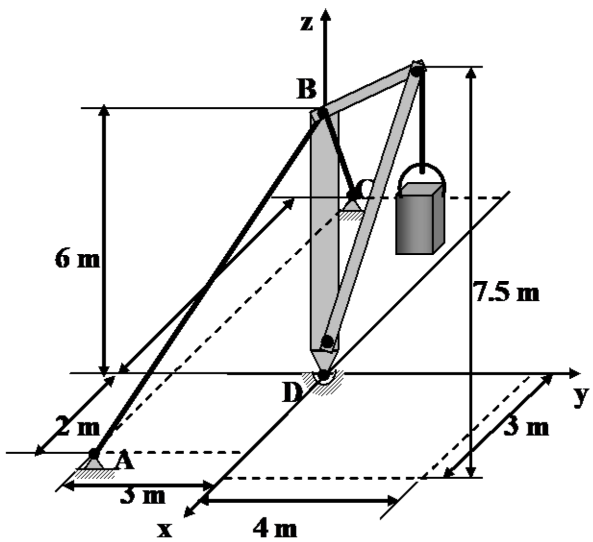
Problem 5.16

5.17 The boom supports a load of W = 89 lb. Determine the components of the reaction at A and the tension in the cables BC and DE.

5.18 The stiff-leg derrick used on ships is supported by a ball-and-socket at D and two cables BA and BC. The cables are attached to a smooth collar ring at B, which allows rotation of the derrick about the z-axis. If the derrick supports a crate having a weight of 1 kN, determine the tension in the supporting cables and the components of the reaction at D.



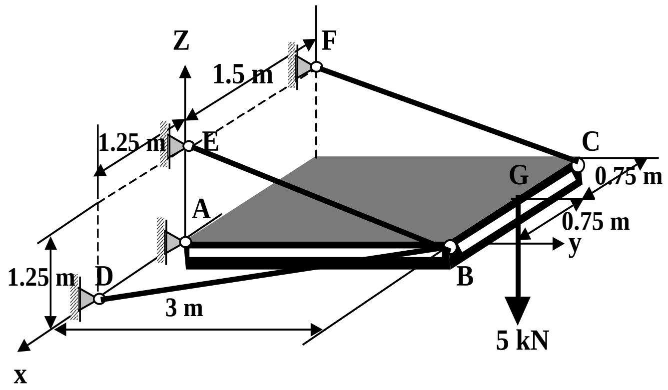
Problem 5.17



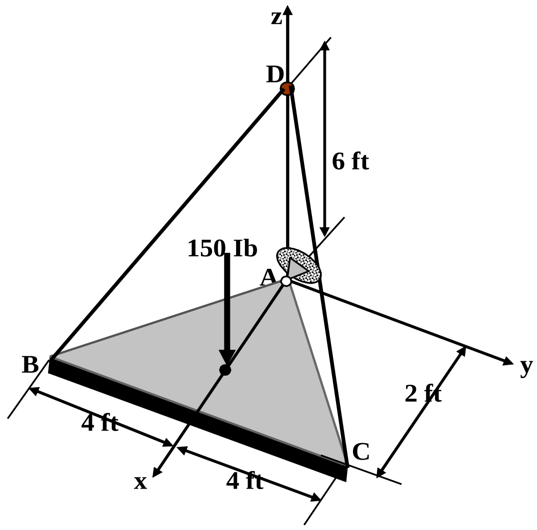
Problem 5.18

5.19 The rigid L-shaped member ABC is supported by a ball-and -socket at A and by three cables. Determine the tension in each cable and the reaction at A caused by the 5-kN load applied at C.

5.20 The triangular plate supports a 150 1b load. Determine the tension in cables BD and CD and x, y, z components of reaction at the ball-and-socket at A.



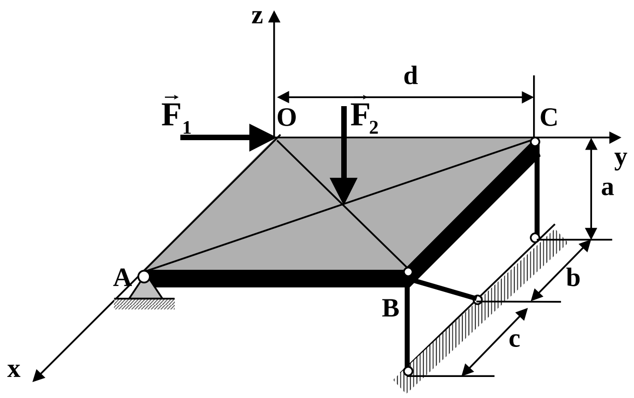
Problem 5.19



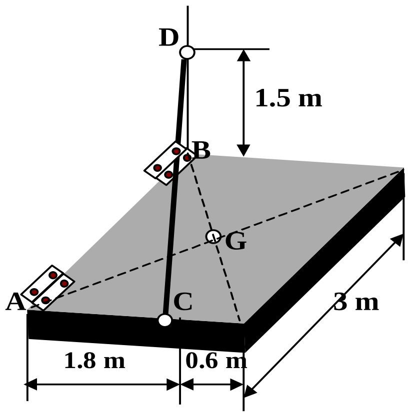
Problem 5.20

5.21 A plate of negligible weight is supported by a ball and socket at A, two short links at B and one short link at C. If d= 4 a and a = b = c, find the reaction at A and the force in each link when the applied forces are: F_1=2 kN and F_2 = 1 kN.

5.22 A 200 kg platform 2.4 by 3 m is held in a horizontal position by two horizontal hinges at A and B and by a cable CD attached to a point D located 1.5 m directly above B. Determine the tension in the cable.



Problem 5.21



Problem 5.22

CHAPTER (6)

ANALYSIS OF PLANE TRUSSES

6.1 Definition of a Truss

The truss is one of the major types of engineering structures. It provides both a practical and an economical solution to many engineering situations, especially in the design of bridges and buildings. A **truss** is a structure composed of slender straight members joined together at their end points (joints); a typical truss is shown in Fig.6.1.

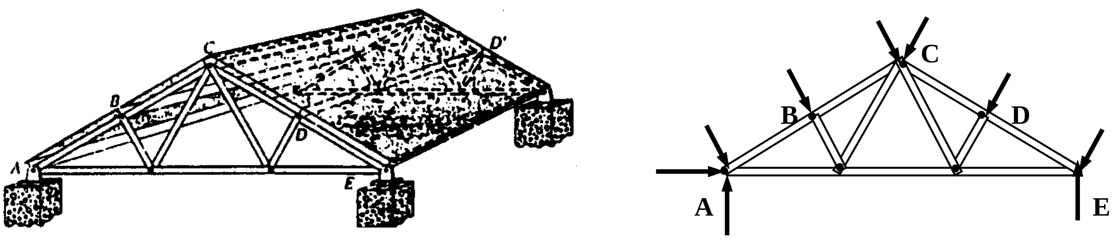


Fig. 6.1

The joints of the truss are labelled by capital letters A, B, C ..., etc. While the members are labelled by the two letters, indicating the two joints at a member ends. The members of a plane truss and the external forces acting on it lie in one plane.

<u>Assumptions for Design</u>.

To design both the members and the connections of a truss, it is necessary first to determine the force developed in each member when the truss is subjected to a given loading. To do this we will make two important assumptions:

• **All loadings are applied at the joints.** In most situations, such as for bridge and roof trusses, this assumption is true. Frequently the weight of the members is neglected because the force supported by each member is usually much larger than its weight. However, if the weight is to be included in the analysis, it is generally satisfactory to apply it as a vertical force, with half of its magnitude applied at each end of the member.

• **The members are joined together by smooth pins.** The joint connections are usually formed by bolting or welding the ends of the members to a common plate, called a gusset plate, as shown in Fig. 6.2(a), or by simply passing a large bolt or pin through each of the members, Fig. 6.2(b). We can assume these connections act as pins provided the center lines of the joining members are concurrent.

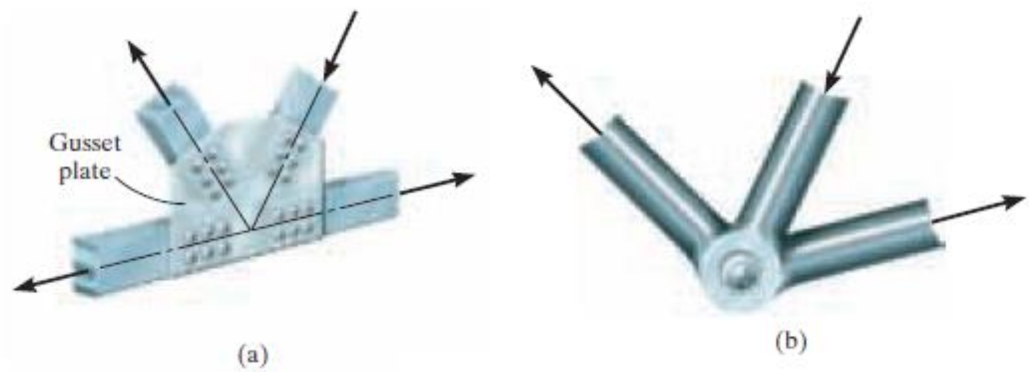


Fig. 6.2

Because of these two assumptions, each truss member will act as a **two-force membe**r, and therefore the force acting at each end of the member will be directed along the axis of the member. If the force tends to elongate the member, it is a **tensile force (T)**, Fig. 6.3(a); whereas if it tends to shorten the member, it is a **compressive force (C)**, Fig. 6.3(b). In the actual design of a truss it is important to state whether the nature of the force is tensile or compressive.

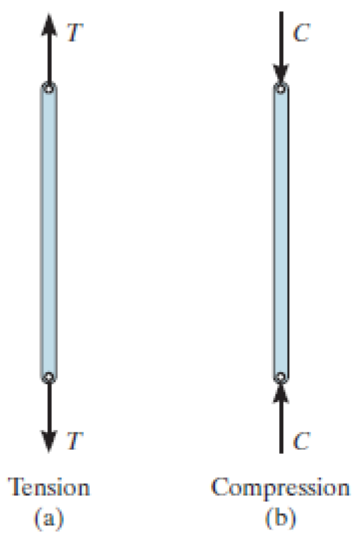


Fig. 6.3

Often, compression members must be made thicker than tension members because of the buckling or column effect that occurs when a member is in compression.

6.2 Truss Analysis

In most cases, the first step in truss analysis, it is required to determine the reactions at the supports. The truss is treated as a rigid body in equilibrium under the applied forces and the reactions. These forces form a system of non-concurrent plane forces. Assuming that the plane of the truss is x-y, So, the equilibrium equations in two dimensions (5.3) must be satisfied:

$$\sum F_x = 0, \qquad \sum F_y = 0 \qquad \sum M_o = \sum M_z = 0$$

The second step is to determine the internal force in each member of the truss. From various methods used to determine internal forces in truss members, the following two methods will be discussed:

6.3 The Method of Joints

If a truss is in equilibrium, then each of its joints also will be in equilibrium. The method of joints is based on this fact. Because the truss members are all straight two - force members lying in the same plane, the forces acting at each joint is coplanar and concurrent. Consequently, it is only necessary to satisfy force equilibrium conditions, $\sum F_x = 0$ and $\sum F_y = 0$. The analysis should start at a joint having at least one known force and at most two unknown forces, since, the application of equilibrium equations at a joint yields two algebraic equations which can be solved for two unknowns. The following procedure provides a typical means for analysing a truss using the method of joints:

Step 1: Draw the free-body diagram of a proper joint.

Step 2: Orient the x and y axes such that the forces on the free body diagram can be easily resolved into their x and y components and then apply the two-force equilibrium equations:

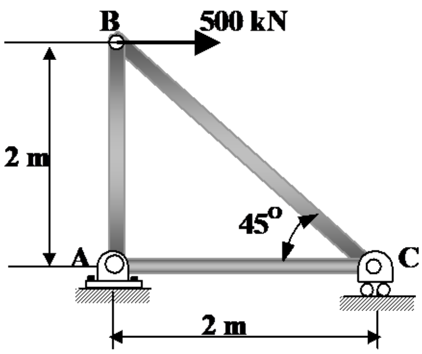
$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

Step 3: Solve for the two-unknown member forces and verify their correct sense.

Step 4: Continue to analyse each of the other joints. Realize that once the force in a member is found from the analysis of a joint at one of its ends, the result can be used to analyse the forces acting on the joint at its other end.

Example 6.1

Determine the force in each member of the shown truss and indicate whether the members are in tension or compression.

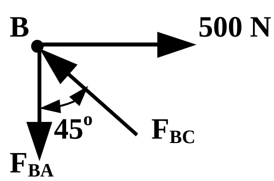


Solution

By inspection, there are two unknown member forces at joint B, two unknown member forces and an unknown reaction force at joint C, and two unknown member forces and two unknown reaction forces at joint A. Hence, we must begin the analysis at joint **B.**

Joint B

The free body diagram of pin at B indicates that there are three forces acting on the bin:



The external force of 500 N and the two unknown forces developed by members BA and BC. Applying the equations of joint equilibrium, we have:

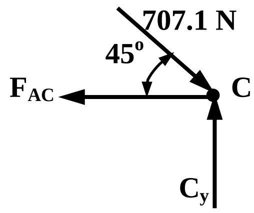
$\sum F_x = 0; \quad 500 - F_{BC}\sin 45^o = 0 \quad F_{BC} = 707.1 \quad N \quad (C)$

$\sum F_y = 0; \quad F_{BC}\cos 45^o - F_{BA} = 0 \quad F_{BA} = 500 \quad N \quad (T)$

Since the force in member BC has been calculated, we can proceed to analyse joint C.

Joint C

From the free-body diagram of joint C, we have:

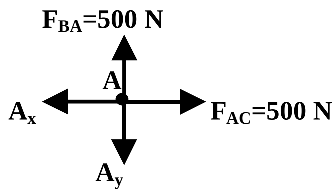


$\sum F_x = 0; \quad -F_{AC} + 707.1\cos 45^o = 0 \quad F_{AC} = 500 \quad N \quad (T)$

$\sum F_y = 0; \quad C_y - 707.1\sin 45^o = 0 \quad C_y = 500 \quad N$

Joint A

Although not necessary, we can determine the support reactions at joint A using the obtained results. From the free-body diagram of pin at joint A, we have:

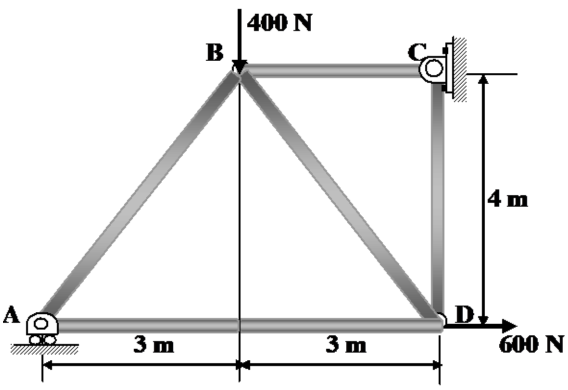


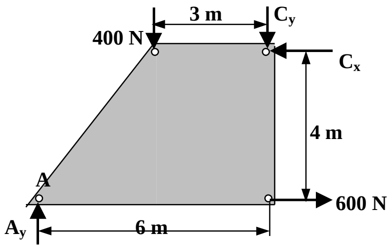
$\sum F_x = 0; \quad 500 - A_x = 0 \quad A_x = 500 \quad N$

$\sum F_y = 0; \quad 500 - A_y = 0 \quad A_y = 500 \quad N$

Example 6.2

Determine the force in each member of the shown truss. Indicate whether the members are in tension or compression.





Solution

No joint can be analysed until the support reactions are determined.

Support Reactions

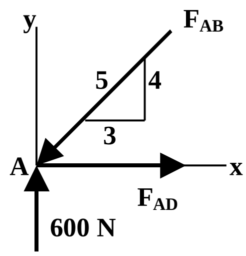
The free body diagram of the entire truss is drawn at first. Applying the equations of equilibrium, we have

$\sum F_x = 0; \quad 600 - C_x = 0 \qquad C_x = 600 \quad \text{N}$

$\sum M_C = 0; \quad -A_y(6) + 400(3) + 600(4) = 0 \qquad A_y = 600 \quad \text{N}$

$\sum F_y = 0; \quad 600 - 400 - C_y = 0 \qquad C_y = 200 \quad \text{N}$

Joint A

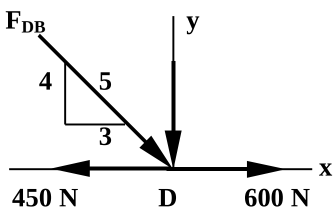


The inclination of $\vec{\mathbf{F}}_{AB}$ is determined from the geometry of the truss. Applying the equations of equilibrium of pin at joint A, we have

$\sum F_x = 0; \quad 600 - \frac{4}{5}F_{AB} = 0 \qquad F_{AB} = 750 \quad \text{N} \quad \text{(C)}$

$\sum F_y = 0; \quad F_{AD} - \frac{3}{5}(750) = 0 \qquad F_{AD} = 450 \quad \text{N} \quad \text{(T)}$

Joint D



The force in AD is known and the unknown forces in BD and DC can be determined. Applying the equations of equilibrium of the pin at joint D, we have :

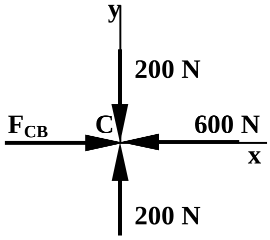
$\sum F_x = 0; \quad -450 + \frac{3}{5}F_{DB} + 600 = 0 \qquad F_{DB} = -250 \quad \text{N} \quad \text{(T)}$

$\sum F_y = 0; \quad -F_{DC} - \frac{4}{5}(-250) = 0 \qquad F_{DC} = 200 \quad \text{N} \quad \text{(C)}$

The negative sign indicates that the force $\mathbf{F_{DB}}$ acts in the opposite sense to that shown in figure. Hence, $\mathbf{F_{DB} = 250\ N}$ **(T)**

<u>Joint</u> C

The equilibrium of the pin at joint C gives:



$\sum F_x = 0; \quad F_{CB} - 600 = 0 \quad F_{CB} = 600 \text{ N} \quad (C)$

$\sum F_y = 0; \quad 200 - 200 = 0 \quad (\text{Check})$

6.4 Zero-Force Members

Truss analysis using the method of joints is greatly simplified if we can first identify those members which support no loading. These zero-force members are used to increase the stability of the truss during construction and to provide added support if the loading is changed.

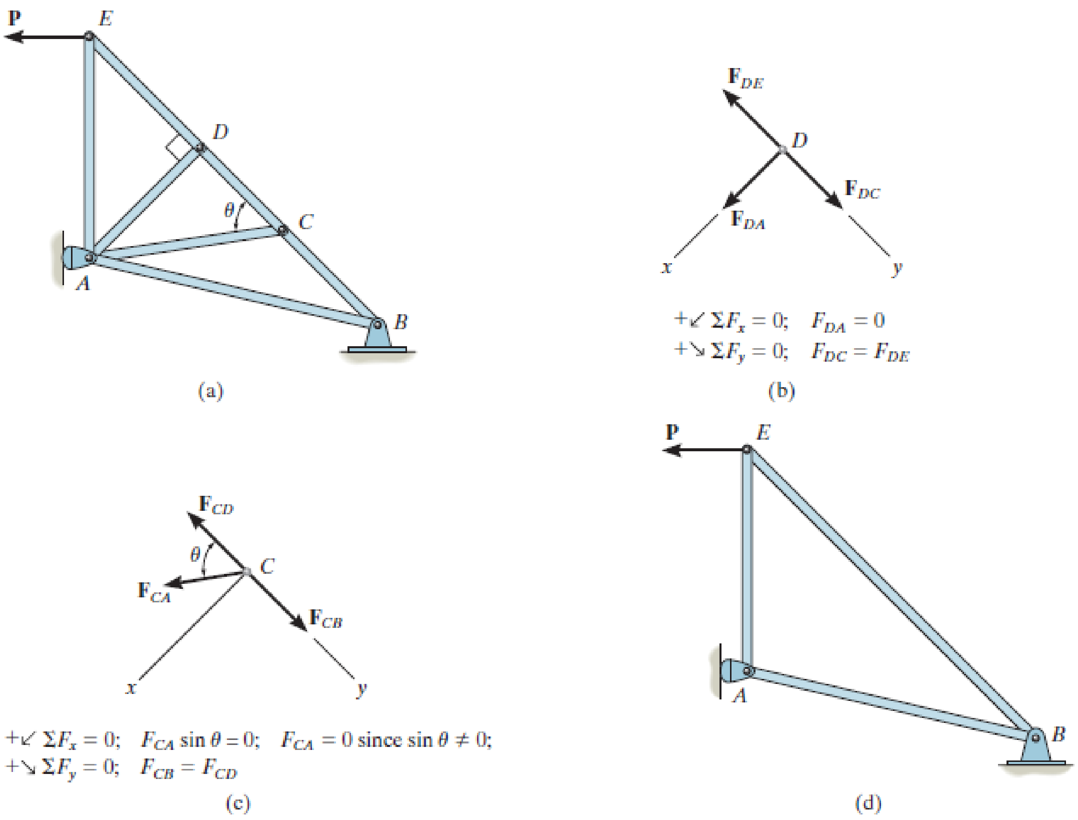


Fig. 6.4

The zero-force members of a truss can generally be determined by inspection of each of its joints. Consider the truss shown in Fig.6.4(a). The free body diagram of the pin at joint D is shown in Fig.6.3(b). By orienting the y-axis along the members DC and DE and x axis along member DA, it is seen that DA is a zero-force member. Note that this is also the case for member CA, Fig.6.4(c). The truss shown in Fig. 6.4(d) is therefore suitable for supporting the load P.

In general, **if three members of a truss are connected at a joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint.**

Also, consider the truss shown in Fig. 6.5(a). If a free body diagram of the pin at joint A is drawn, Fig. 6.5(b), it is seen that members AB and AF are zero-force members. (We could not have come to this conclusion if we had considered the free-body diagrams of joints F or B simply because there are five unknowns at each of these joints.) In a similar manner, consider the free body diagram of joint D, Fig. 6.5(c). Here again it is seen that DC and DE are zero-force members. From these observations, we can conclude that **if only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero-force members**. The load on the truss in Fig. 6.5(a) is therefore supported by only five members as shown in Fig. 6.5(d).

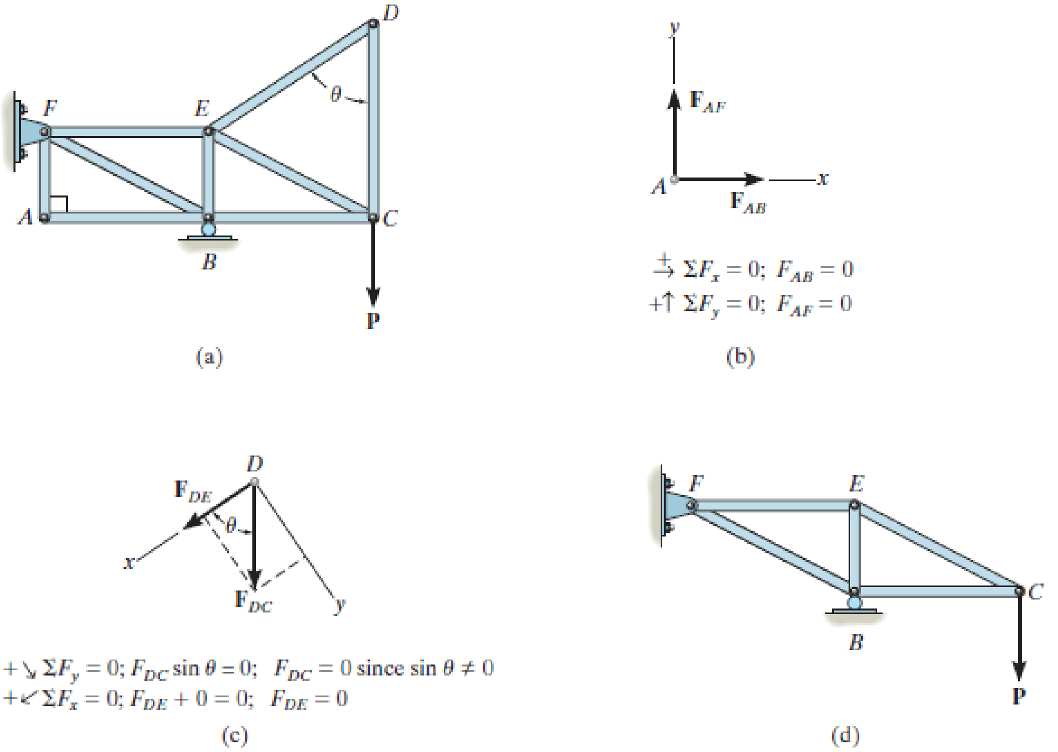


Fig. 6.5

6.5 The Method of Sections

The method of sections is used to determine the loading acting within a body. It is based on the principle that if a body is in equilibrium, then any part of the body is also in equilibrium. To apply this method, one passes an imaginary section through the body, thus cutting it into two parts. When a free body diagram of one of the parts is drawn, the loads acting at the section must be included on the free body diagram, Fig.6.6. One then applies the equations of equilibrium to the part in order to determine the loading at the section.

The three unknown member forces $\mathbf{F_{BC}}$, $\mathbf{F_{GC}}$ and $\mathbf{F_{GF}}$ can be obtained by applying the three equilibrium equations to the free body diagram, of the left-hand part, shown in Fig.6.6(b). If, however, the free body diagram of the right-hand part shown in Fig.6.6(c) is considered, the three support reactions $\mathbf{D_x}$, $\mathbf{D_y}$ and $\mathbf{E_x}$ will have to determine at first.

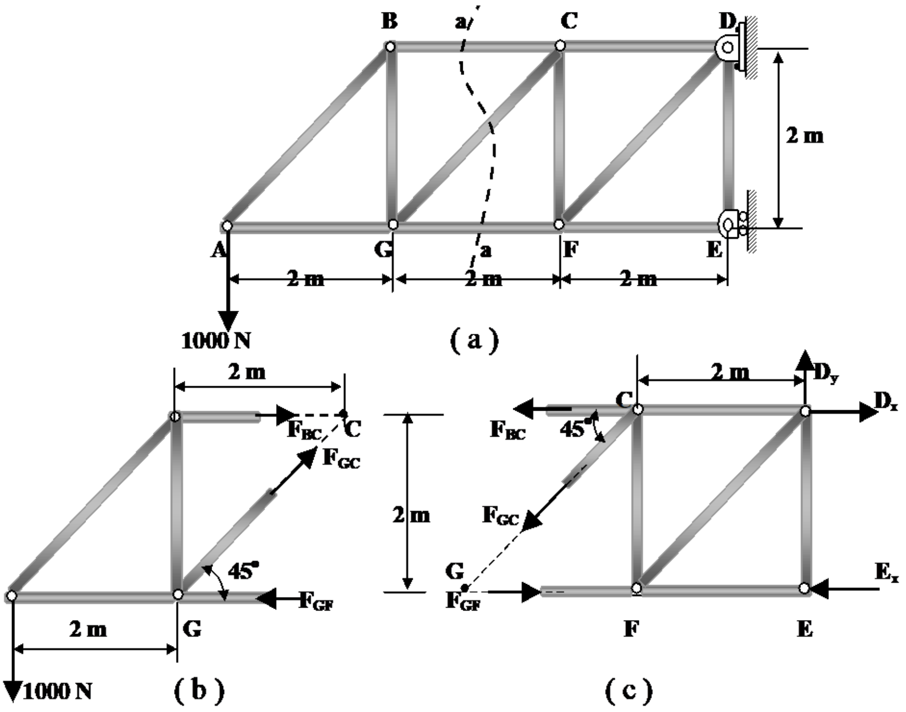
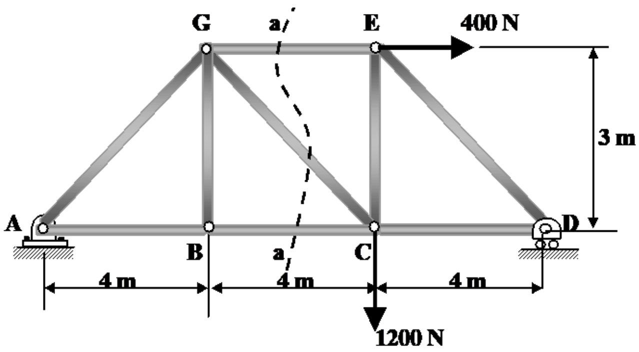


Fig. 6.6

When applying the equilibrium equations, one should consider ways of writing the equations so as to yield a direct solution for each of the unknowns, rather than having to solve simultaneous equations. For example, summing moments about C in Fig.6.6(b) would yield a direct solution for $\mathbf{F}_{GF}$, since $\mathbf{F}_{BC}$ and $\mathbf{F}_{GC}$ have no moment about C. Likewise, $\mathbf{F}_{BC}$ can be directly obtained by summing moments about G. Finally, $\mathbf{F}_{GC}$ can be found directly from a force summation in the vertical direction.

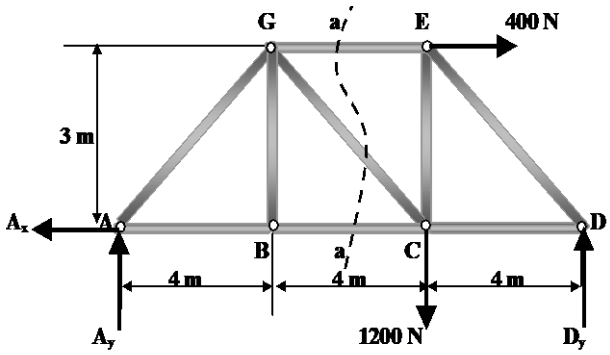
Example 6.3

Determine the force in members GE, GC and BC of the truss shown. Indicate whether the members are in tension or compression.

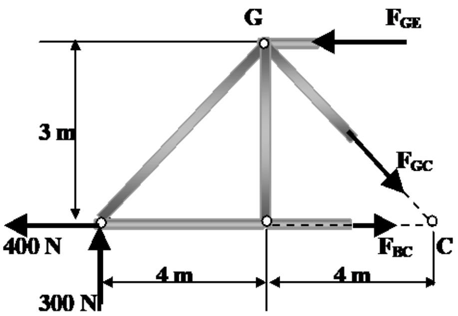


Solution

Section a-a is chosen since it cuts through the three members whose forces are to be determined.



It is first necessary to determine the external reactions at A or D. Considering the free-body diagram of the entire truss and applying the equations of equilibrium, we have



$\sum F_x = 0; \qquad 400 - A_x = 0$

$A_x = 400 \quad N$

$\sum M_A = 0; \quad -1200(8) - 400(3) + D_y(12) = 0$

$D_y = 900 \quad N$

$\sum F_y = 0; \qquad A_y - 1200 + 900 = 0$

$A_y = 300 \quad N$

The free body diagram of the left-hand part of the truss will be used since it involves the least number of forces.

Summing moments about G, we have:

$\sum M_G = 0; \quad -300(4) - 400(3) + F_{BC}(3) = 0$

$F_{BC} = 800 \quad N \qquad (T)$

By summing moments about C, we have:

$\sum M_C = 0; \quad -300(8) + F_{GE}(3) = 0 \qquad F_{GE} = 800 \quad N \qquad (C)$

Finally, summing forces in the y direction, we get:

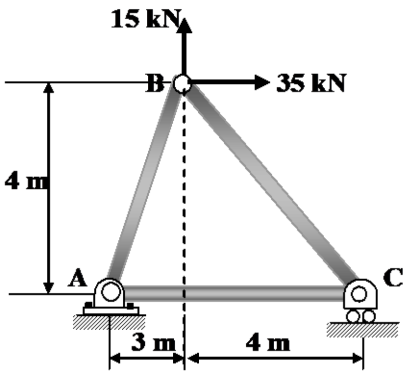
$\sum F_y = 0; \qquad 300 - \frac{3}{5} F_{GC} = 0 \qquad F_{GC} = 500 \quad N \qquad (T)$

PROBLEMS

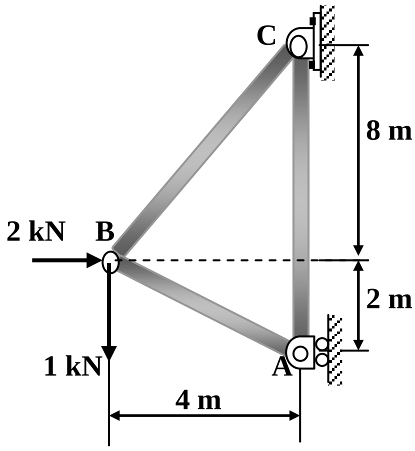
For the given trusses, Problem 6.1 to Problem 6.10, determine:

a) the components of the reactions at supports,

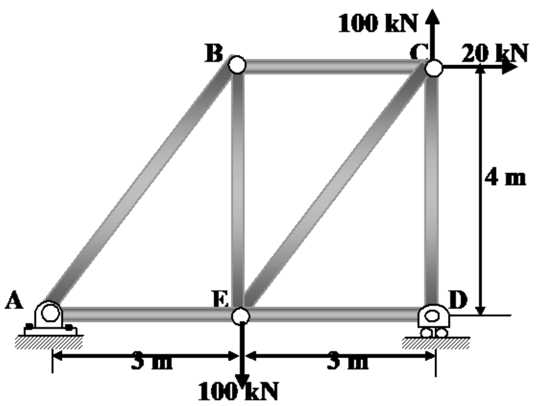
b) the forces in all members, using the method of joints. Indicate whether the members are in tension or compression.



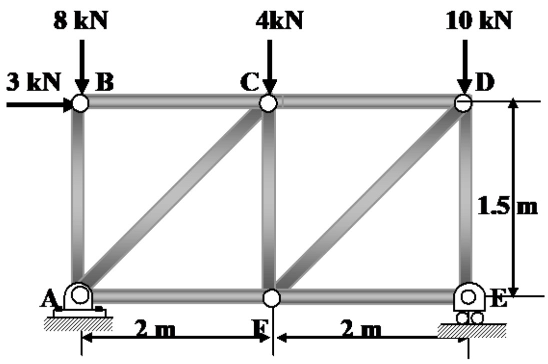
Problem 6.1



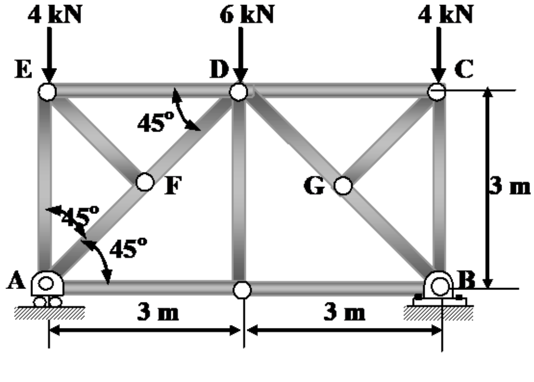
Problem 6.2



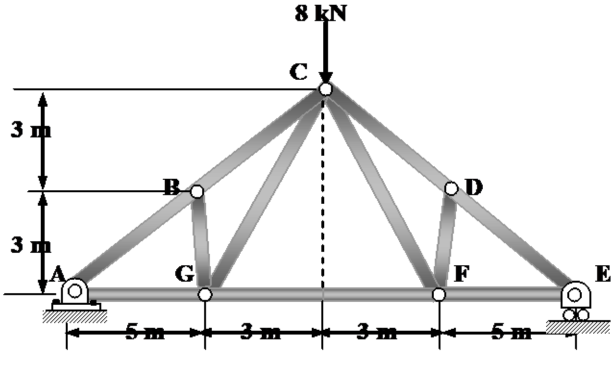
Problem 6.3



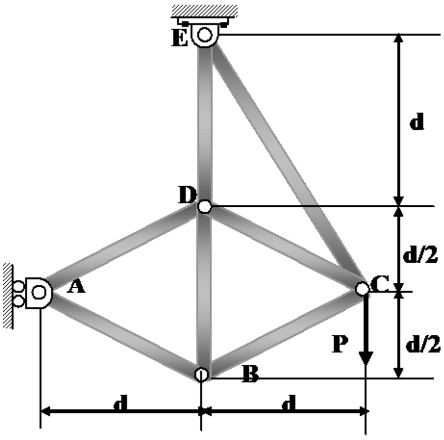
Problem 6.4



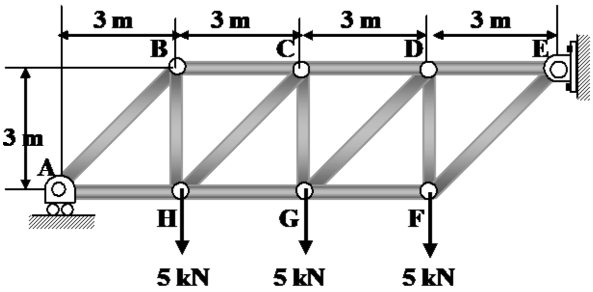
Problem 6.5



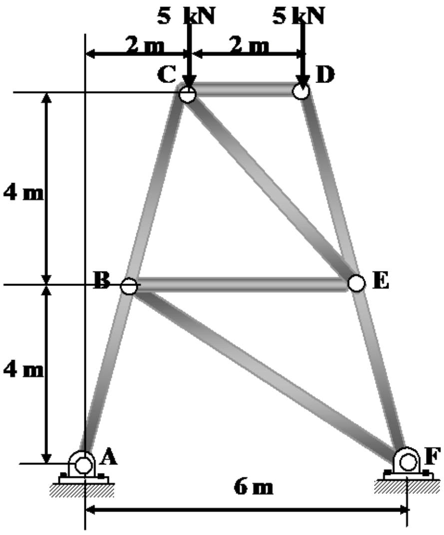
Problem 6.6



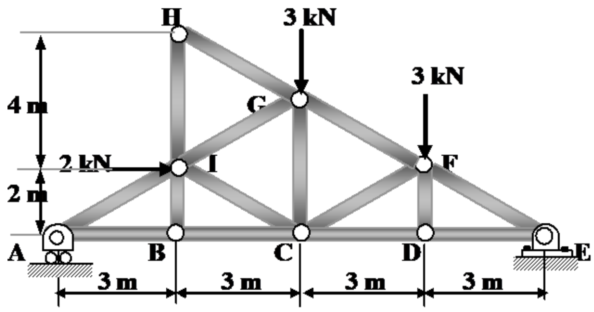
Problem 6.7



Problem 6.8

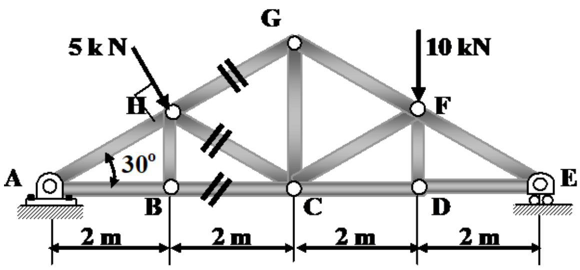


Problem 6.9

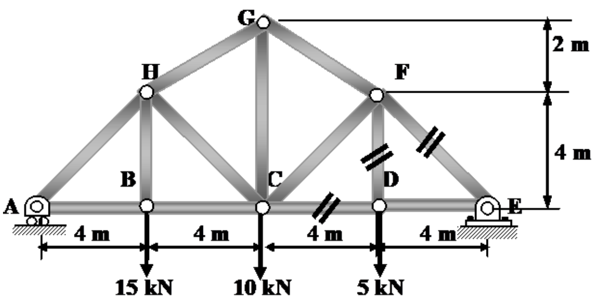


Problem 6.10

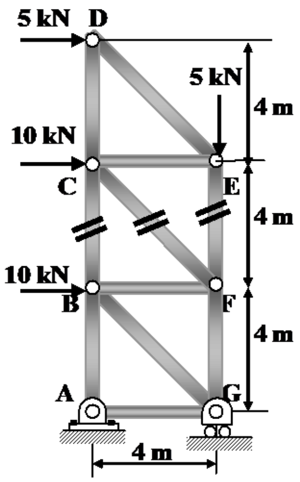
Using the method of sections, determine the force in indicated members of the given trusses (Problem 6.11 to Problem 6.16). State whether the members are in tension or compression



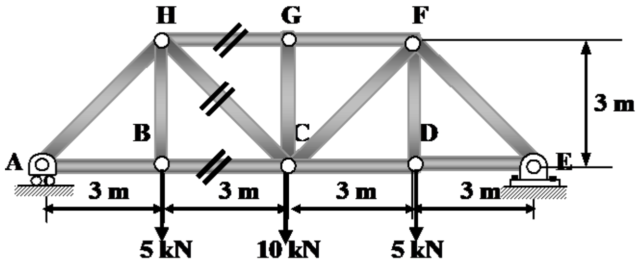
Problem 6.11



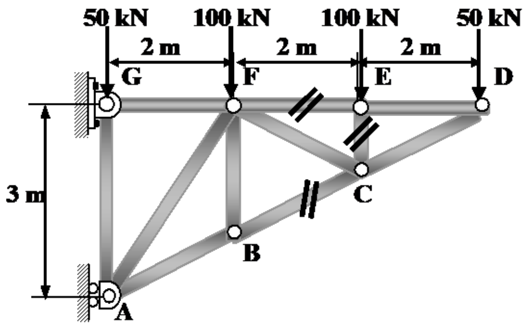
Problem 6.12



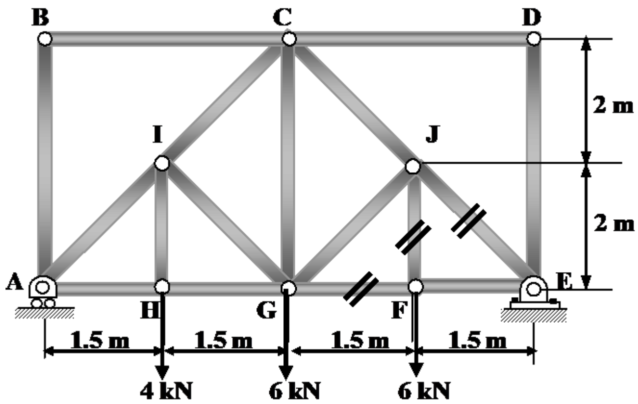
Problem 6.13



Problem 6.14



Problem 6.15



Problem 6.16

CHAPTER (7)

BEAMS
Supports, Loads and Reactions

7.1 Types of Supports

Three types of supports are recognized for beams loaded with forces acting in the same plane:

i- Roller support or link

In this case, the support is capable of resisting a force in only one direction, Fig. 7.1.

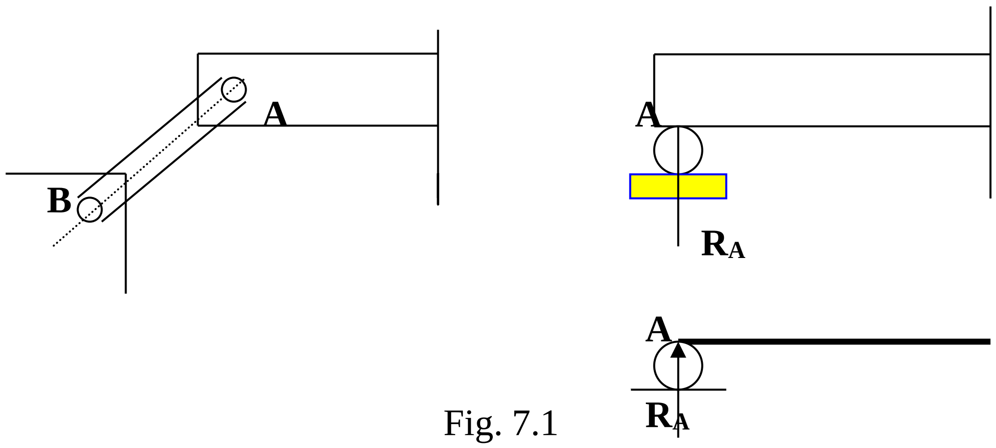


Fig. 7.1

ii- Hinged support or pin

The hinged support is capable of resisting a force acting in any direction of the plane. In general, the reaction may be resolved into two components, one in the horizontal direction and the other in the vertical direction, Fig. 7.2.

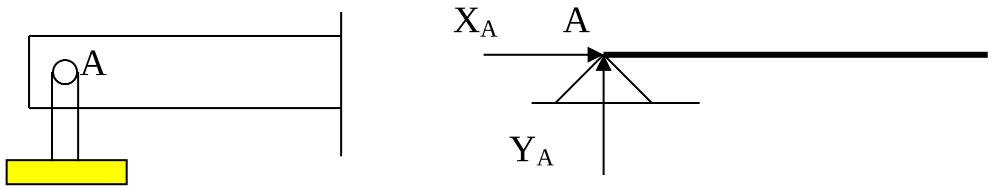


Fig. 7.2

iii- **Fixed support (built-in)**

The fixed support is capable of resisting a force in any direction and also is capable of resisting a couple or a moment. A system of three actions can exist at this support, two force components and a moment (Fig .7.3).

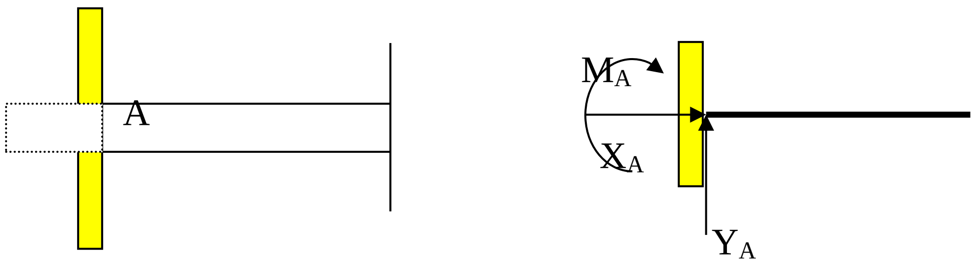


Fig. 7.3

7.2 Types of Loads

Beams can support or carry a variety of transverse loads and moments in their central plane:

i- Concentrated forces

In this case the forces are applied over very limited portions of the beam, Fig . 7.4.

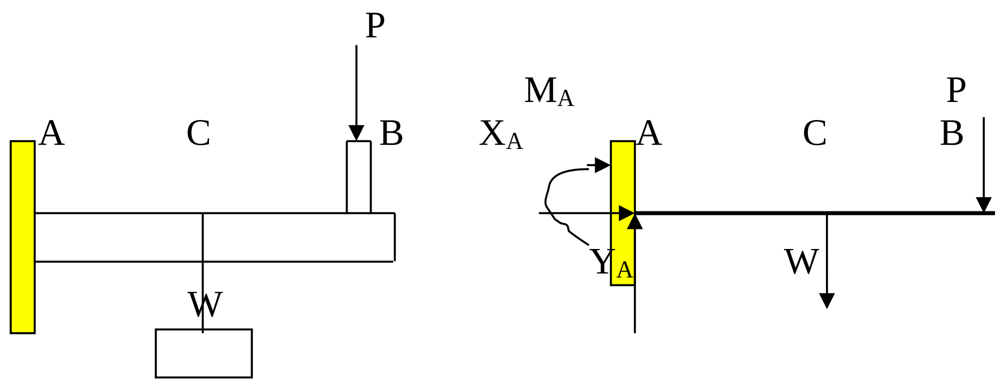
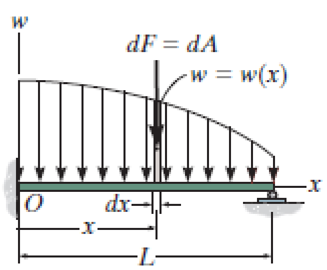


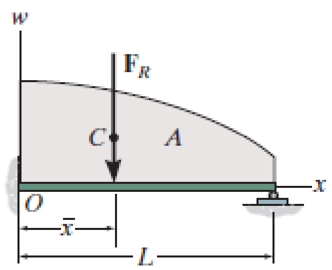
Fig. 7.4

ii- Distributed loads

The most common type of distributed loading encountered in engineering practice can be represented along a single axis. For example, consider the beam (or plate) in Fig. 7.5(a) that has a constant width and is subjected to a load that varies only along the x axis. This loading can be described by the function $w = w(x)$ N/m or lb/ft., that indicates the intensity of the loading along the length of a member. We can replace this coplanar parallel force system with a single equivalent resultant force F_R acting at a specific location on the beam. This resultant force is equivalent to the area under the loading diagram, and has a line of action that passes through the centroid or geometric center of this area, Fig. 7.5(b).



(a)



(b)

Fig. 7.5

iii- Concentrated moment (couple)

One possible arrangement of many, for applying a concentrated moment is given in Fig. 7.6.

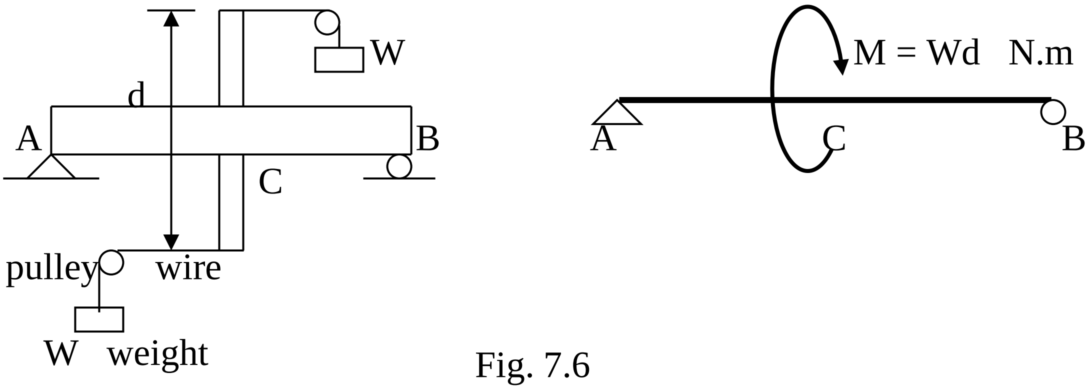


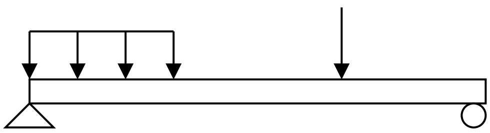
Fig. 7.6

7.3 Classification of Beams

Beams are classified into several groups, depending primarily on the kind of used supports :

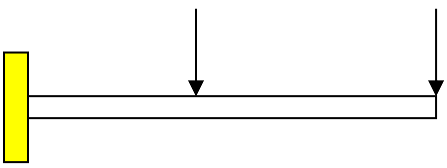
i- Simply supported beam

The supports are at the two beam ends and are either hinges or rollers.



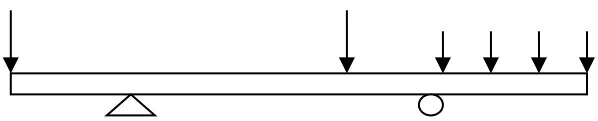
ii- Cantilever beam

One end is fixed and the other end is completely free.



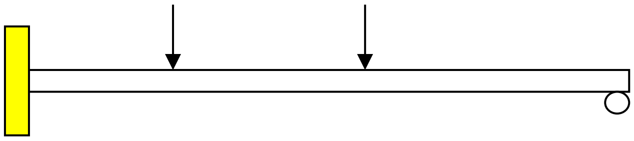
iii- Over hanging beam

The beam, in this case, projects beyond two intermediate supports.



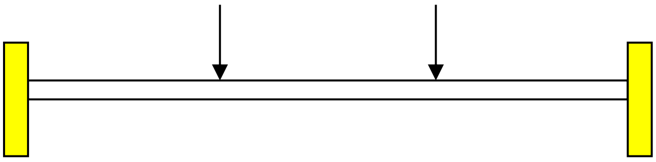
iv- Restrained beam

One end is fixed and the other end is simply supported.



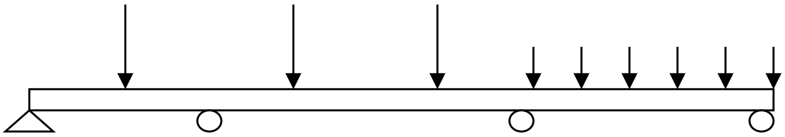
v- Fixed beam (fixed–ended beam)

The two ends of the beam are fixed.



vi- Continuous beam

Intermediate supports (more than two) are provided for a physically continuous member acting as a beam .

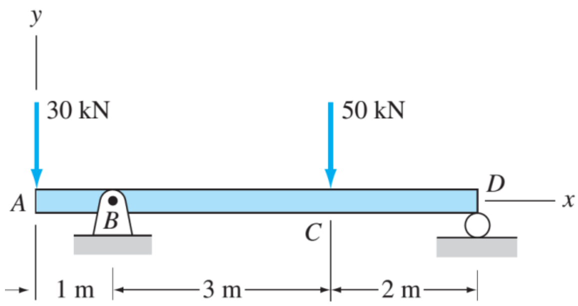


7.4 Determination of Reactions

A structure is kept in equilibrium under the action of the applied loads and the reactions at the supports. A general plane system of forces is in equilibrium if the resultant force and the resultant moment at any point (p) in the plane are zeros. Hence, the system of applied forces and reactions is in equilibrium if:

$$\begin{aligned} \Sigma\ F_x &= 0 \\ \Sigma\ F_y &= 0 \\ \Sigma\ M_p &= 0 \end{aligned} \qquad (7.1)$$

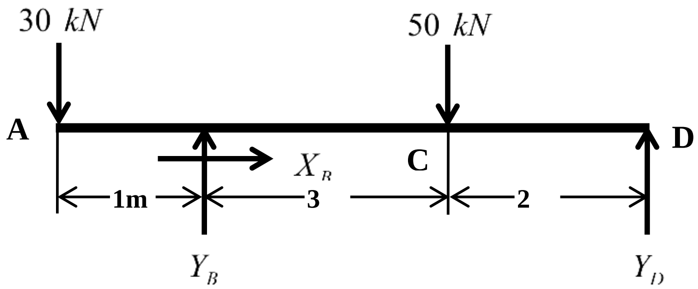
To determine the reactions of the beam, apply the three equations of equilibrium (7.1) for the system of forces composed of the known loads and the unknown three reaction components .



Example 7.1

For supported beam shown, determine the reactions at B and D.

Solution



$\sum M_B = 0:$

$30 \times 1 - 50 \times 3 + Y_D \times 5 = 0$

$30 - 150 + 5Y_D = 0$

$5Y_D = 120$

$Y_D = 24\ \text{kN}$

$\sum F_x = 0: \ X_B = 0$

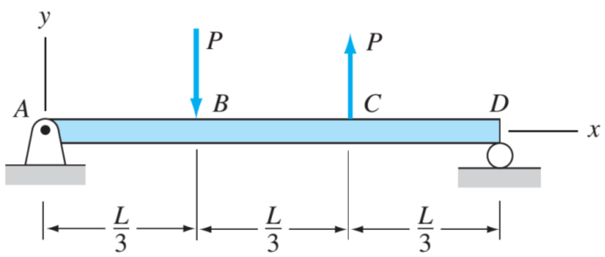
$\sum F_y = 0: \ Y_B + Y_D - 30 - 50 = 0$

$Y_B + Y_D = 80$

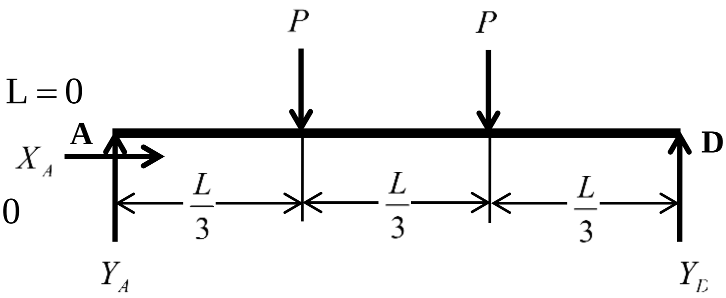
$Y_B + 24 = 80$ $\qquad Y_B = 56\ \text{kN}$

Example (7.2)

For supported beam shown, determine the reactions at A and D.



Solution



$$\sum M_A = 0: \quad -P \times \frac{L}{3} - P \times \frac{2L}{3} + Y_D \times L = 0$$

$$-P\left[\frac{L}{3} + \frac{2L}{3}\right] + Y_D \times L = 0$$

$$-P \times L + Y_D \times L = 0$$

$$Y_D = P$$

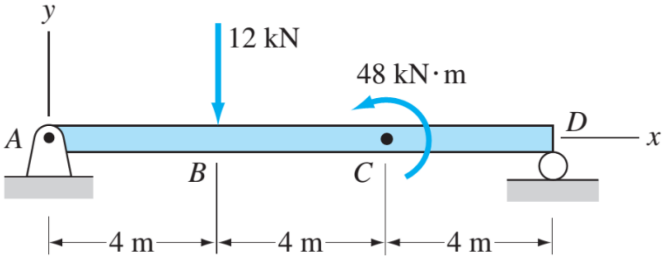
$$\sum F_x = 0: \quad X_A = 0$$

$$\sum F_y = 0: \quad Y_A + Y_D - P - P = 0$$

$$Y_A + P = 2P$$

Example (7.3)

For supported beam shown, determine the reactions at A and D.



Solution

$$\sum M_A = 0: \quad 48 - 12 \times 4 + Y_D \times 12 = 0$$

$$48 - 48 + 12Y_D = 0$$

$$12Y_D = 0$$

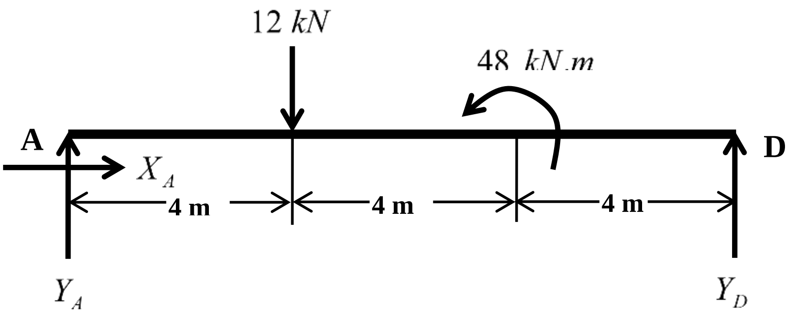
$$Y_D = 0$$

$$\sum F_x = 0: \quad X_A = 0$$

$$\sum F_y = 0: \quad Y_A + Y_D - 12 = 0$$

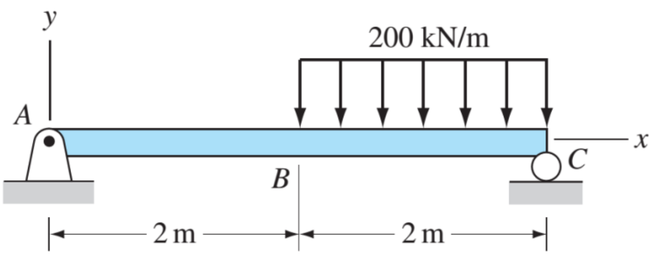
$$Y_A + 0 = 12$$

$$Y_A = 12 \text{ kN}$$



Example (7.4)

For supported beam shown, determine the reactions at A and C.

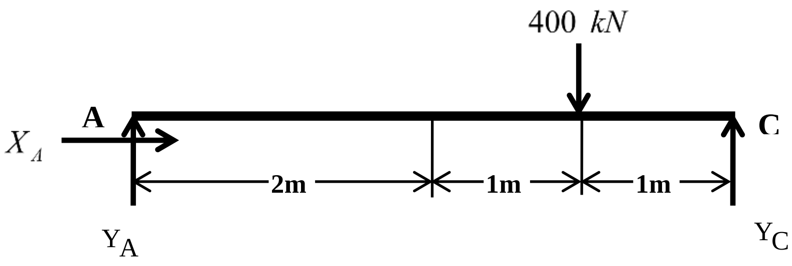


Solution

$$\sum M_A = 0: \quad -400 \times 3 + Y_C \times 4 = 0$$

$$4Y_C = 1200$$

$$Y_C = 300 \text{ kN}$$



$$\sum F_x = 0: \quad X_A = 0$$

$$\sum F_y = 0: \quad Y_A + Y_C - 400 = 0$$

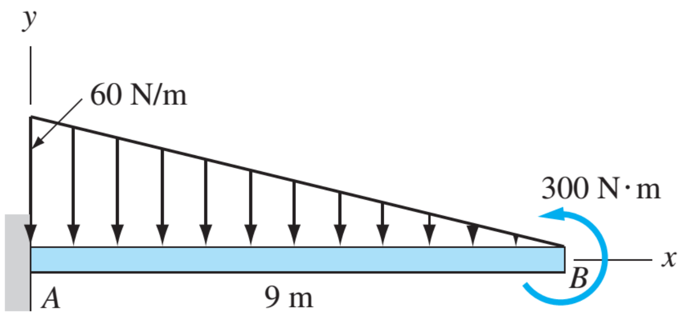
$$Y_A + Y_C = 400$$

$$Y_A + 300 = 400$$

$$Y_A = 100 \text{ kN}$$

Example (7.5)

For supported beam shown, determine the reactions at A.

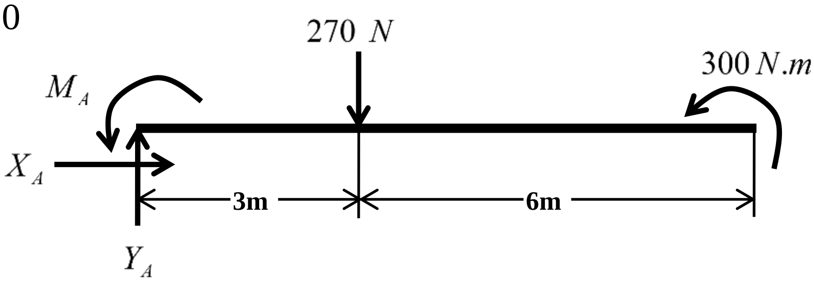


Solution

$$\sum M_A = 0: \quad 300 - 270 \times 3 + M_A = 0$$

$$300 - 810 + M_A = 0$$

$$M_A = 510 \text{ N.m}$$



$$\sum F_x = 0: \quad X_A = 0$$

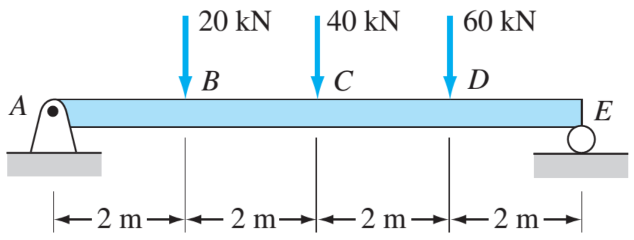
$$\sum F_y = 0: \quad Y_A - 270 = 0$$

$$Y_A = 270 \text{ N}$$

PROBLEMS

Determine the reaction for the given loaded beams

1.



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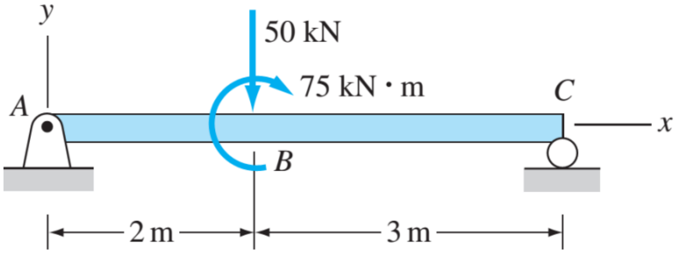

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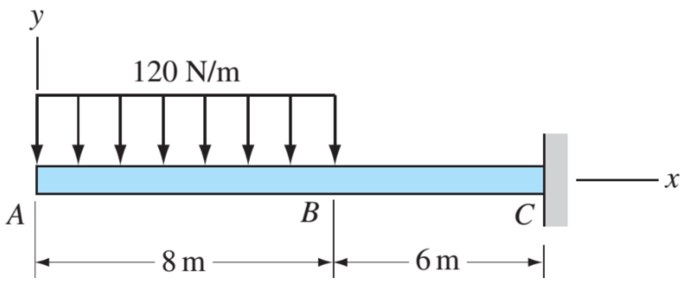

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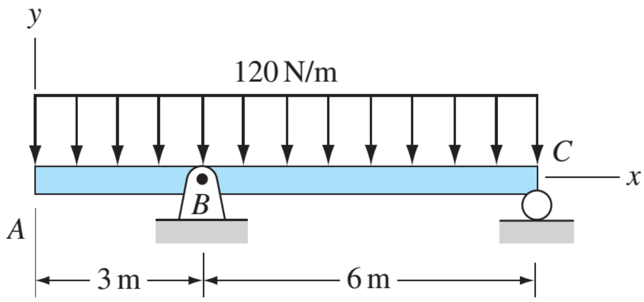

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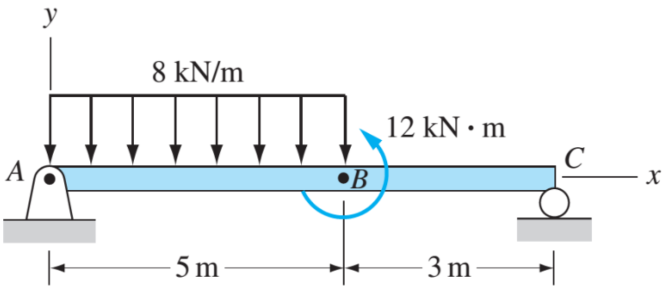
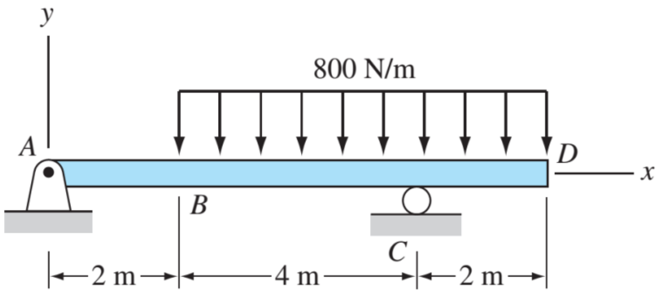
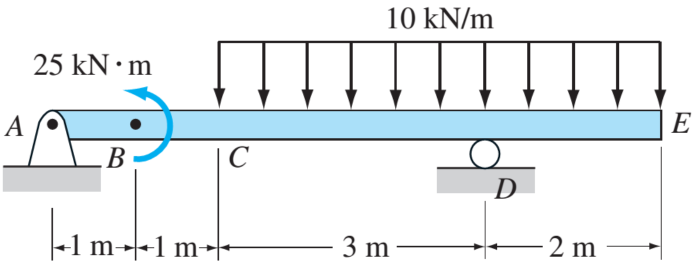


6.



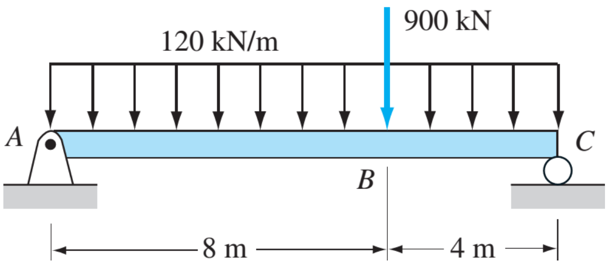
y
800 N/m
A
D
x
B
C
2 m
4 m
2 m

7.



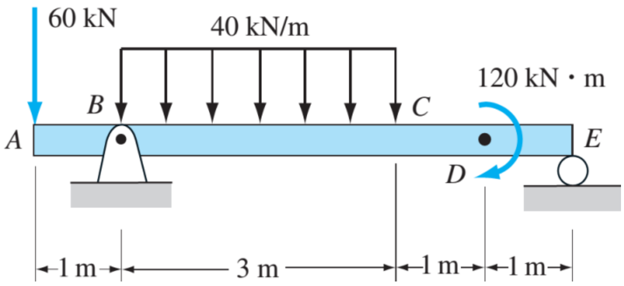
10 kN/m
25 kN · m
A
E
B
C
D
1 m
1 m
3 m
2 m

8.



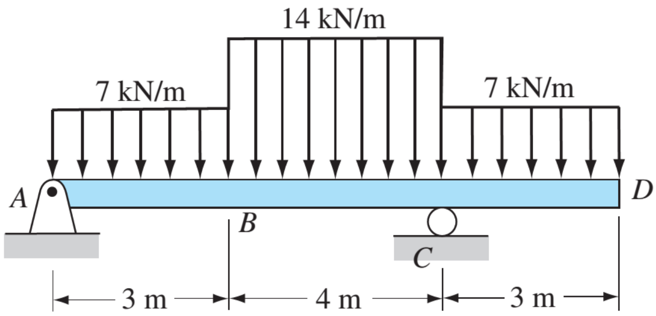
900 kN
120 kN/m
A
C
B
8 m
4 m

9.



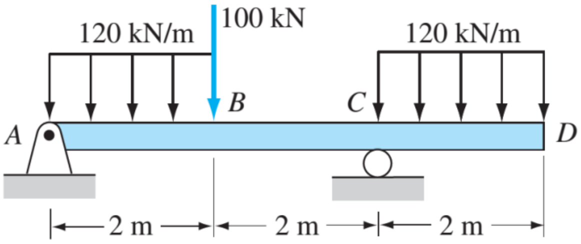
60 kN
40 kN/m
120 kN · m
B
C
A
E
D
1 m
3 m
1 m
1 m

10.



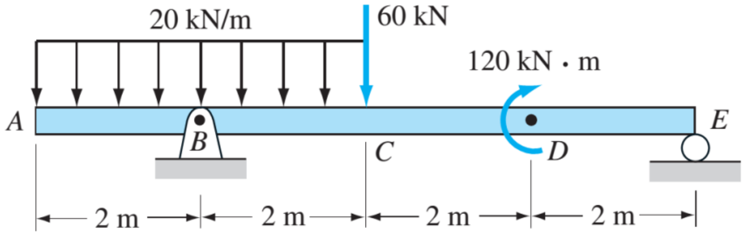
14 kN/m
7 kN/m
7 kN/m
A
D
B
C
3 m
4 m
3 m

11.



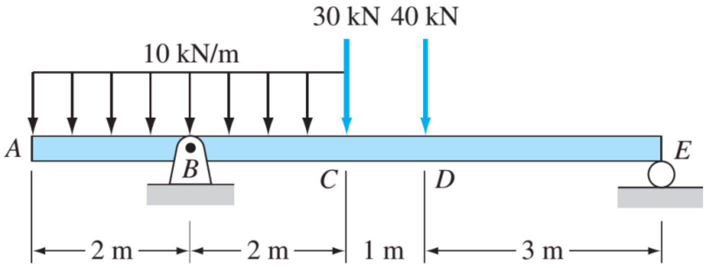
120 kN/m
100 kN
120 kN/m
A
B
C
D
2 m
2 m
2 m

12.



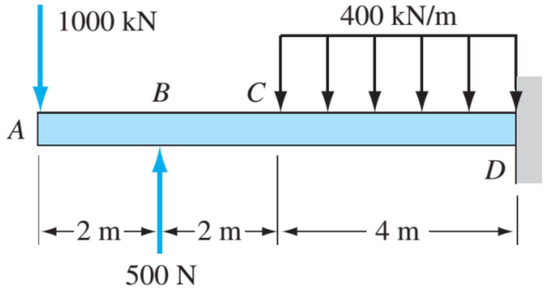
20 kN/m
60 kN
120 kN · m
A
B
C
D
E
2 m
2 m
2 m
2 m

13.



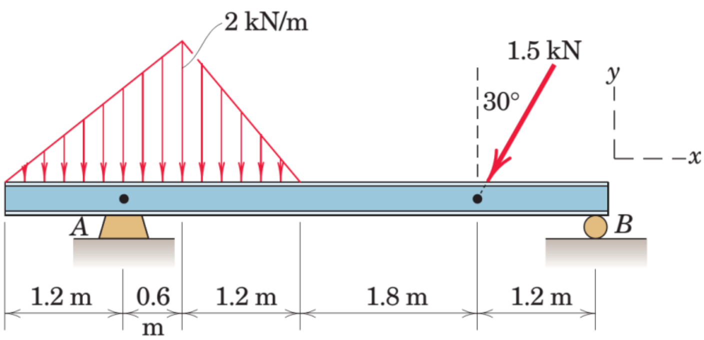
30 kN
40 kN
10 kN/m
A
B
C
D
E
2 m
2 m
1 m
3 m

14.



1000 kN
400 kN/m
A
B
C
D
2 m
2 m
4 m
500 N

15.



2 kN/m
1.5 kN
30°
y
x
A
B
1.2 m
0.6 m
1.2 m
1.8 m
1.2 m

CHAPTER (8)

MECHANISMS

One can use the graphical solution for problems involving plane forces in equilibrium. This method is advantageous in case of plane mechanisms due to the complexity of geometry of such systems. In graphical solution, the body (or the system) must be drawn to a suitable scale of length m_L. Moreover, a convenient scale of forces m_F is chosen to draw the force polygons.

It should be mentioned that if the whole system is in equilibrium, then the individual members must be, also, in equilibrium. During the solution one can find several bodies under different loading conditions. We may have two force body, three force body, … etc. So, at first we shall study the different possible cases of bodies under equilibrium.

8.1 Equilibrium of a Two Force Body

When a rigid body is subjected to two forces, it is commonly called a "two force body". If a two force body is in equilibrium, then the two forces must have the same magnitude, same line of action and opposite sense.

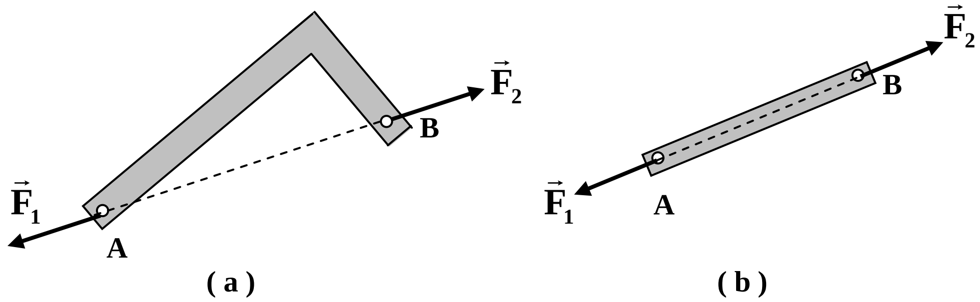


Fig. 8.1

The proof is self-evident. Consider a rigid body in equilibrium under the action of $\vec{F}_1$ and $\vec{F}_2$ (Fig.8.1). Then, for equilibrium, the sum of moments about any point must be zero. Take $\sum M_A = 0$, so we can conclude that the line of action of $\vec{F}_2$ must pass through A. Similarly, by applying $\sum M_B = 0$, we see that the line of action of $\vec{F}_1$ must pass through B. Thus, the two forces have the same line of action. From $\sum F_x = 0$ or $\sum F_y = 0$, it can be proved that the two forces have the same magnitude and opposite sense.

Note that if the body is a straight member as shown in Fig. 8.1(b), then the line of action of F_1 and F_2 coincides with the line AB which is the axis of the member itself.

8.2 Equilibrium of a Three Force Body

If a body is in Equilibrium under the action of three forces, then these forces must be coplanar and their lines of actions intersect at one point. If we consider that two forces constitute one plane, then the third one must lie in the same plane to ensure that the sum of components of the forces in a direction perpendicular to that plane is zero. Thus, the three forces are coplanar.

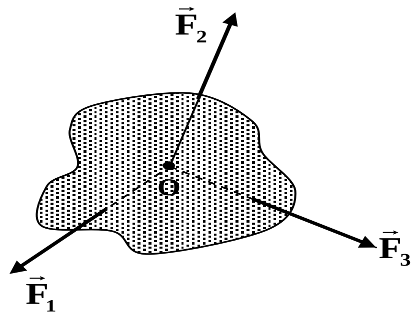


Fig. 8.2

As shown in Fig. 8.2, three forces $\vec{F}_1$, $\vec{F}_2$ and $\vec{F}_3$ act at different points A, B and C, respectively. Suppose that $\vec{F}_1$ and $\vec{F}_2$ intersect at one point O.Take $\sum M_O = 0$. This equation will be satisfied if the line of action of $\vec{F}_3$ passes through point O. Thus, the three forces must intersect at the same point O. an exceptional case is when the three forces are parallel. So, no point of intersection can be obtained.

Example 8.1

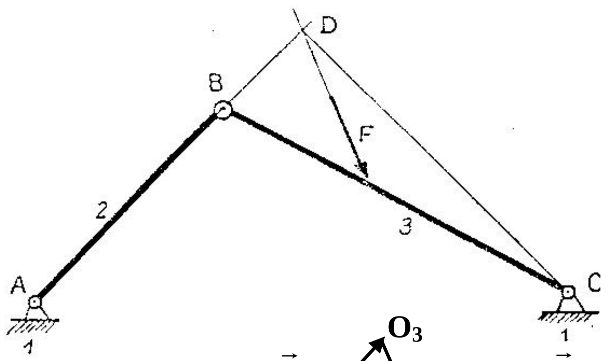
The shown structure supports a force F = 5000 N. Determine, graphically, the reactions at A and C as well as the pin force at B.

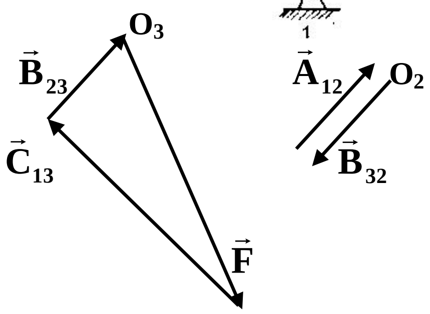
Scale of length m_L = 10 cm/cm

Solution:

The body 2 is a two force body. So, it is either in tension or compression. It

m_L= 10 cm/cm
m_F = 1000 N/cm





A	B	C (N)
1800	1800	4500

is acted upon by two forces $\vec{A}_{12}$ and $\vec{B}_{32}$ where:

$\vec{A}_{12}$ is the action of frame 1 on body 2,

$\vec{B}_{32}$ is the action of body 3 on body 2,

Then; $\vec{A}_{12} + \vec{B}_{32} = 0$

These forces have the line of action AB.

The Body 3 is a three force body,

Then; $\vec{B}_{23} + \vec{F} + \vec{C}_{13} = 0$

These forces intersect at one point. This point can be obtained by knowing two lines of action. The line of action of $\vec{B}_{23}$ is along AB. Then, the three forces acting on body 3 intersect at point D. The corresponding triangle of forces is shown, from which we can get the magnitudes of $\vec{B}_{23}$ and $\vec{C}_{13}$.

For body 2: $\vec{B}_{32} = -\vec{B}_{23}$ and $\vec{A}_{12} = -\vec{B}_{32}$

These two forces are also shown.

The results are indicated in a table in which we write the magnitude of the reaction at any joint.

8.3 Equilibrium of Two Forces and a Couple

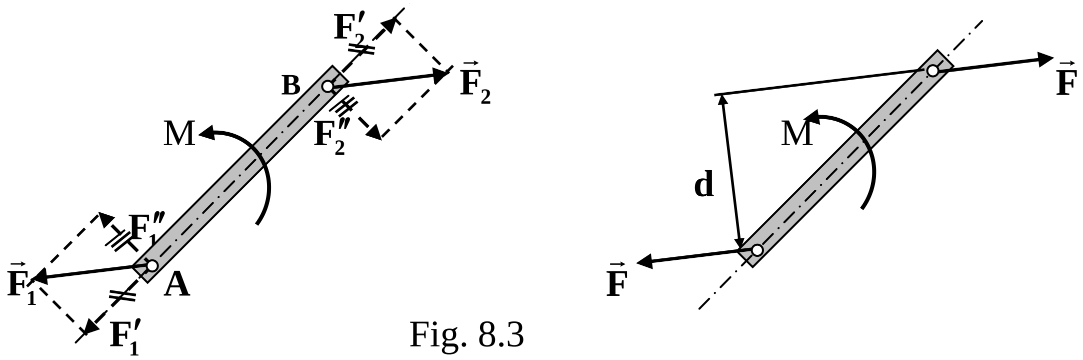


Fig. 8.3

Suppose that the beam AB, of length L, is in equilibrium under the action of a couple of moment M and two forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ acting at A and B as shown (Fig. 8.3). Each force can be resolved into two components: F\` along the axis of the beam and F" perpendicular to that axis. Take the sum of forces in direction AB, then $F_1` = F_2`$ and have opposite sense.

Apply $\sum M_A = 0$ then, $F_2'' = M/L$

$$\text{Apply } \sum M_B = 0 \qquad \text{then,} \qquad F_1'' = M/L$$

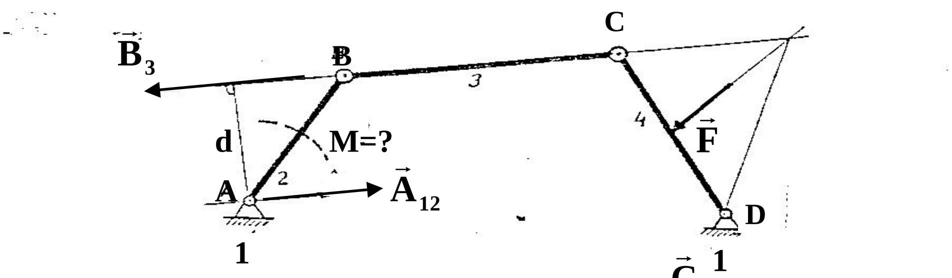
F_1'' and F_2'' have also, opposite sense. Therefore, the two forces $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude, and opposite in sense, such that they form a couple whose moment is equal to the given applied moment.

<u>Example 8.2</u>

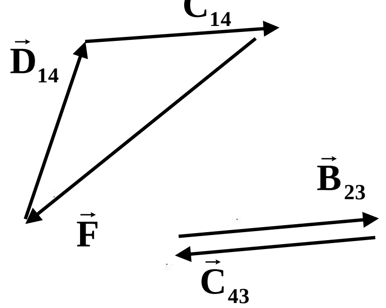
A force F = 6000 N acts on link 4 of the four joint mechanism as shown in figure. Determine the magnitude of the moment M which must be applied to link 2 to maintain the equilibrium of the mechanism. Find also, the reactions at the joints.

$m_L = 10$ cm/cm; $\qquad m_F = 1000$ N/cm

<u>Solution</u>:



A	B	C	D (N)	M (N cm)
3000	3000	3000	4100	75000



<u>Link 2</u> is in equilibrium under 2 forces ($\vec{A}_{12}$, $\vec{B}_{32}$) and the required couple.

$$\vec{A}_{12} + \vec{B}_{32} = 0$$

Link 3 is a two force body

$$\vec{B}_{23} + \vec{C}_{43} = 0$$

Link 4 is a three force body

$$\vec{F} + \vec{C}_{34} + \vec{D}_{14} = 0$$

At first we solve for link 4 to get $\vec{D}_{14}$ and $\vec{C}_{34}$ (and hence $\vec{C}_{43}$). Thus, $\vec{B}_{23}$ (and hence $\vec{B}_{32}$) can be determined by analyzing link 3. For link 2:

$$M = B_{32} \times d$$

Acting in clockwise direction. The distance d is the perpendicular distance between the lines of action of $\vec{B}_{32}$ and $\vec{A}_{12}$.

8.4 Culmann's line

A particular case of equilibrium which is of considerable interest is that of a rigid body subjected to four forces such that: One of them is known in magnitude and direction; and the other Three forces are known only in direction (known lines of action). Such system of forces can be solved graphically using the "Culmann's line".

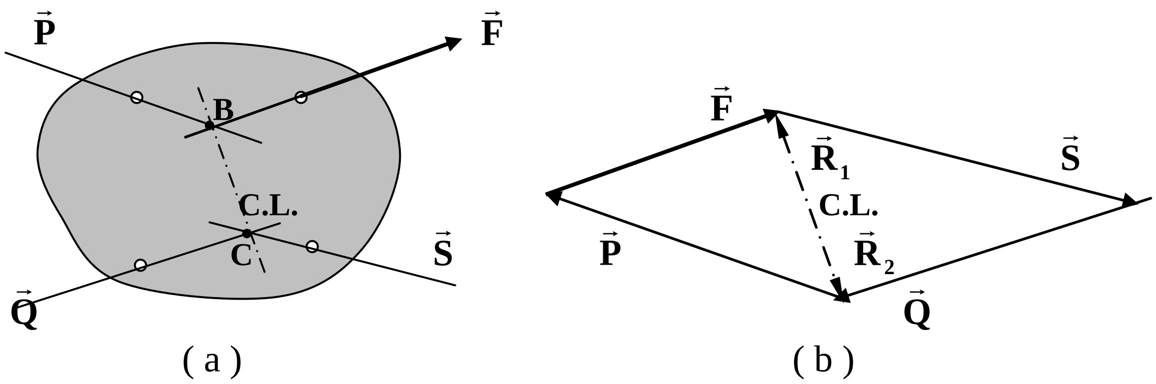


Fig. 8.4

The shown body (Fig. 8.4) is in equilibrium under the action of $\vec{F}$, $\vec{P}$, $\vec{Q}$ and $\vec{S}$. the force $\vec{F}$ is known.

$$\vec{F} + \vec{P} + \vec{Q} + \vec{S} = 0$$

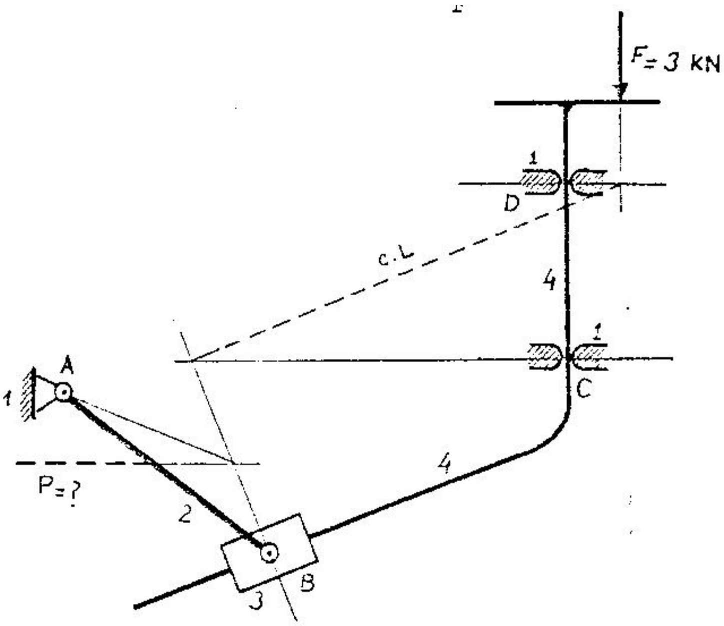
Suppose that any two of them (e.g $\vec{\mathbf{F}}$ and $\vec{\mathbf{P}}$) intersect at point B. Therefore, their resultant $\vec{R}_1 = \vec{F} + \vec{P}$ passes through B. the other two forces ($\vec{Q}$ and $\vec{S}$) intersect at C.

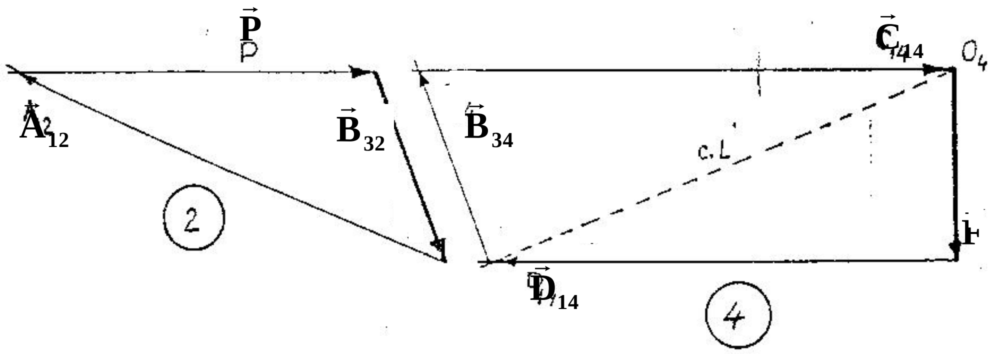
Then, their resultant $\vec{R}_2 = \vec{Q} + \vec{S}$ passes through C. Therefore, the four forces are reduced to $\vec{R}_1$ and $\vec{R}_2$ acting at B and C, respectively. Thus, the lines of action of $\vec{R}_1$ and $\vec{R}_2$ must be the same and it is the line BC. This line is called "Culmann's line" C.L. As shown in Fig. 8.4 (b), this line is used to construct the force polygon.

Example 8.3

The shown mechanism is acted upon by a force F = 3 kN. Find the force P, acting on member 2, to maintain the equilibrium. Find also the reactions in the joints.

m_L= 10 cm/cm m_F= 1000 N/cm

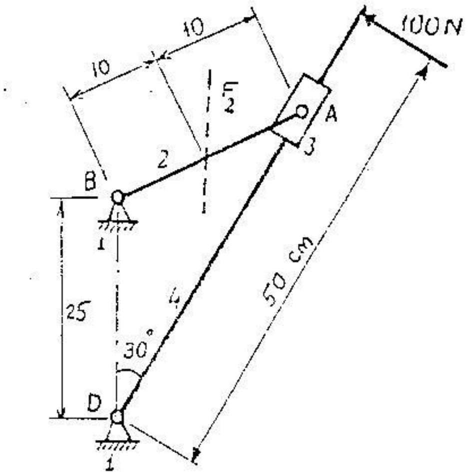




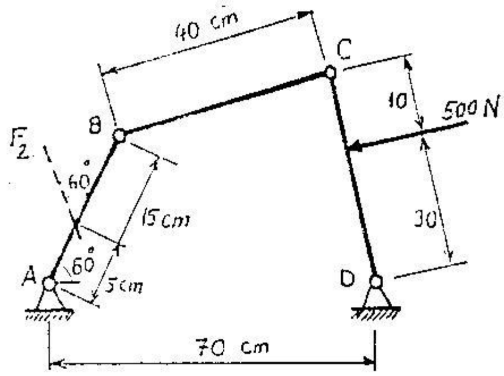
A	B	C	D	P (N)
7800	3200	8500	7400	6000

PROBLEMS

8.1 For the shown mechanisms find the magnitude of the force F_2 required to maintain the equilibrium. Find also the reactions in the connections.

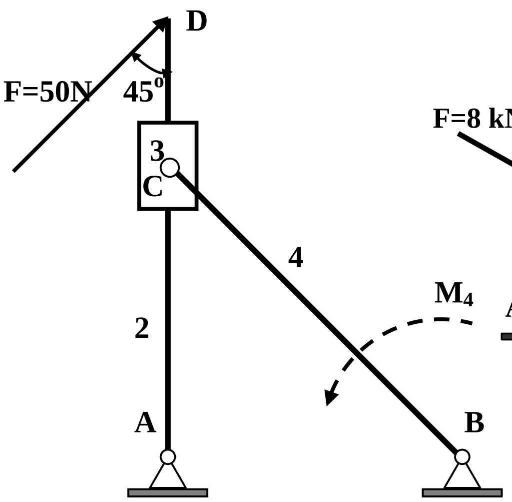


a)

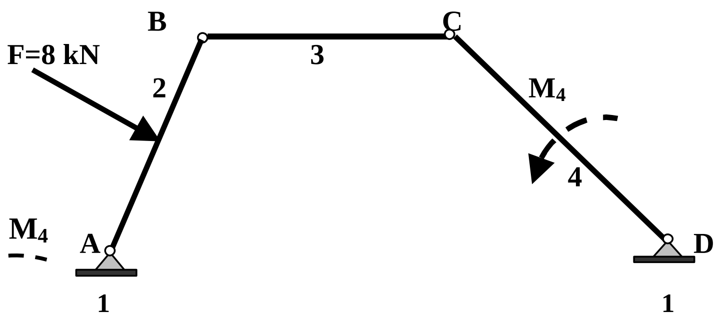


b)

8.2 For the shown mechanisms find the magnitude of the couple M_4 required to maintain the equilibrium. Find also the reactions in the connections. Scale of length $m_L = 10$ cm/cm.

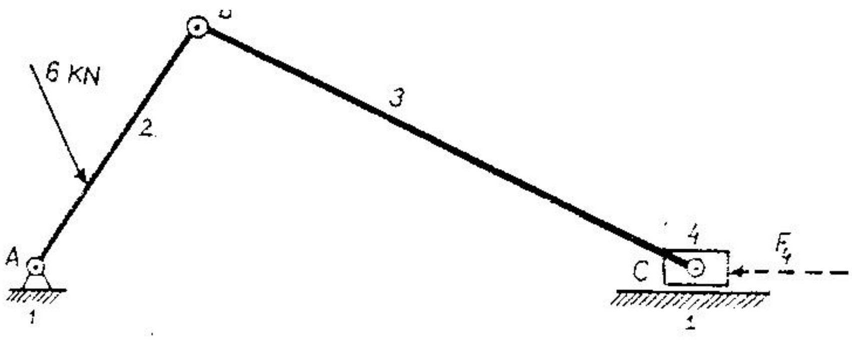


a)

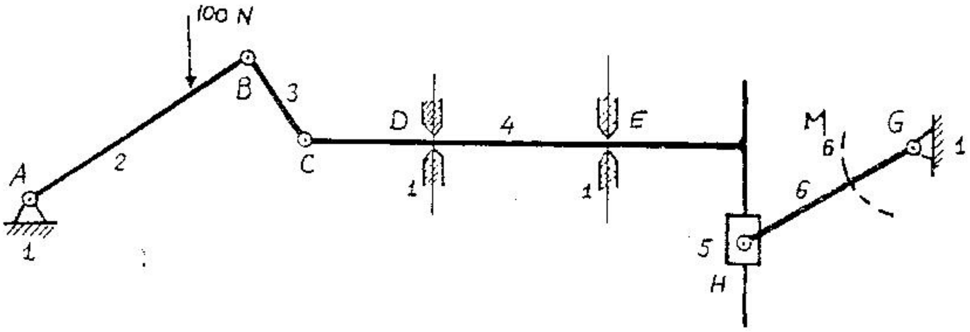


b)

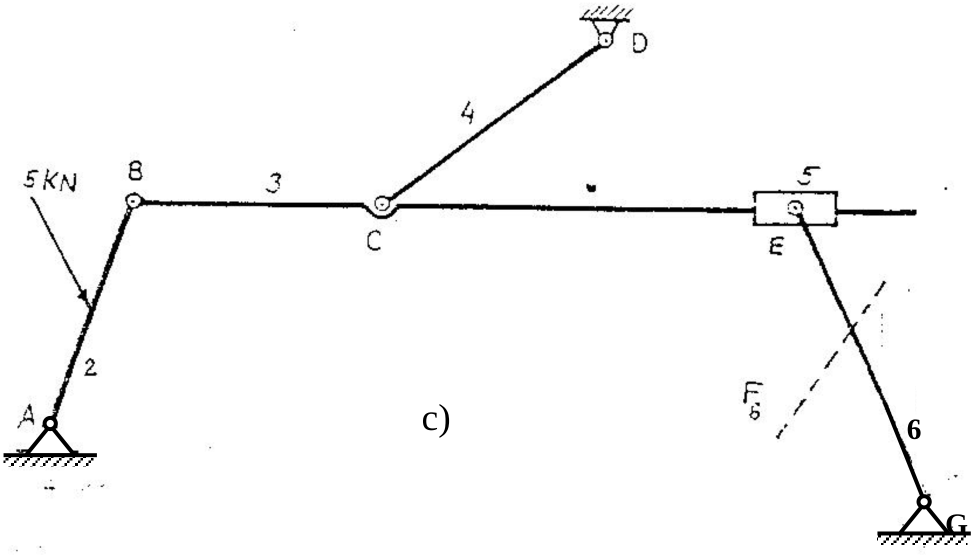
8.3 The following mechanisms are drawing to scale m_L = 10 cm/cm. For each case find the necessary values, drawn in dashed lines, to keep the equilibrium. Find also the reactions in all joints.



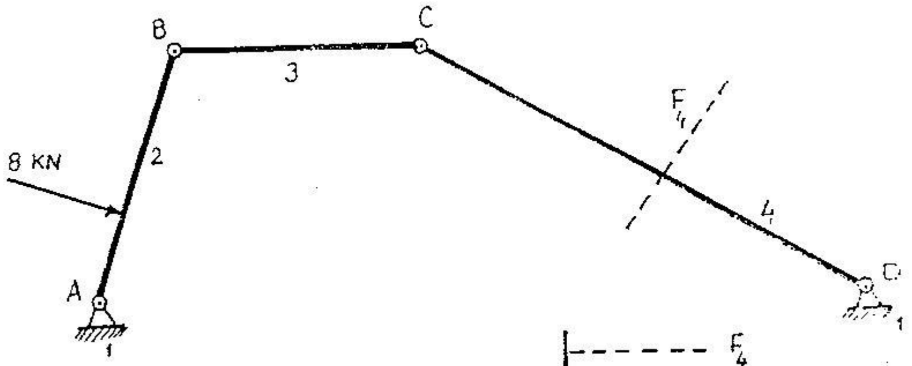
a)



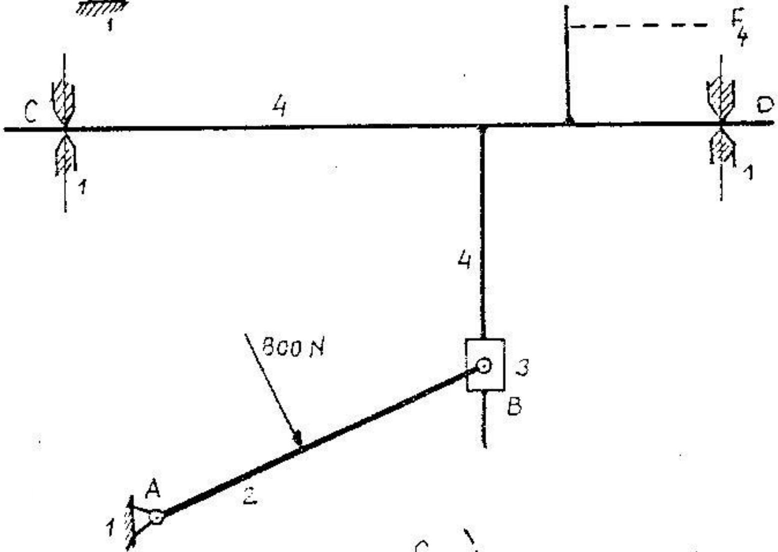
b)



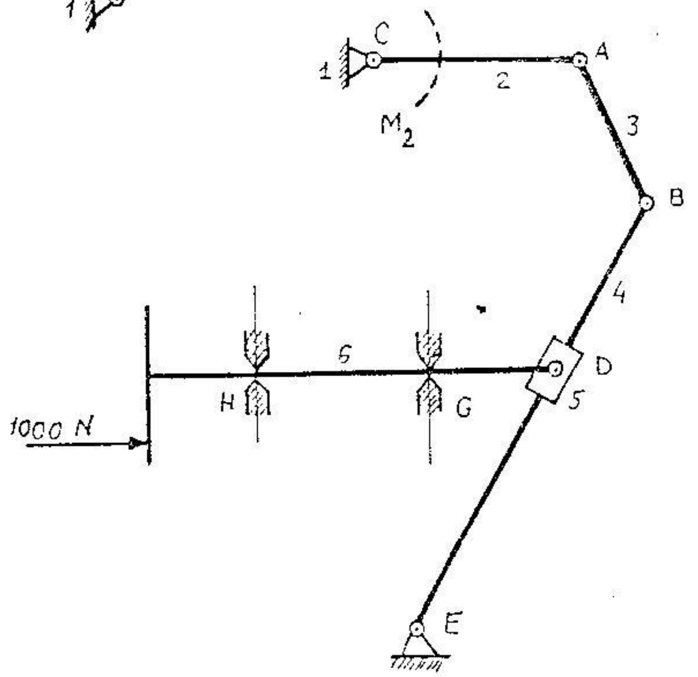
c)


B
C
3
F4
8 KN
2
4
A
D
1
1

d)


F4
C
4
D
1
1
4
800 N
3
B
A
2
1

e)


C
A
1
2
M2
3
B
4
6
D
H
G
5
1000 N
E

a)

f)

CHAPTER (9)

FRICTION

The resistance to motion of two bodies, in contact with each other, is determined by friction. Friction is a complex physical phenomenon and the amount of it depends on number of factors. There are different types of friction; dry, fluid, rolling, sliding, static and kinetic. In the present chapter, sliding friction will be studied.

9.1 Basic Laws of Sliding Friction

Make the following simple experiment. Place a known weight on a small square plate lying on a horizontal surface, Fig. 9.1. Attach a spring dynamometer, to measure the force, to the plate by a cord, and try to put the system in motion by pulling the dynamometer.

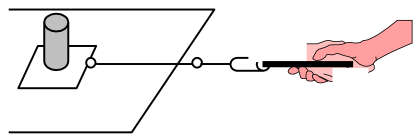


Fig. 9.1

It will require a definite force to make the plate starting motion. Then, smaller force is required to move the plate at a certain speed.

The following laws of sliding friction have been established; experimentally:

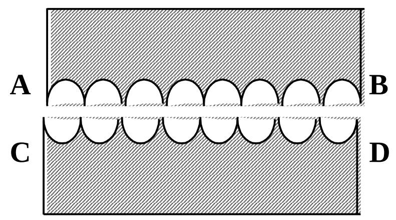
1. The friction force is proportional to normal reaction on the contact surface.

2. Changing the plate area does not change the force necessary to move the plate. Therefore; the force of friction does not depend on the area of contact.

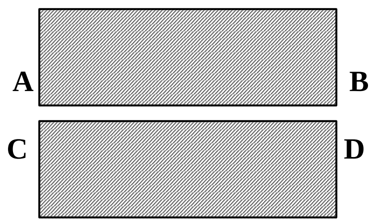
3. The amount of friction will change if either the plate or the horizontal surfaces are of different materials. Therefore; keeping normal reaction unchanged, friction force will depend on the material of the contact bodies and the finish of their surfaces.

4. Friction force does not depend on sliding velocity, although, the force necessary at the start of sliding (limit static friction), is greater than that required when sliding (motion) has been achieved. For which reason, it must be differentiated between static and kinetic friction.

9.2 Dry and Liquid Friction



(a) Dry Friction



(b) Fluid Friction

Fig. 9.2

The force of friction depends on the condition of contact surfaces. If the surfaces are dry, they will come into direct contact with each other, Fig. 9.2(a). This type of contact will give rise to cohesion and resist the relative motion of both surfaces, such resistance is called dry friction.

Now let us assume there is a layer of lubricant between the contagious surfaces, as shown in Fig. 9.2(b). The lubricant layer will completely separate the surfaces AB and CD and their irregularities will not come in contact with each other. There will be interaction between the particles of the lubricant. This kind of friction is called fluid friction. In such a case, there will be less resistance to relative motion than in the case of dry friction.

9.3 Static and Kinetic Friction

If a block of weight W is placed on a rough horizontal surface, Fig. 9.3(a). In this case, the block is held in equilibrium due to the normal reaction N, from the surface, such that N = W.

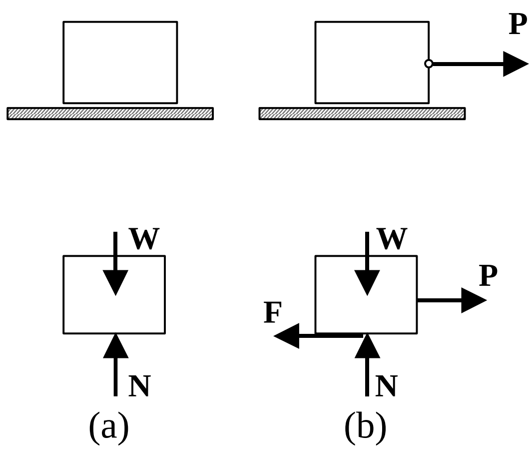


Fig. 9.3

Let a horizontal force P act on the block, such that its value increasing gradually, starting from zero. The block remains in equilibrium, for smaller values of P. In this case, an opposing force F must be created (friction force) to maintain equilibrium in horizontal direction.

Increasing the value of the force P, the friction force F increases, having the same value of P to maintain equilibrium, until a certain limit after which the block starts to move in direction of the force P. The maximum value of friction force reached just before the motion of the block is called the limit friction force, F_{lim}, Fig. 9.4.

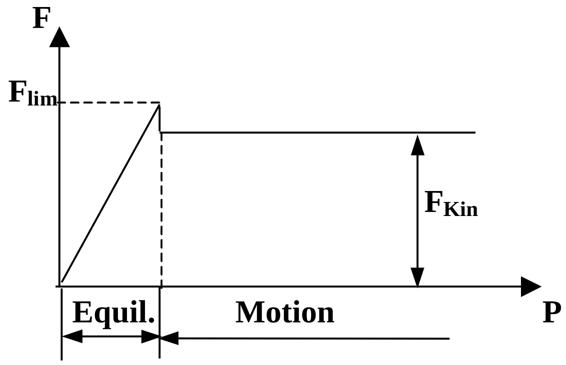


Fig. 9.4

As the block starts to move, a smaller value of friction force is observed, which remains unchanged with increasing the value of the force P. The friction force, in this case of motion, is called kinetic friction force $\mathbf{F_{Kin}}$.

9.4 Coefficients of Static and Kinetic Friction

The limit friction force F_{lim} is proportional to the normal reaction N. Hence, they are related as:

$$F_{lim} = \mu_s N \qquad (9.1)$$

Where: μ_s is called the coefficient of static friction.

In the case of motion, the kinetic friction force F_{kin} is also proportional to the normal reaction N, but with smaller value. Hence, a similar relation, to equation (9.1), can be written as:

$$F_{kin} = \mu_k N \qquad (9.2)$$

Where: μ_k is called the coefficient of kinetic friction.

It may be observed that equation (9.1) is only valid for limit equilibrium (when the motion is about to occur).

For the same contact bodies, the kinetic friction force is less than the limit static one;

$$F_{kin} < F_{lim} \tag{9.3}$$

Keeping in mind the relations (9.1) and (9.2), then the coefficient of kinetic function is less than the coefficient of static one;

$$\mu_K < \mu_S \tag{9.4}$$

9.5 Angle of Friction

The angle of friction ϕ is known as the angle between the normal reaction N and the total reaction R, Fig. 9.5. It may be observed that,

$$\tan\phi = F / N$$

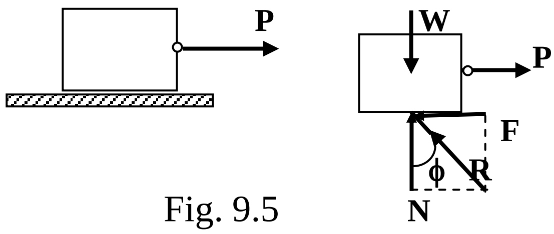


Fig. 9.5

In the case of limit static friction, the angle ϕ in this case is called the angle of static friction, which is denoted by ϕ_s which may be obtained as:

$$\phi_s = \tan^{-1} (F_{lim} / N) \tag{9.5}$$

Substituting the value of F_{lim} from equation (9.1) into equation (9.5), we have:

$$\phi_s = \tan^{-1} (\mu_s N / N) = \tan^{-1} \mu_s$$

or $$\tan\phi_s = \mu_s \tag{9.6}$$

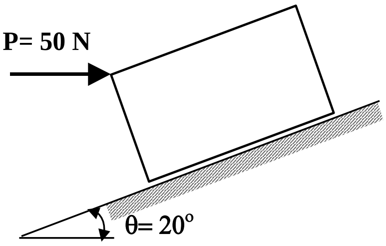
In a similar manner; the angle of kinetic friction ϕ_k can be expressed as:

$$\tan \phi_k = \mu_k \qquad (9.7)$$

It may be stated that the angle of static friction tan ϕ_s and that of kinetic friction tan ϕ_k are usually used in graphical solution of friction problems

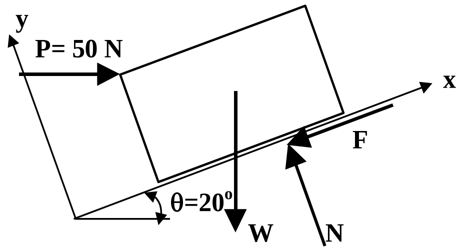
Example 9.1

Determine the magnitude and direction of the friction force acting on the shown block of weight 100 N, if the coefficient of static friction between the block and the inclined plane is $\mu_s = 0.2$.



Solution:

We, first, draw the free body diagram for the block, assuming the block is in equilibrium and tending to move up the inclined plane, against friction force F.



Write down the two equations of equilibrium of the block along the inclined plane, direction x, and normal to it, direction y.

$$\sum F_x = 0$$

$$P \cos \theta - W \sin \theta - F = 0$$

or $$50 \cos 20^o - 100 \sin 20^o = F$$

Hence, $$F = 12.8 \text{ N}$$

$$\sum F_y = 0$$

$$N - P \sin\theta - W\cos\theta = 0$$

Or $$N = 50 \sin 20° + 100 \cos 20°$$

$$N = 111 \text{ N}$$

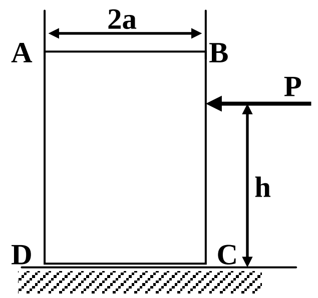
Hence the limit friction force,

$$F_{lim} = \mu_s N = 0.2\,(111) = 22.2 \quad \text{N}$$

Since, the friction force on the block F = 12.8 N is less than the limit friction force F_{lim} = 22.2 N, then the block is in equilibrium and the friction force, on it, is 12.8 N down the inclined plane.

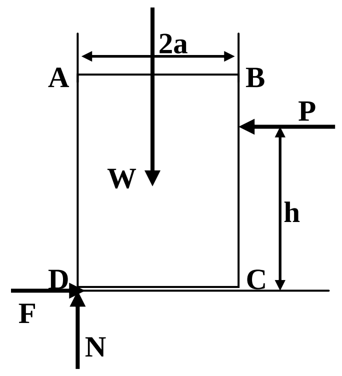
Example 9.2

A block of weight W rests on a horizontal plane with coefficient of friction μ_s. If the block is acted upon by a force P, as shown, determine the conditions for sliding of the block without overturning.



Solution:

Draw the free body diagram for the block in the case of overturning, in which the normal reaction N acts at the edge of the block D.



To make the block sliding, the force P must be greater than the limit friction force, hence,

$$P \geq F_{lim}$$

For equilibrium in vertical direction; N = W;

But $F_{lim} = \mu_s N = \mu_s W$

Hence $P \geq \mu_s W$ or $(P / W) \geq \mu_s$ which is the first condition.

For no turning of the block about D;

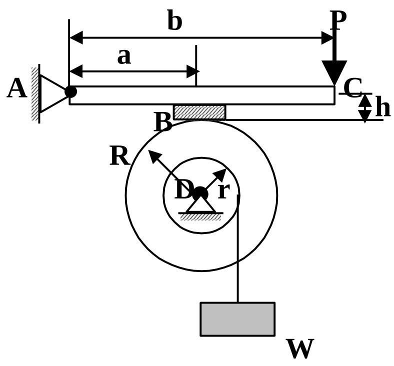
$$P h \leq W a$$

or $(h / a) \leq (W / P)$,

hence the second condition is $(h / a) \leq (1 / \mu_s)$.

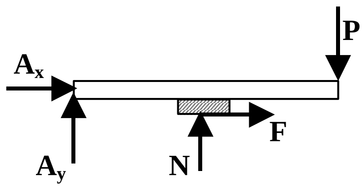
Example 9.3

For the given lever brake, find the smallest value of the force P required to maintain the weight W in equilibrium. Given the coefficient of static friction, at contact surface B, as μ_s For equilibrium of the lever AC ,



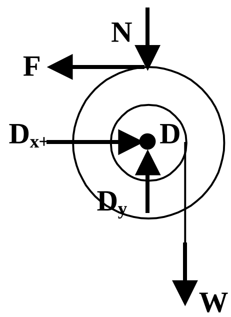
$$\sum M_A = 0$$

$$P b - F h - N a = 0$$



Or $P = (F h + N a) / b$ (1)

$$\sum F_x = 0$$



$$A_x + F = 0$$

or $A_x = - F$ (2)

$$\sum F_y = 0$$

$A_y + N - P = 0$ or $A_y = P - N$ (3)

For equilibrium of the drum

$$\sum M_D = 0$$

$$W r - F R = 0 \qquad \text{or} \qquad F = (r / R) W \qquad (4)$$

$$\sum F_x = 0$$

$$D_x - F = 0 \qquad \text{or} \qquad D_x = F \qquad (5)$$

$$\sum F_y = 0$$

$$D_y - N - W = 0 \qquad \text{or} \qquad D_y = N + W \qquad (6)$$

In the case of limit static friction (the motion is impending), the friction force $F = \mu_s N$

From equation (4):

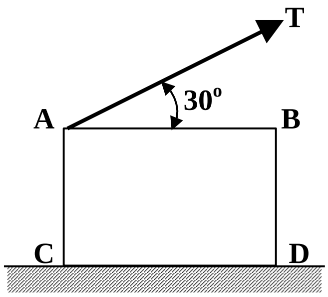
$$\mu_s N = (r / R) W \qquad \text{or} \qquad N = (r W / R \ \mu_s) \qquad (7)$$

Substituting the value of N into equations, (1) we have:

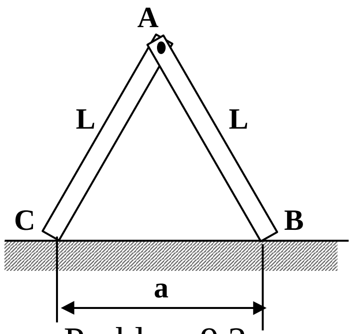
$$P = \{ h + (a / \mu_s)\} (r / R b) W$$

PROBLEMS

9.1 A square crate of mass 200 kg is pulled by a rope, as shown. If the coefficient of static friction between the crate and the ground is μ_s = 0.3, determine the tension T required to move the crate. Discuss the possibility of tipping of the crate.



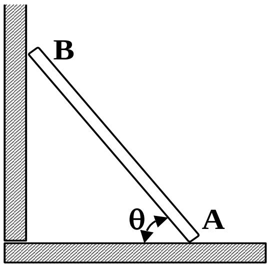
Problem 9.1



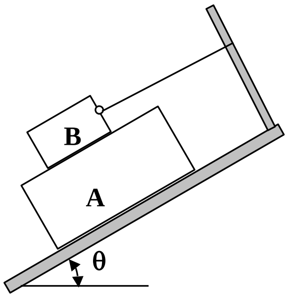
Problem 9.2

9.2 A double–leg ladder, with leg length L = 3 m, rests on a horizontal rough surface such that the maximum distance between the two contact points B and C is a = 2m . Find the coefficient of static friction between the ladder and the surface.

9.3 A ladder of weight W rests on the ground and a vertical well, as shown. If the coefficient of static friction between the ladder and each of the ground and the wall is μ_s = 0.2, find the minimum inclination angle of the ladder with the horizontal such that a man of weight 7W can ascend to the top of the ladder without ladder slipping.



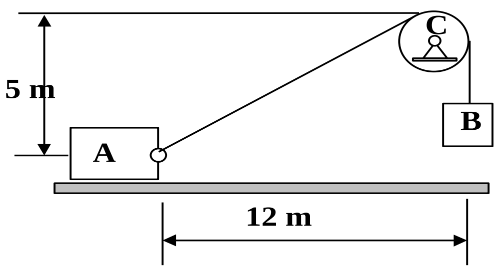
Problem 9.3



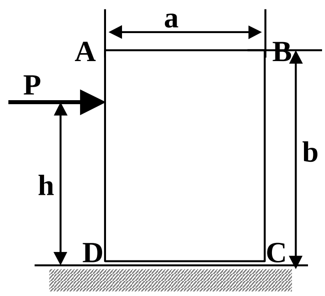
Problem 9.4

9.4 A block A of mass 9 kg is placed on a rough inclined plane with angle θ. A block B of mass 3kg is attached by a cord, fixed at the top of the plane, as shown. If the coefficient of static friction between all contact surfaces is $\mu s = 1/3$, find the value of the angel θ for which the block A is impending.

9.5 A block A of mass 200 kg is placed on a horizontal surface and is attached by a rope passing around a smooth pulley C. Another block B of mass 130 kg, attached to the other end of the rope, as shown. If the coefficients of static and kinetic friction between the block A and the surface are: $\mu s = 0.5$, $\mu k = 0.4$. Discuss whether the block A is removed and what is the value of the friction force.



Problem 9.5

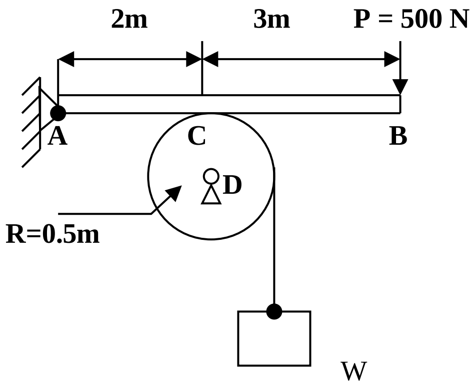


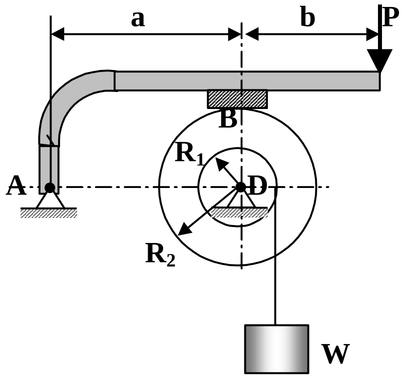
Problem 9.6

9.6 A force P is applied to a crate of weight W, as shown. If the coefficient of static friction between the crate and the horizontal surface is μ_s, determine the limiting values of h so that the crate will slide without tipping either the front edge C or the rear edge D.

Ans. $h_{max, min} = 0.5 \{b - (W / p)\} (b \pm a)\}$

9.7 Find the maximum weight of the block W which can be suspended from the pulley D without motion of the pulley, if the coefficient of static friction between the rod A B and the pulley D is $\mu_s = 0.3$.



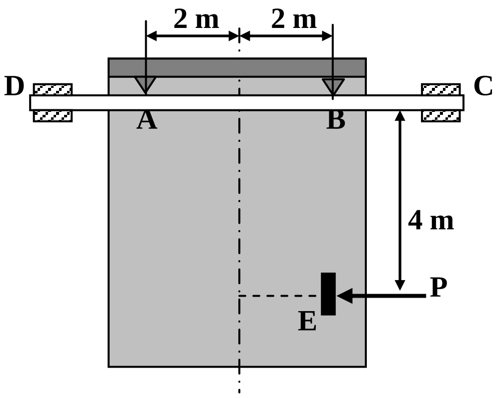


Problem 9.7

Problem 9.8

9.8 Find the least value of the force P which prevents the pulley D from motion, if a = 0.6m, R_1 = 0.1m b=0.8m, R_2 = 0.3 m and μ_s = 0.5 (at B). Calculate the horizontal and vertical reactions on the two pins A and D.

9.9 A heavy door of weight 800 N can slide on a rail CD, as shown. If the coefficients of static friction between the door and the rail at A and B are 0.15 and 0.25, respectively, find the value of the horizontal for P, acts on the handle E, required to move the door to the left.



Problem 9.9 & 9.10

9.10 In the previous problem (9.9) find the force required to move the door, to the right. Compare between the two forces obtained.

CHAPTER (10)

GEOMETRICAL PROPERTIES OF AN AREA

10.1 First Moment of an Area

Consider a differential area element dA with coordinate x and **y**, Fig. 10.1. We define the first moment of area A with respect to x-axis by the integral:

$$\int_A y \, dA \qquad (10.1)$$

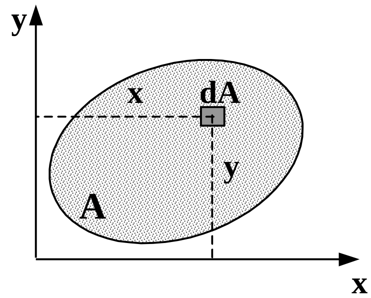


Fig. 10.1

Similarly, the first moment of the area A, with respect to y-axis, is defined by the integral:

$$\int_A x \, dA \qquad (10.2)$$

These integrations must be performed over the total surface area. The units of the first moments are [m^3].

10.2 Centroid of an Area

The centroid of an area is the geometrical center of the surface area. It can be defined as the point C of the coordinates x_C and y_C, Fig. 10.2, which satisfy the relations:

$$x_C = \frac{\int x \, dA}{\int dA} \qquad (10.3)$$

$$y_C = \frac{\int y \, dA}{\int dA} \qquad (10.4)$$

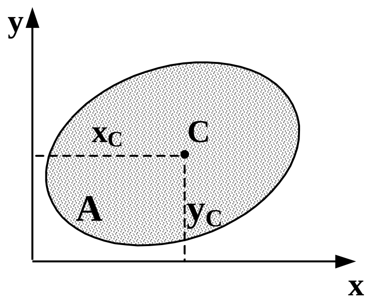


Fig. 10.2

Centroid of a symmetrical area

Consider the area A of Fig. 10.3 which is symmetric with respect to the y-axis, we observe that to every element of area dA of abscissa +ve x corresponds an element of area dA' of abscissa –ve x. It follows from the integral (10.3) that $x_c = 0$.

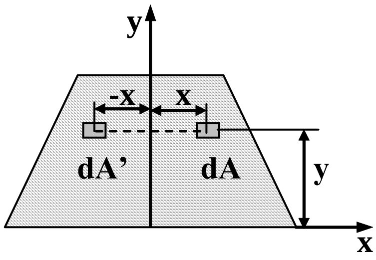
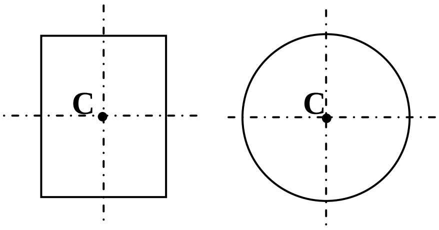


Fig. 10.3

Thus, **if an Area A has an axis of symmetry, its centroid C is located on that axis.**



(a) (b)

Fig. 10.4

Since, a retangular Area possesses two axes of symmety, Fig. 10.4(a), the centroid **C** of a rectangular area locates at the point of intersection of the two axes of symmety. Similarly, the centroid of a circular area coincides with the center of the circle, , Fig. 10.4(b)

Example 10.1:

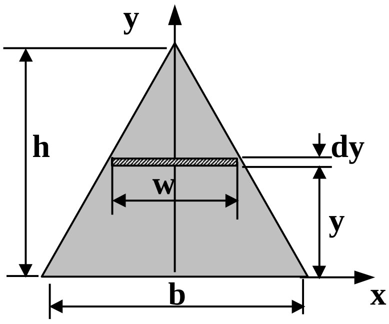
Locate the centroid of the isosceles triangular area with base b and height h.

solution:

y is axis of symmetry for this triangle, so, its centroid must lie on this axis.

$x_C = 0$

To determine y_C, a differential strip of area dA= w dy at height y is chosen. By similarity of triangles:



$$\frac{w}{b} = \frac{h-y}{h}$$

or $$w = (\frac{h-y}{h})\, b = (1 - \frac{y}{h})\, b$$

Then $\qquad dA = (\frac{h-y}{h})\, b\, dy$

Applying Eq. (10.4) we have

$$y_c = \frac{\int y\, dA}{\int dA} = \frac{\int_o^h y\,(1-\frac{y}{h})\, b\, dy}{\int_o^h (1-\frac{y}{h})\, b\, dy}$$

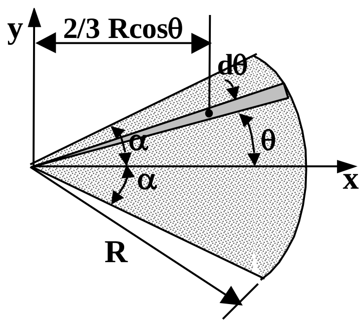
$$y_c = \frac{h}{3}$$

The centroid of the isosceles triangle is located at the point (0, h/3).

Example 10.2:

Locate the centroid of the area of a circular sector with respect to its vertex o.

Solution:



The x axis is axis of symmetry (C locates at this axis) and thus y_c = 0. The area may be covered by swinging a triangle of differential area about the vertex and through the total angle of the sector. This triangle (differential area element) has an area

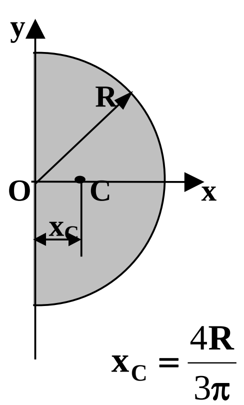
$$dA = (1/2)\, R.\, R\, d\theta,$$

and its centroid is located at

$$x = (2/3)\, R \cos\theta,$$

thus $\qquad x_c = \frac{\int x\, dA}{\int dA} = \frac{\int_{-\alpha}^{\alpha} (2/3)\, R\cos\theta\, (1/2) R^2 d\theta}{\int_{-\alpha}^{\alpha} (1/2) R^2 d\theta}$

or $\qquad x_c = \frac{2R\, \mathrm{Sin}\alpha}{3\alpha}$



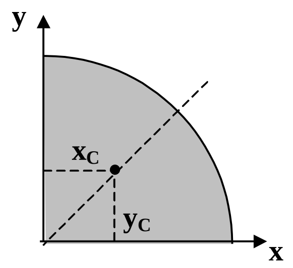
$$\mathbf{x_C} = \frac{4\mathbf{R}}{3\boldsymbol{\pi}}$$

For a hemi-circular area, α=π/2

Hence, $x_C = \dfrac{4R}{3\pi}$

For a quarter-circular area, $\alpha = \pi/4$

Hence, $x_C = y_C = \dfrac{4R}{3\pi}$



10.3 Centroid of a Composite Area

The composite area is a compound surface area that consists of number of simple surfaces of known areas and known centroids. Fig. 10.5 shows an area A, which may be divided into simple geometrical figures, A_1, A_2 and A_3 with known centroids C_1, C_2 and C_3, respectively. The coordinates x_C and y_C of the centroid C, of the given composite area A, can be obtained as:

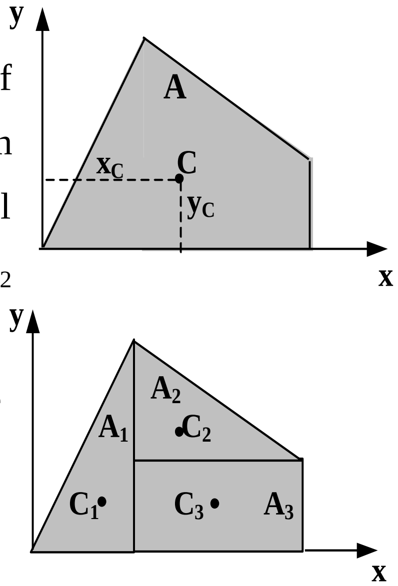


Fig. 10.5

$$x_c = \frac{A_1 x_{c1} + A_2 x_{c2} + A_3 x_{c3}}{A_1 + A_2 + A_3}$$

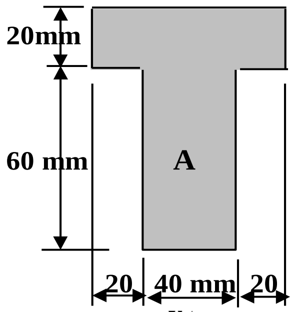
$$y_c = \frac{A_1 y_{c1} + A_2 y_{c2} + A_3 y_{c3}}{A_1 + A_2 + A_3}$$

In general, the centroid C of a composite area consists of (n) number of simple areas can be determined as:

$$x_c = \frac{\sum_{i=1}^{i=n} A_i x_{ci}}{\sum_{i=1}^{i=n} A_i}, \qquad y_c = \frac{\sum_{i=1}^{i=n} A_i y_{ci}}{\sum_{i=1}^{i=n} A_i} \qquad (10.5)$$

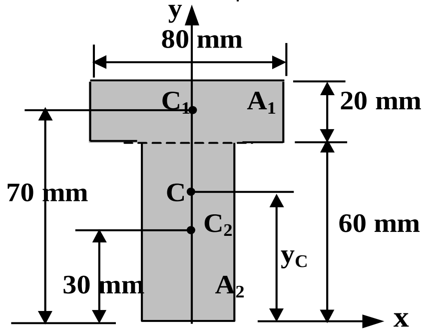
Example 10.3:

Determine the centroid of the given T sectional Area.



Solution:

Selecting the coordinate axes shown, we note that the centroid C must be located on the axis of symmetry y, thus $x_c = 0$.



Dividing the area A into two component parts A_1, A_2, we use Eq. (10.5) to determine the coordinate y_c of the centroid. The computation is best carried out in tabular form

Part	Area: A_i mm^2	Y_i mm	$A_i y_i$ mm^3
1	(20) (80) =1600	70	$112x10^3$
2	(40) (60) =2400	30	$72x10^3$
Σ	4000		184×10^3

$$y_C = \frac{\sum A_i y_i}{\sum A_i} = \frac{184x10^3}{4x10^3} = 46 \text{ mm}$$

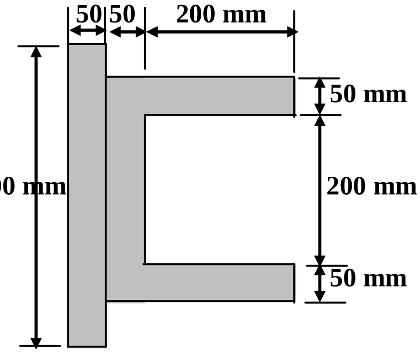
Example 10.4:

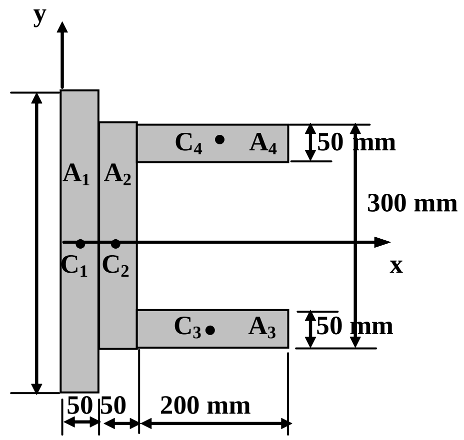
Determine the centroid of the given area.

Solution:

Choose the coordinate system as shown, where the **x**-axis is an axis of symmetry, thus $y_c = 0$. Divide the given area into four parts, as shown. Carrying out the computation in tabular form, we have

Part	**Area: A_i (cm^2)**	**X_{ci} (cm)**	**$A_i\, xc_i$ (cm^3)**
1	(40)(5)= 200	2.5	500
2	(30)(5)=150	7.5	1125
3	(20)(5)= 100	20	2000
4	(20)(5) = 100	20	2000
Σ	550		5625





Thus,

$$x_c = \frac{\sum A_i x_{ci}}{\sum A_i} = \frac{5625}{550} = 10.23 \qquad \text{Cm}$$

10.4 Moments of Inertia of an Area

The integral $I_S = \int_A e^2\, dA$, Fig.10.6, with respect to an axis s, where e is the perpendicular distance from the element of area dA to the axis s, sometimes referred to as the second moment of the area about an axis (the s axis) , but more often it is called the **moment of inertia of the area**.

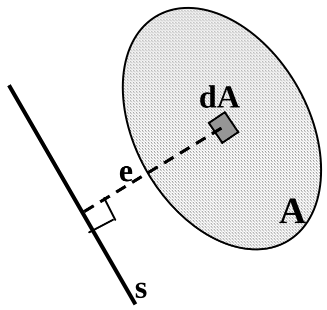


Fig. 10.6

The word "inertia" is used here since the formulation is similar to the mass moment of inertia, $I_S = \int_A e^2 dm$, which is a dynamical property will be described in engineering mechanics (3). Although for an area this integral has no physical meaning, it often arises in formulas used in fluid mechanics, mechanics of materials, structural mechanics, and mechanical design, and so the engineer needs to be familiar with the methods used to determine the moment of inertia.

According to the above definition, the moment of inertia of the area A, Fig. 10.7, with respect to the x and y axes are given, respectively, as:

$$I_x = \int_A y^2 \, dA \qquad (10.8)$$

$$I_y = \int_A x^2 \, dA \qquad (10.9)$$

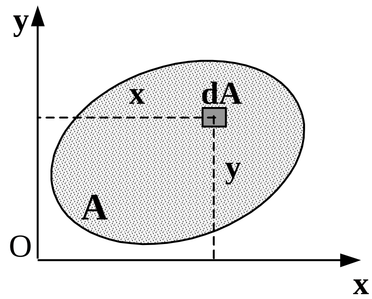


Fig. 10.7

The moment of inertia of an area A, with respect to z axis (perpendicular to the plane of the area A), Fig.10.8, is given as:

$$I_z = \int_A r^2 \, dA \qquad (10.10)$$

Denoting that: $r^2 = x^2 + y^2$

we can write;

$$I_z = \int_A (x^2 + y^2) \, dA$$

$$= \int_A x^2 dA + \int_A y^2 dA$$

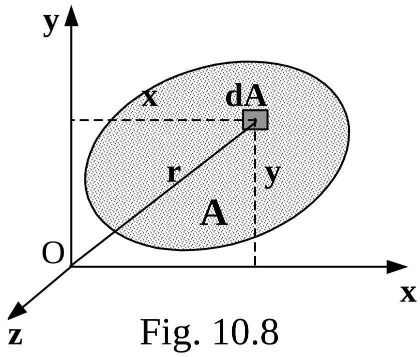


Fig. 10.8

$$I_z = I_x + I_y \qquad (10.11)$$

The moment of inertia I_Z may be denoted as the polar moment of inertia of the area A with respect to the point O, as:

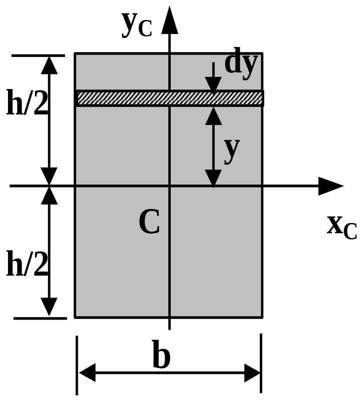
$$I_Z = I_O = I_X + I_y \qquad (10.12)$$

We note that the moments of inertia of an area are always positive quantities and they are expressed in m^4.

Example 10.5

Determine the moments of inertia of the rectangular area about the centroidal axes x_C , y_C and the centroidal polar axis through C.

Solution:



For the calculation of the moment of inertia I_{XC}, a horizontal strip of area dA= b dy is chosen so that all elements of the strip have the same y coordinate, thus

$$I_{xc} = \int_A y^2\, dA \quad , \qquad dA = b\, dy$$

$$I_{xc} = \int_{-h/2}^{h/2} y^2\, b\, dy = \frac{bh^3}{12}$$

By interchanging symbols, the moment of inertia about the centroidal axis y_C is

$$I_{yc} = \frac{hb^3}{12}$$

The centroidal polar moment of inertia is

$$I_O = I_Z = I_X + I_y = \frac{1}{12}(b\, h^3 + hb^3) = \frac{A}{12}(b^2 + h^2)$$

where A = bh = area of the rectangle.

Example 10.6:

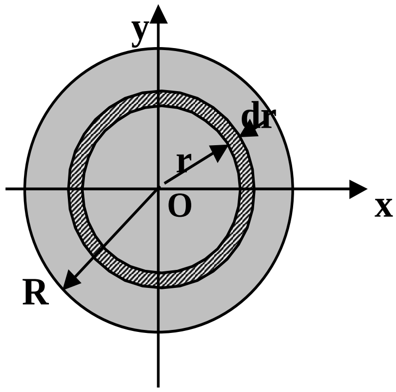
Calculate the moments of inertia of the area of the circle about a diametric axis and about the polar axis through the center.

Solution:

An element of area in the form of a circular ring, as shown in the figure, may be used for the calculation of the moment of inertia about the polar axis through O since all elements of the ring are equidistant from O.

Thus dA=2 π r dr

$$I_o = \int_A r^2 \, dA = \int_0^R r^2 (2\pi r) dr$$



$$I_o = \frac{\pi}{2} R^4$$

By symmetry, $I_X = I_y$

According to Eq. (10.11), then $I_O = I_X + I_y$,

or $\quad I_X = I_y = I_O / 2$

Thus; $\quad I_X = I_y = \dfrac{\pi R^4}{4}$

10.5 Radius of Gyration of an Area

The radius of gyration of an area A with respect to an axis s is defined by the quantity r_S, which satisfies the relation.

$$I_S = A\ r_s^2 \qquad (10.13)$$

Where I_s is the moment of inertia of the area A with respect to the s-axis.

Solving Eq. (10.13) for r_S , we have :

$$r_s = \sqrt{\frac{I_s}{A}}$$

Accordingly, the radius of gyration with respect to x, y and z axes are:

$$r_x = \sqrt{\frac{I_x}{A}}$$

$$r_y = \sqrt{\frac{I_y}{A}}$$

$$r_z = r_o = \sqrt{\frac{I_z}{A}} \qquad (10.14)$$

We note that the radius of gyration of an area is expressed in m.

10.6 Parallel-Axis Theorem

The parallel-axis theorem can be used to find the moment of inertia of an area about any axis that is parallel to an axis passing through the centroid and about which the moment of inertia is known. To develop this theorem, consider the moment of inertia I_X of an area A with respect to an arbitrary x-axis, Fig. 10.9.

According to equation 10.8, we can write:

$$I_X = \int_A y^2 dA$$

If x_C is a centroidal axis, parallel to the x-axis and d is the perpendicular distance between the two axes x_c and x, we write:

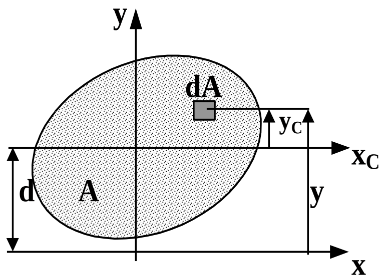


Fig. 10.9

$$y = y_C + d$$

Substituting for **y** in the above integral representing I_X, we have:

$$I_X = \int_A (y_C + d)^2 \, dA$$

$$I_X = \int_A y_c^2 \, dA + 2d \int_A y_c \, dA + d^2 \int_A dA$$

The first integral represents the moment of inertia I_{XC} of the area A with respect to the centroidal axis x_C. The second integral is equal to zero, since the centroid C of the area is located on that axis. Finally, we observe that the last integral is equal to the total area A. Therefore, we have:

$$I_x = I_{xc} + A\,d^2 \qquad (10.15)$$

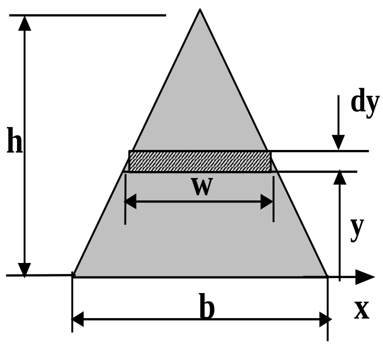
This result is known as the parallel-axis theorem. We should note that the parallel axis theorem may be used, only, if one of the two parallel axes is a centroidal axis.

Example 10.7:

Determine the moments of inertia of the triangular area, with base b and height h, with respect to an axis passing through its base and an axis passing through its centroid C.

Solution:

A strip of area parallel to the base is selected, as shown, with area dA = w dy from similarity of triangles, then:



Thus,

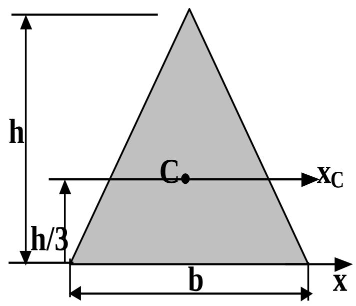
$$\frac{w}{b} = \frac{h - y}{h}$$

$$dA = \left(\frac{h - y}{h}\right) b\, dy$$

and

$$I_x = \int_A y^2 dA = \int_0^h y^2 \left(\frac{h - y}{h}\right) b\, dy$$

$$I_x = \frac{bh^3}{12}$$



Applying the theorem of parallel-axis (10.15), we have:

$$I_x = I_{xc} + \left(\frac{h}{3}\right)^2$$

$$\frac{b\,h^3}{12} = I_{xc} + \left(\frac{h}{3}\right)^2$$

Solving the above equation for I_{xc}, we have:

$$I_{xc} = \frac{bh^3}{36}$$

Example 10.8

Determine the moment of inertia of the shown semicircular section with respect to its principal centroidal axes x_C, y_C.

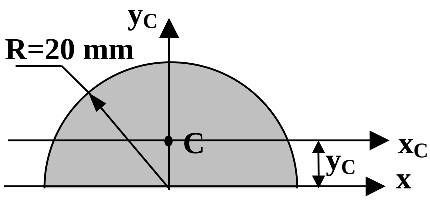
Solution:

The moment of inertia of the semicircular area with respect to the x-axis is one half of that for a complete circle about the same axes. Thus,

$$I_x = \frac{1}{2}\left(\frac{\pi}{4}R^4\right) = 2\pi \ x10^4 \quad mm^4$$

Position of centroid C is given by (Example 10.2):

$$y_c = \frac{4R}{3\pi} = \frac{80}{3\pi} \quad mm$$



Applying parallel axis theorem, we have

$$I_{xc} = I_x - Ay_c^{\,2}$$

$$I_{xc} = 2\pi \ x10^4 - \frac{1}{2}\pi(20)^2\left(\frac{80}{3\pi}\right)^2 = 1.75x10^4 \quad mm^4$$

Also, the moment of inertia of the semicircular area with respect to the y_c axis is one half of that for a complete circle about the same axes. Thus,

$$I_{yc} = \frac{1}{2}\left(\frac{\pi}{4}R^4\right) = 2\pi \ x10^4 \quad mm^4$$

10.7 Moment of Inertia for Composite Areas

A composite area consists of a series of connected "simpler" parts or shapes, such as rectangles, triangles, and circles. Provided the moment of inertia of each of these parts is known or can be determined about a common axis, then the moment of inertia for the composite area about this axis equals the **algebraic sum** of the moments of inertia of all its parts.

Procedure for analysis

The moment of inertia for a composite area about a reference axis can be determined using the following procedure.

- **Composite Parts.**

Using a sketch, divide the area into its composite parts (n parts) and indicate the perpendicular distance from the centroid of each part to the reference axis.

- **Parallel-Axis Theorem.**

If the centroidal axis for each part does not coincide with the reference axis, the parallel-axis theorem, $I = I_C + Ad^2$, should be used to determine the moment of inertia of the part about the reference axis. For the calculation of I_C, use the table at the end of this chapter.

- **Summation.**

The moment of inertia of the entire area about the reference axis is determined by summing the results of its composite parts about this axis.

$$I = \sum_{i=1}^{i=n} I_i$$

Note: If a composite part has an empty region (hole), its moment of inertia is found by subtracting the moment of inertia of this region from the moment of inertia of the entire part including the empty region.

Example 10.9

Determine the moments of inertia I_{xC} and I_{yC} of the area shown with respect to its centroidal axes x_C and y_C .

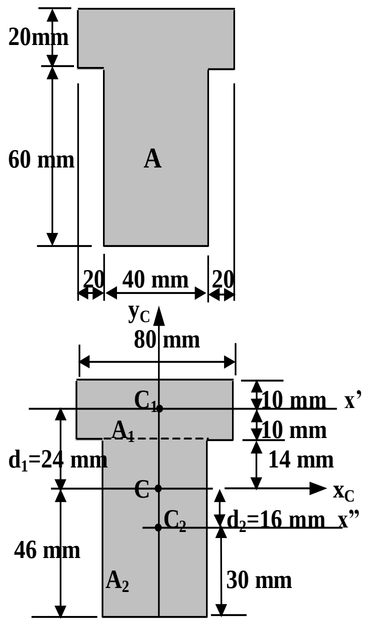
Solution:

Location of centroid.

The centroid C of the area must first be located. However, this has been done in example 10.3 for the given area. We recall from that example that C is located on y axis, 46 mm above the lower edge of the area A.

Computation of Moments of Inertia.

We divide the area A into two rectangular areas A_1 and A_2 and compute the moments of inertia of each with respect to the x_C axis and y_C axis and then making summation for the entire area A.



$$I_{xc} = [I_{xc}]_1 + [I_{xc}]_2$$

$$[I_{xc}]_1 = [I_{x'} + A_1 d_1^2] = \left[\frac{(80)(20)^3}{12} + 1600(24)^2\right]$$

$$= 97.5 x 10^4 \quad mm^4$$

$$[I_{xc}]_2 = [I_{x''} + A_2 d_2^2] = \left[\frac{(40)(60)^3}{12} + 2400(16)^2\right]$$

$$= 133.4 x 10^4 \quad mm^4$$

$$I_{xc} = [I_{xc}]_1 + [I_{xc}]_2$$

or $I_{xC} = 97.5 \text{ x}10^4 + 133.4 \text{ x } 10^4$

$= 230.9 \text{ x } 10^4 \quad mm^4$

$$I_{yc} = [I_{yc}]_1 + [I_{yc}]_2$$

$$= \left[\frac{(20)(80)^3}{12}\right] + \left[\frac{(60)(40)^3}{12}\right]$$

or $I_{yc} = 85.3x10^4 + 8x10^4 = 93.3x10^4 \quad mm^4$

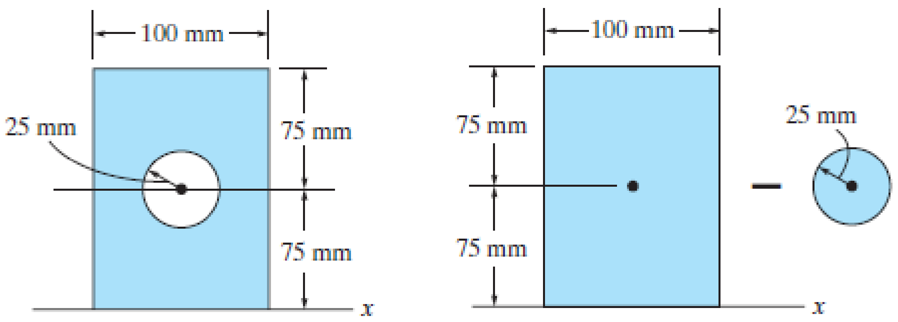
Example 10.10

Determine the moment of inertia of the area shown in figure about the x axis.

Solution

Composite Parts. The area can be obtained by subtracting the circle from the rectangle as shown. The centroid of each area is located in the figure.

Parallel-Axis Theorem. The moments of inertia about the x axis are determined using the parallel-axis theorem and the geometric properties formulae for circular and rectangular areas $I_{xc} = \frac{\pi}{4} R^4$ and $I_{xc} = \frac{bh^3}{12}$



Circle

$I_x = I_{xc} = + A\, d_y^2$

$= \frac{\pi}{4}(25)^4 + \pi(25)^2(75)^2 = 11.4x(10)^6 \quad mm^4$

Rectangle

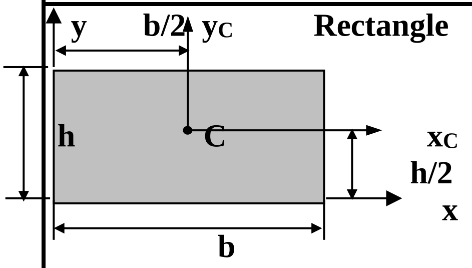
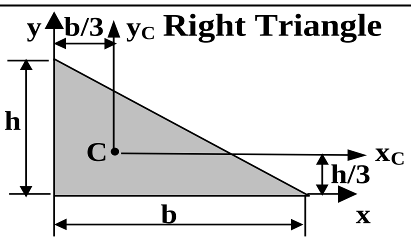
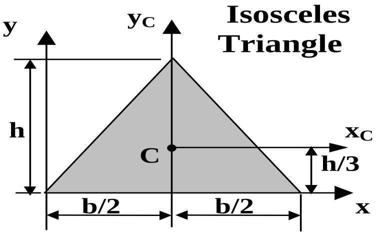
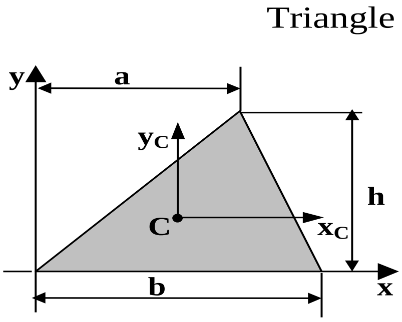
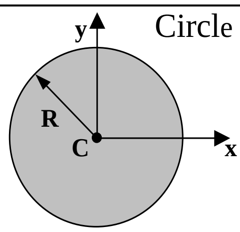
$I_x = I_{xc} = + A\, d_y^2$

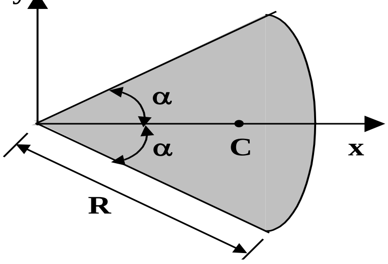
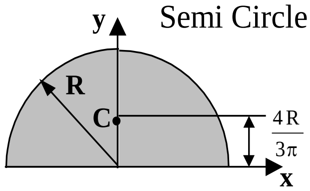
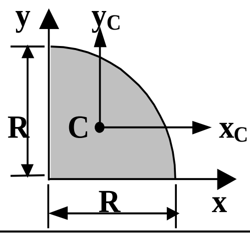
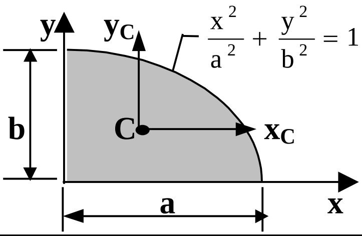
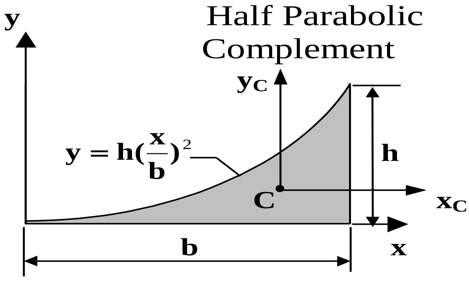
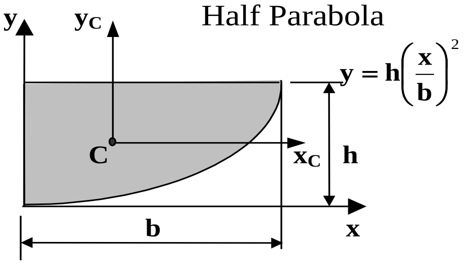
$= (100)(150)^3/12 + (100)(150)(75)^2 = 112.5(10)^6 \quad mm^4$

Summation. The moment of inertia for the area is therefore

$I_X = -11.4x(10)^6 + 112.5\, x(10)^6 = 101x(10)^6 \quad mm^4$

10.8 Centroid and Moment of Inertia of Some Simple Areas.

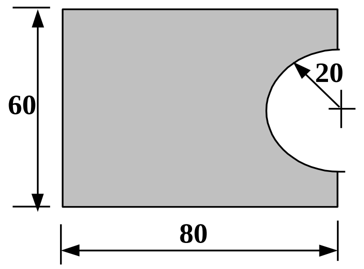
Area	Centroid	Area Moment of Inertia
 	$x_C = b/2$ $y_C = h/2$	$I_{xC} = \frac{bh^3}{12}$, $I_{yC} = \frac{b^3h}{12}$ $I_x = \frac{bh^3}{3}$, $I_y = \frac{b^3h}{3}$ $I_{x_Cy_C} = 0$, $I_{xy} = \frac{b^2h^2}{4}$
 	$x_C = b/3$ $y_C = h/3$	$I_{x_C} = \frac{bh^3}{36}$, $I_{y_C} = \frac{b^3h}{36}$ $I_x = \frac{bh^3}{12}$, $I_y = \frac{b^3h}{12}$ $I_{x_Cy_C} = -\frac{b^2h^2}{72}$, $I_{xy} = \frac{b^2h^2}{24}$
 	$x_C = 0$ $y_C = h/3$	$I_{x_C} = \frac{bh^3}{36}$, $I_{y_C} = \frac{b^3h}{48}$ $I_x = \frac{bh^3}{12}$, $I_y = \frac{7}{48}b^3h$ $I_{x_Cy_C} = 0$, $I_{xy} = \frac{b^2h^2}{12}$
 	$x_C = \frac{a+b}{3}$ $y_C = h/3$	$I_{x_C} = \frac{bh^3}{36}$, $I_{y_C} = \frac{bh}{36}(a^2 - ab + b^2)$ $I_x = \frac{bh^3}{12}$, $I_y = \frac{bh}{12}(a^2 + ab + b^2)$ $I_{x_Cy_C} = \frac{bh^2}{72}(2a - b)$, $I_{xy} = \frac{bh^2}{24}(2a + b)$
 	$x_C = 0$ $y_C = 0$	$I_x = I_y = \frac{\pi R^4}{4}$ $I_C = \frac{\pi R^4}{2}$, $I_{xy} = 0$

Area	Centoid	Area Moment of Inertia
Circular Sector 	$x_C = \frac{2R\sin\alpha}{3\alpha}$ $y_C = 0$	$I_x = \frac{R^4}{8}(2\alpha - \sin 2\alpha)$ $I_y = \frac{R^4}{8}(2\alpha + \sin 2\alpha)$ $I_{xy} = 0$
Semi Circle 	$x_C = 0$ $y_C = \frac{4R}{3\pi}$	$I_{x_C} = 0.1098R^4$ $I_x = I_y = \frac{\pi R^4}{8}$ $I_{x_C y_C} = I_{xy} = 0$
Quarter Circle 	$x_C = \frac{4R}{3\pi}$ $y_C = \frac{4R}{3\pi}$	$I_{x_C} = I_{y_C} = 0.054888R^4$ $I_x = I_y = \frac{\pi R^4}{16}$ $I_{x_C y_C} = -0.01647R^4$ $I_{xy} = \frac{R^4}{8}$
Quarter Ellipse 	$x_C = \frac{4a}{3\pi}$ $y_C = \frac{4b}{3\pi}$	$I_{x_C} = 0.05488ab^3$ $I_{y_C} = 0.05488a^3b$ $I_x = \frac{\pi ab^3}{16}$, $I_y = \frac{\pi a^3 b}{16}$ $I_{x_C y_C} = -0.01647a^2b^2$ $I_{xy} = \frac{a^2b^2}{8}$
Half Parabolic Complement 	$x_C = \frac{3b}{4}$ $y_C = \frac{3h}{10}$	$I_{x_C} = \frac{37bh^3}{2100}$, $I_{y_C} = \frac{b^3h}{80}$ $I_x = \frac{bh^3}{21}$, $I_y = \frac{b^3h}{5}$ $I_{x_C y_C} = \frac{b^2h^2}{120}$, $I_{xy} = \frac{b^2h^2}{12}$
Half Parabola 	$x_C = \frac{2b}{5}$ $y_C = \frac{5h}{8}$	$I_{x_C} = \frac{8bh^3}{175}$, $I_{y_C} = \frac{19b^3h}{480}$ $I_x = \frac{2bh^3}{7}$, $I_y = \frac{2b^3h}{15}$ $I_{x_C y_C} = \frac{b^2h^2}{60}$, $I_{xy} = \frac{b^2h^2}{6}$

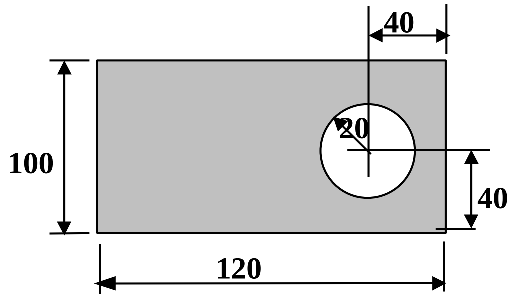
PROBLEMS

(All DIMENSIONS ARE IN mm)

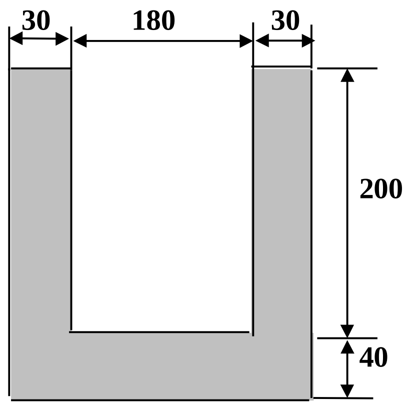
10.1 Determine the location of the centroid of the given Figures:



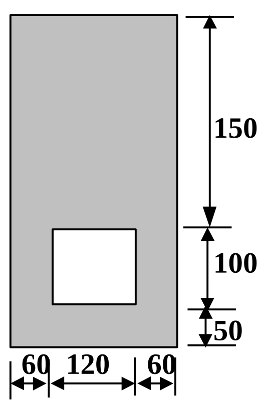
(a)



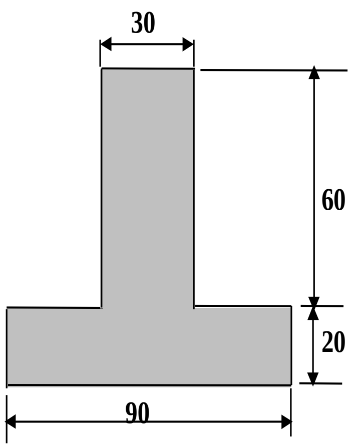
(b)



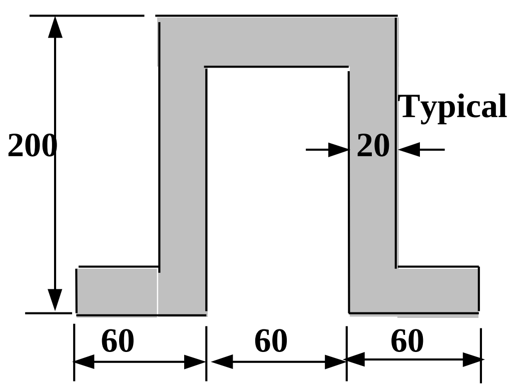
(c)



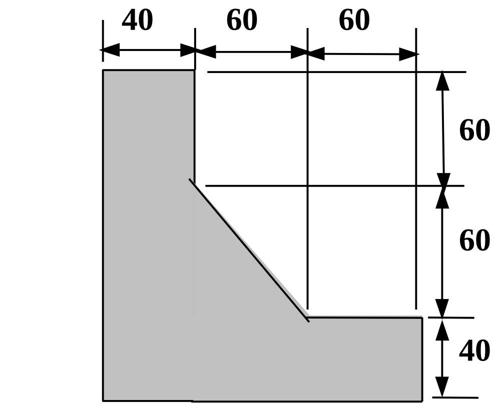
(d)



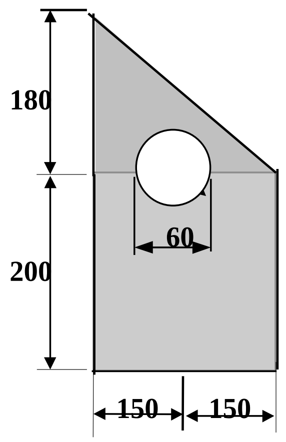
(e)



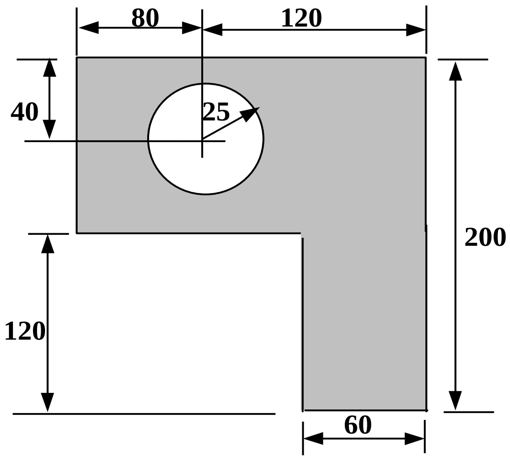
(f)


40
60
60
60
60
40

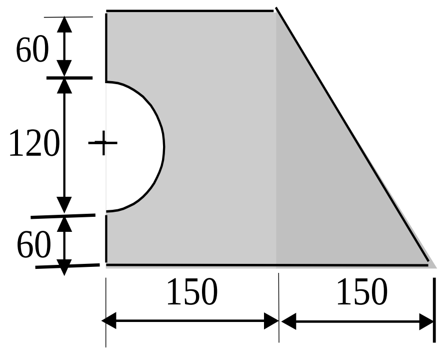
(g)


180
200
60
150
150

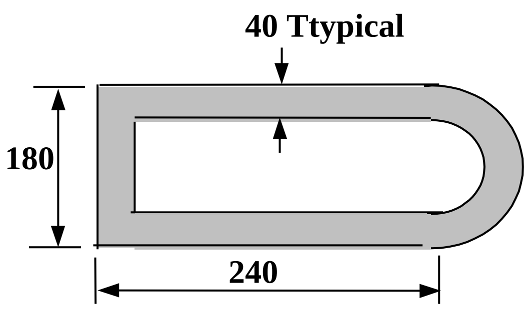
(h)


80
120
40
25
200
120
60

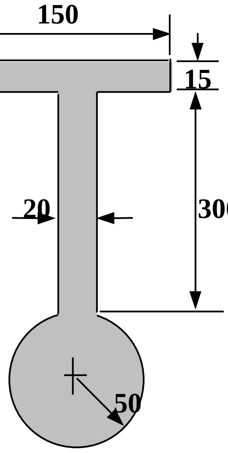
(i)


60
120
60
150
150

(j)

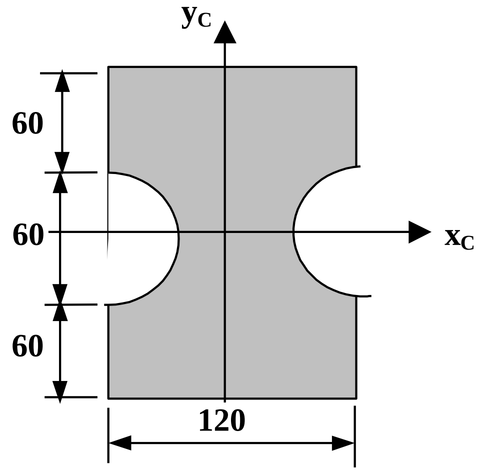

40 Ttypical
180
240

(k)

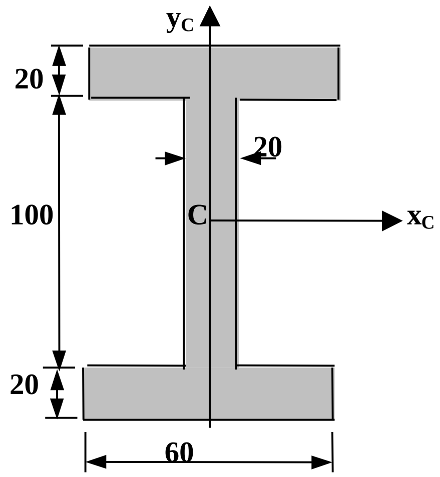

150
15
20
300
50

(l)

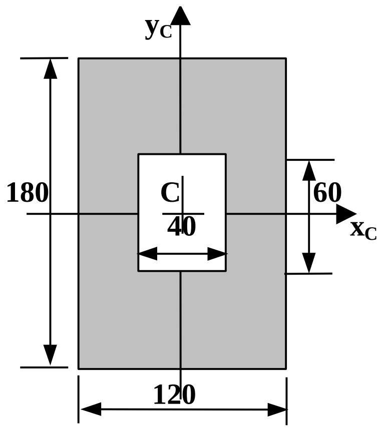
10.2 For the given Figures, determine the moments of inertia about the principal centroidal axes, x_c and y_c .



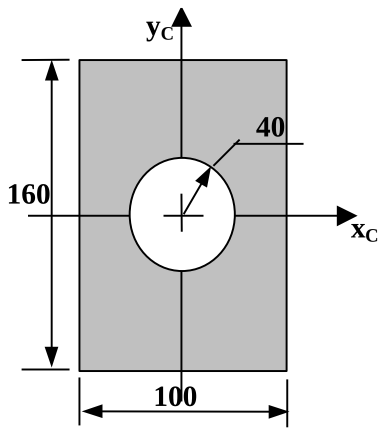
(a)



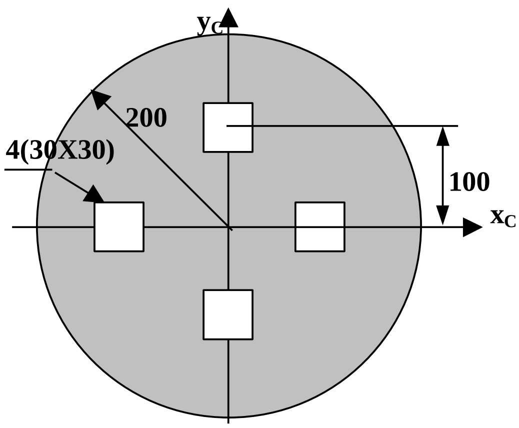
(b)



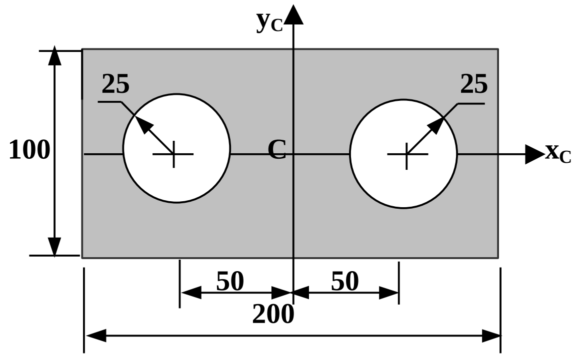
(c)



(d)

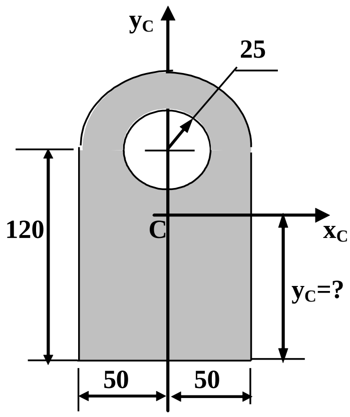


(e)

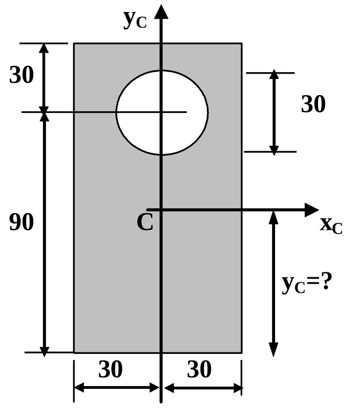


(f)

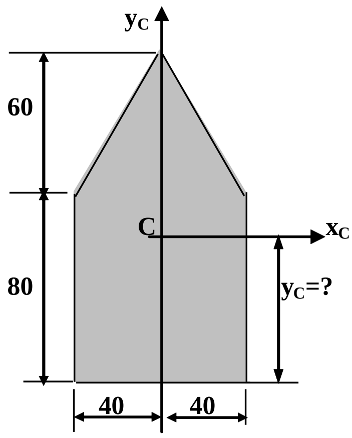
10.3 For the given sections, determine the location of the centroid, $\mathbf{y_c}$, and the principal centroidal moments of inertia. ($\mathbf{I_{xc}}$ and $\mathbf{I_{yc}}$)



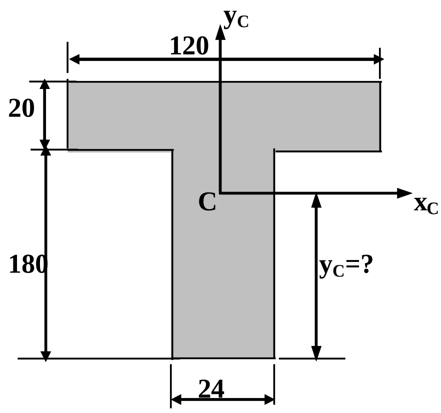
(a)



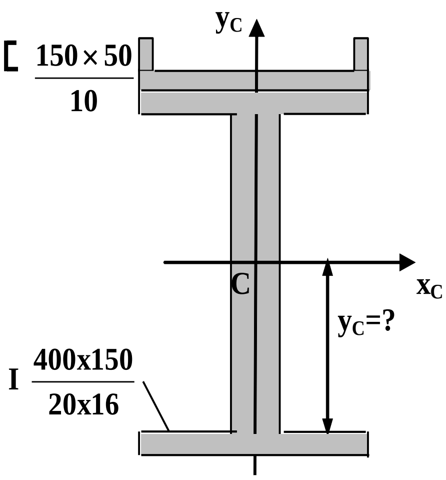
(b)



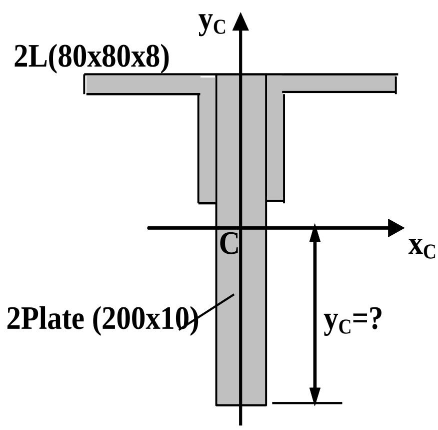
(c)



(d)



(e)



(f)

Appendix A

Mid Term Examinations

Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Academic Year:	2022/2023	Term:	First Term
Exam:	Mid Term	Time:	60 Min.
Subject:	Engineering Mechanics (1)	Code:	ENG 001
Examiners:	Examination Committee	Group:	1 : 24

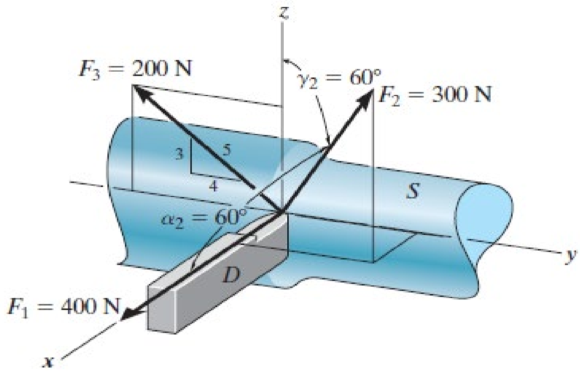
Name:	ID:	Group:

Answer the following questions:

Question (1): (10 Marks)

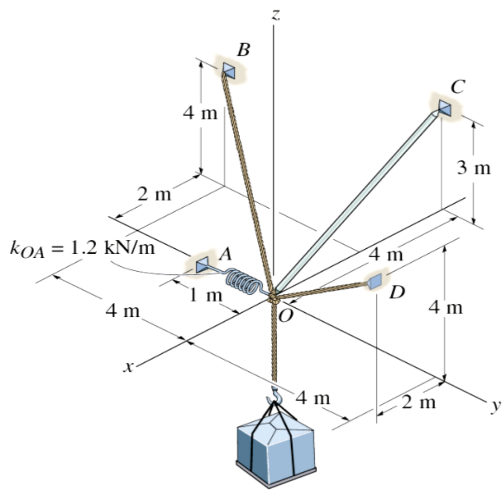
The shaft *S* exerts three force components on the die *D*. Find the magnitude and coordinate direction angles of the resultant force.

Solution



Question (2): (10 Marks)

If the maximum allowable tension in cable OB is 120 N, determine the tension developed in cables OC and OD, required to support the 490.5 N crate. The spring OA has an unstretched length of 0.8 m and a stiffness k_{OA} = 1200 N/m. The force in the strut acts along the axis of the strut.



With my best wishes for success

Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Academic Year:	2022/2023	Term:	Second Term
Exam:	Mid Term	Time:	40 Min.
Subject:	Engineering Mechanics (1)	Code:	ENG 001
Examiners:	Examination Committee	Group:	30 & 61 &62

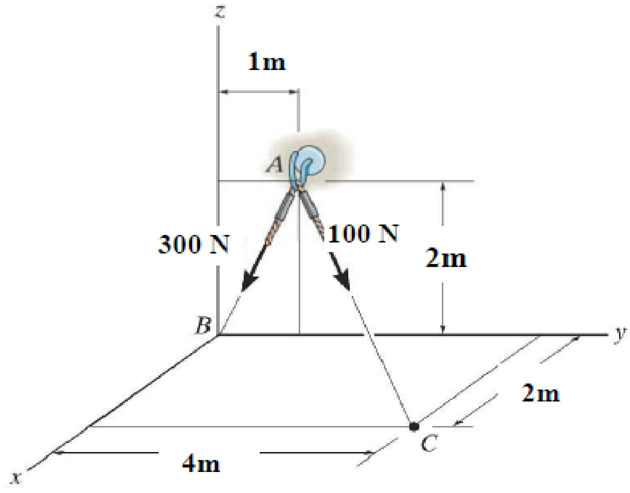
Name:	ID:	Group:	B.N:

Answer the following questions:

Question (1): (10 Marks)

Determine the magnitude and coordinate direction angles of the resultant force.

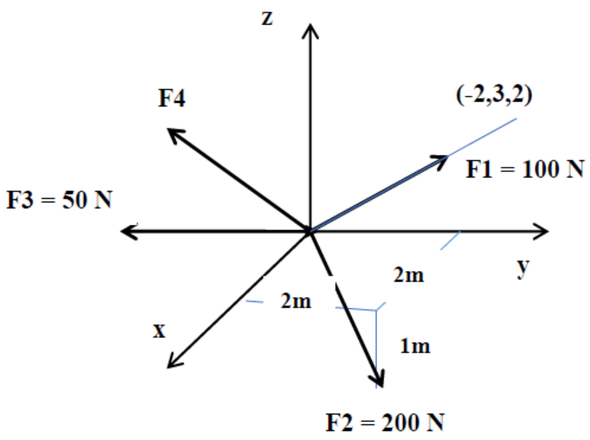
Solution



Question (2): (10 Marks)

Determine the magnitude and coordinate direction angles of the force F4 required to keep the concurrent force system in equilibrium.

Solution



With my best wishes for success

Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Academic Year:	**2022/2023**	**Term:**	**Third Term**
Exam:	**Mid Term**	**Time:**	**60 Min.**
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiners:	**Examination Committee**	**Group:**	**1**

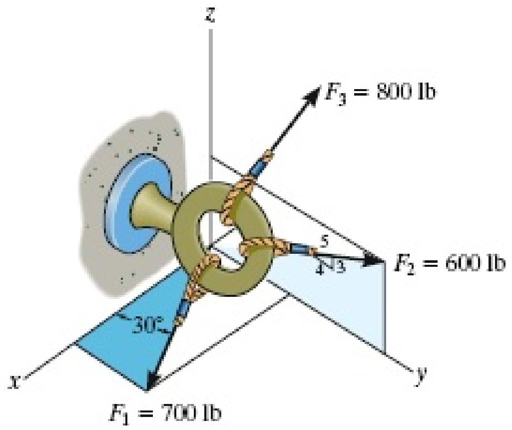
Name:	**ID:**	**Group:**	**B.N:**

Answer the following questions:

Question (1): (LO1., LO2.) (10 Marks)

Determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt. If the coordinate direction angles of F_3 are $\alpha_3 = 120$, $\beta_3 = 45$ and $\gamma_3 = 60$.

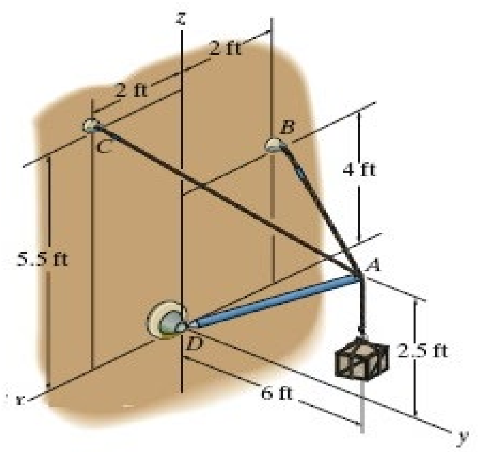
Solution



Question (2): (LO1.,LO3) (10 Marks)

Determine the tension in developed in cables AB, AC and the force developed along arm AD for equilibrium of the 400 lb crate.

Solution



With my best wishes for success

The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Mid Term Exam Evaluation			
Subject:	Engineering Mechanics (1)	Code:	ENG 001
Examiner:	Examination Committee	Time:	60 min
Date:	- / 1 / 2021 (C)	Group:	-----

Answer the following questions:

Question (1): [10 Marks]

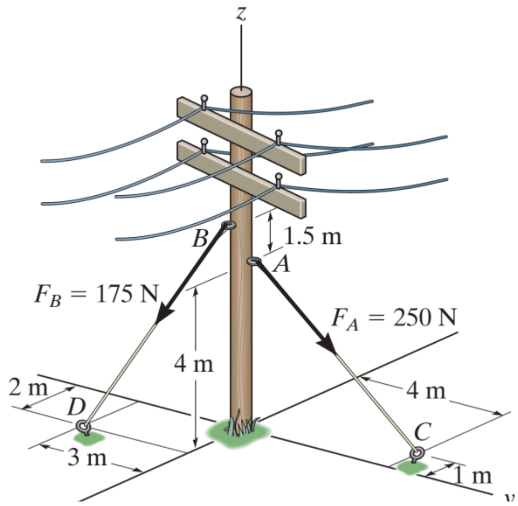
The guy wires are used to support the telephone pole.

Determine:

(a) The resultant force in cartesian vector.

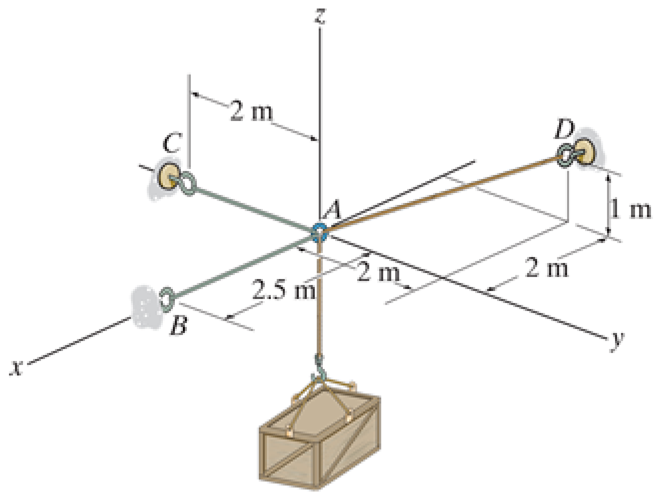
(b) The magnitude of the resultant force.

(c) The coordinate direction angles of the resultant force.



Question (2): [10 Marks]

Determine the tension in the cables in order to support the 100 kg crate in the equilibrium position shown.



Good Luck

Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Academic Year:	2021/2022	Term:	First Term
Exam:	Mid Term	Time:	60 Min.
Subject:	Engineering Mechanics (1)	Code:	ENG 001
Examiners:	Examination Committee	Group:	1 : 40

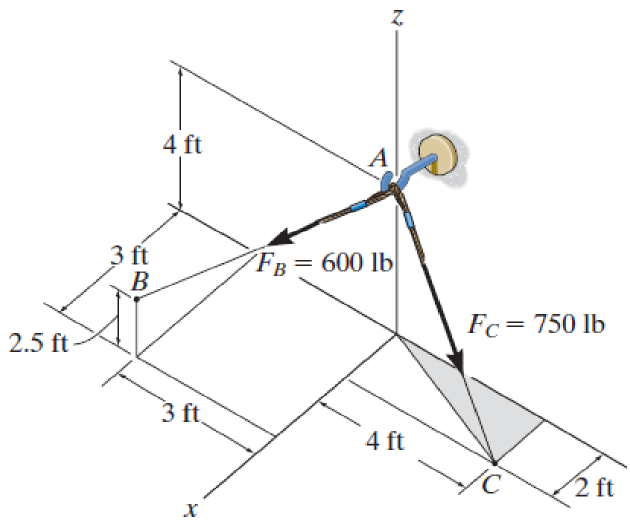
Name:	ID	Group:

Answer the following questions:

Question (1): (10 Marks)

Determine the magnitude and coordinate direction angles of the resultant force acting at *A*.

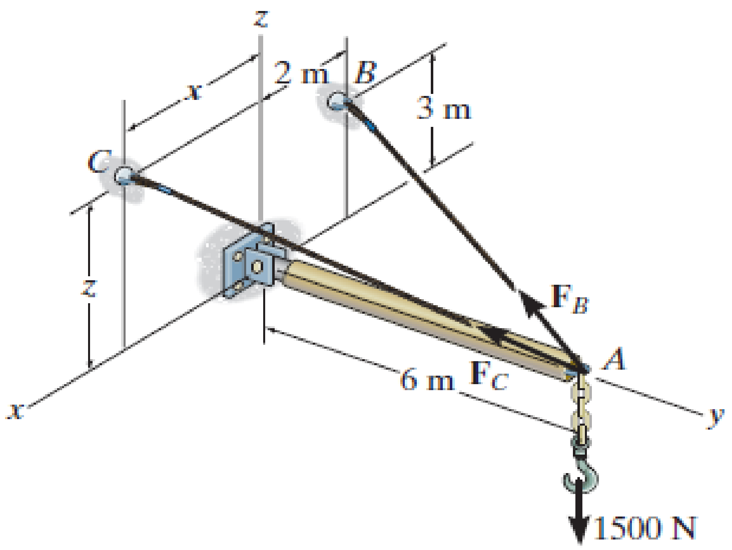
Solution



Question (2): (10 Marks)

Two cables are used to secure the overhang boom in position and support the 1500 N load. **If the resultant force is directed along the boom from point *A* towards *O*,** determine the magnitudes of the resultant force and forces F_B and F_C. **Set x = 3 m and z = 2 m.**

Solution



With my best wishes for success

The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Mid Term Exam Evaluation			
Subject:	Engineering Mechanics (1)	Code:	ENG 001
Examiner:	Examination Committee	Time:	60 min
Date:	23 / 12 / 2020 (A)	Group:	1 - 18

Answer the following questions:

Question (1): **[10 Marks]**

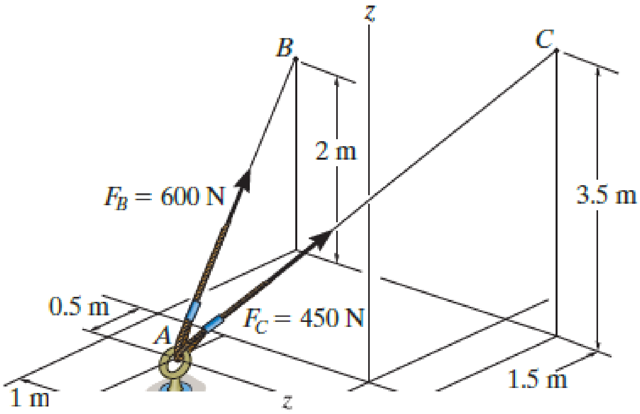
For the two forces shown in Figure.

Determine:

(a) The resultant force in cartesian vector.

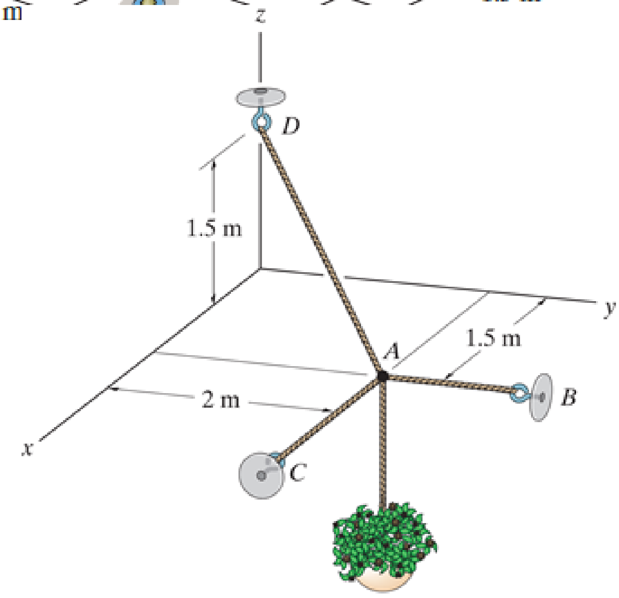
(b) The magnitude of the resultant force.

(c) The coordinate direction angles of the resultant force.



Question (2): **[10 Marks]**

The three cables are used to support the 40 kg flowerpot. Determine the force developed in each cable for equilibrium.



Good Luck

The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Mid Term Exam Evaluation			
Subject:	Engineering Mechanics (1)	Code:	ENG 001
Examiner:	Examination Committee	Time:	60 min
Date:	23 / 12 / 2020 (B)	Group:	19 - 34

Answer the following questions:

Question (1): **[10 Marks]**

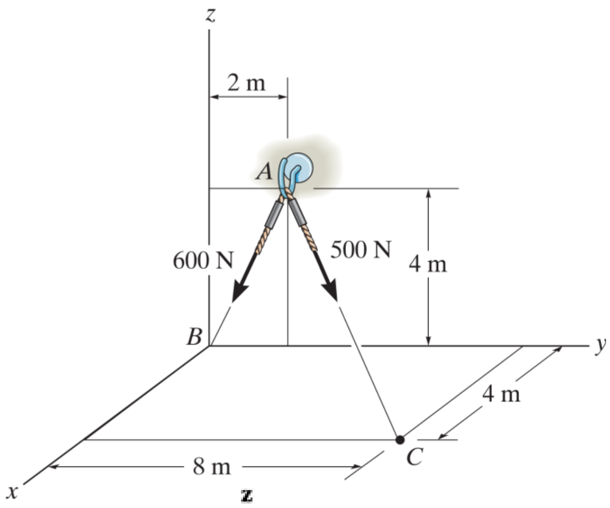
For the two forces shown in Figure.

Determine:

(a) The resultant force in cartesian vector.

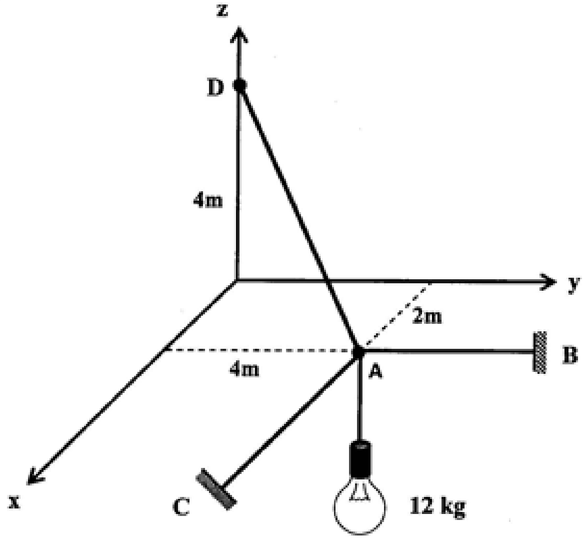
(b) The magnitude of the resultant force.

(c) The coordinate direction angles of the resultant force.



Question (2): **[10 Marks]**

The three cables are used to support 1**2** kg lamp. Determine the Tension force in AD, AB and AC for equilibrium.

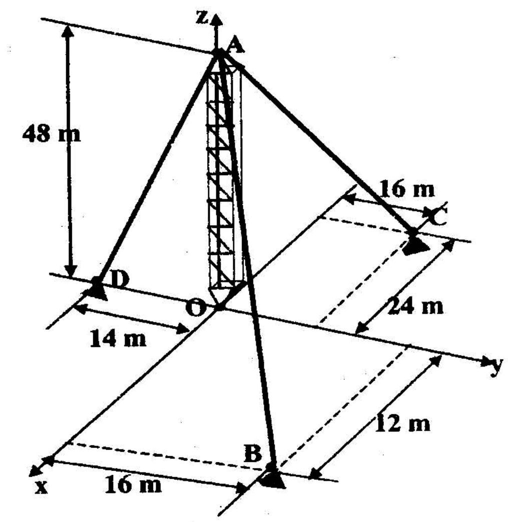


Good Luck

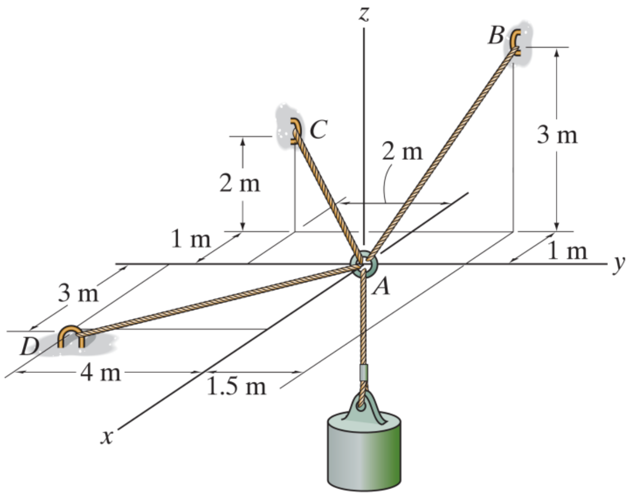
The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Mid Term Exam			
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiner:	**----------------**	**Time:**	**60 min**
Date:	**Oct. 2018**	**Group:**	**---------**

Answer the following questions:

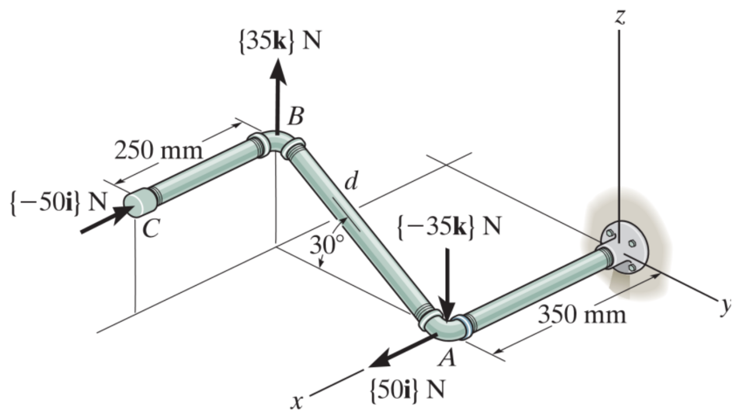
1. Knowing that the tension in the cable AC is $28\,\text{kN}$, determine the required values of tension in AB and AD so the resultant of the three forces applied at A is vertical



2. Determine the tension developed in cables AB, AC and AD required for equilibrium of the $75\,\text{kg}$ cylinder.



3. Determine the resultant couple moment of the two couples that act on the pipe assembly. The distance from A to B is $d = 400\,\text{mm}$. Express the result as a Cartesian vector.

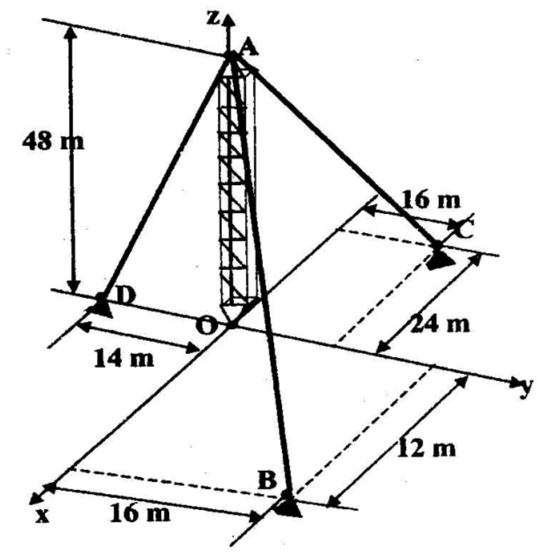


Good Luck

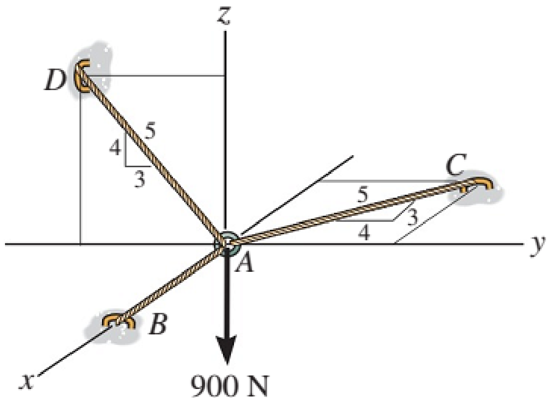
The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Mid Term Exam			
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiner:	**----------------**	**Time:**	**60 min**
Date:	**Oct. 2018**	**Group:**	**---------**

Answer the following questions:

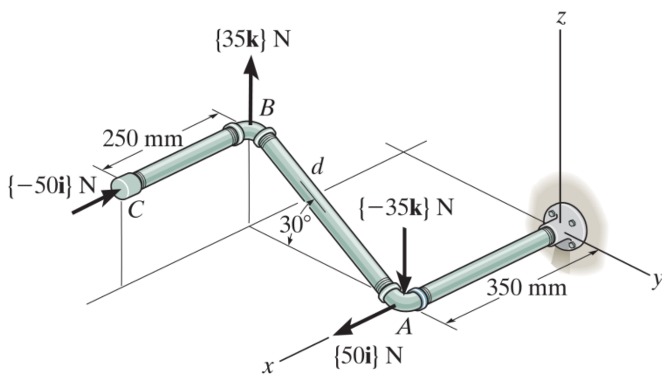
1. Knowing that the tension in the cable AB is 39 kN, determine the required values of tension in AC and AD so the resultant of the three forces applied at A is vertical.



2. Determine the tension developed in cables AB, AC, and AD.



3. Determine the distance d between A and B so that the resultant couple moment has a magnitude of $M_C = 20$ N.m.

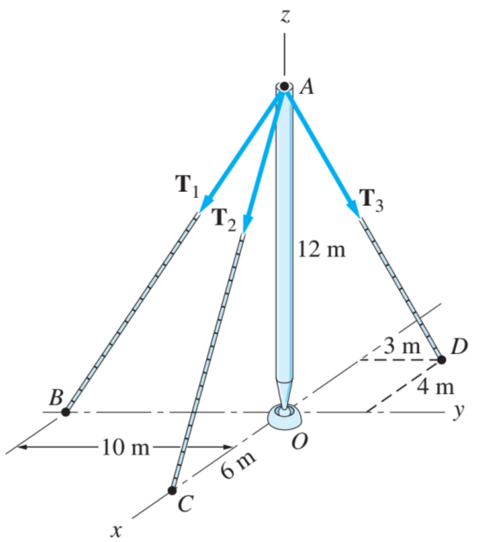


Good Luck

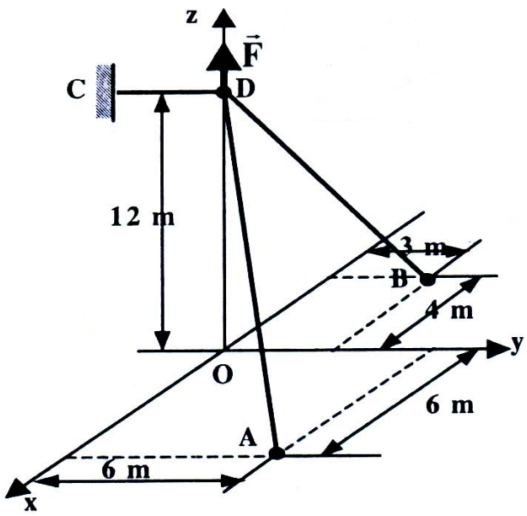
The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Mid Term Exam			
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiner:	**----------------**	**Time:**	**60 min**
Date:	**Oct. 2017**	**Group:**	**---------**

Answer the following questions:

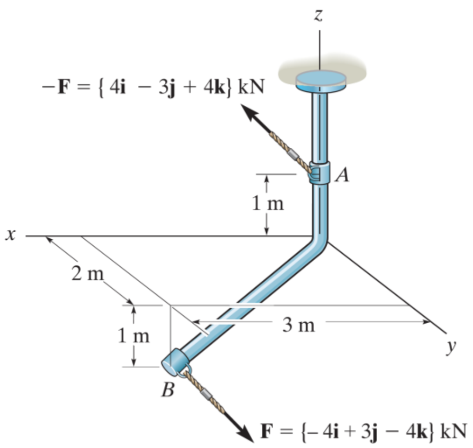
1. Three cable tensions T_1, T_2 and T_3 act at the top of the flagpole. Given that the resultant force for the three tensions is $F_R = -400\,\overrightarrow{k}\ (N)$, find the magnitudes of the cable tensions.



2. Three cables are jointed at point D where an upward force F of magnitude 6 KN is applied. Find the tension in each cable for equilibrium.



3. Express the moment of the couple acting on the rod in Cartesian vector form. What is the magnitude of the couple moment?

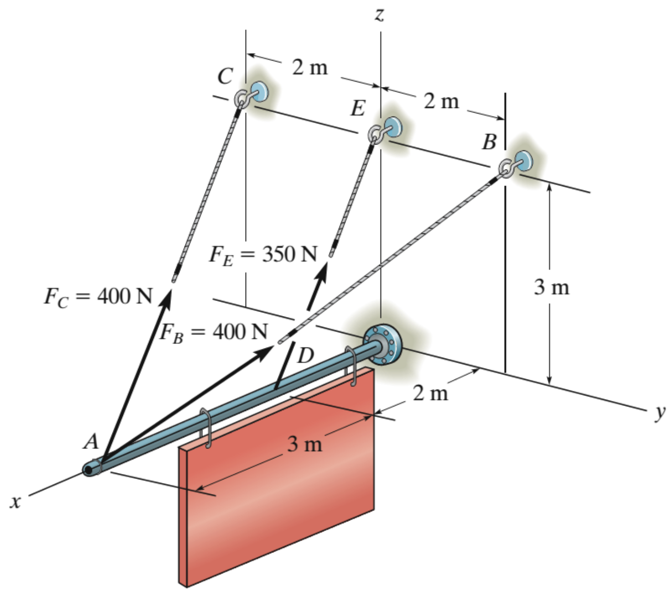


Good Luck

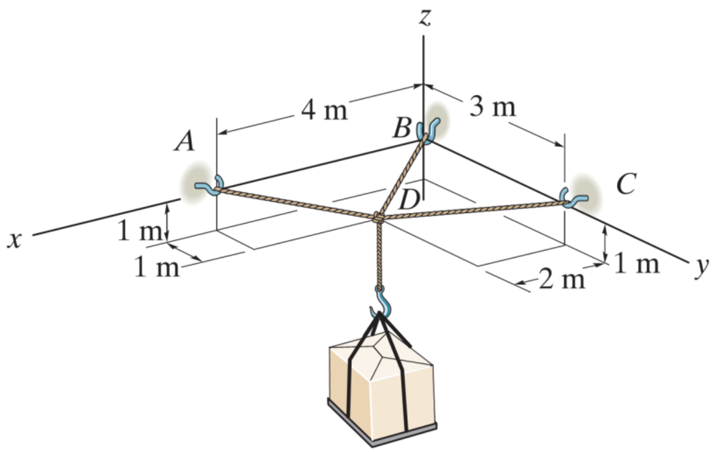
The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Mid Term Exam			
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiner:	**----------------**	**Time:**	**60 min**
Date:	**Oct. 2017**	**Group:**	**---------**

Answer the following questions:

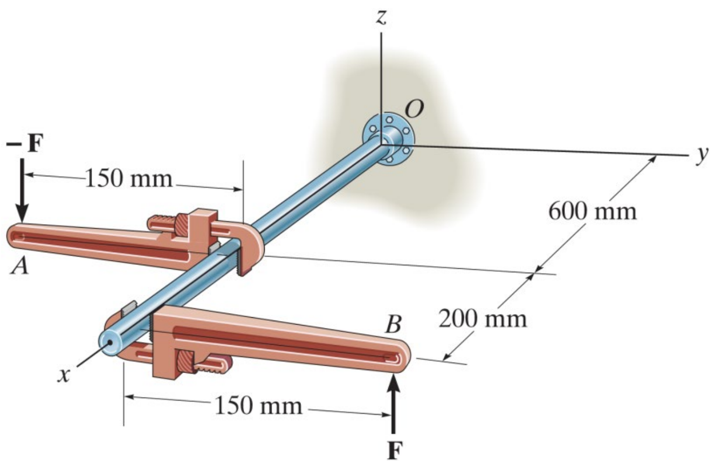
1. The three supporting cables exert the forces shown on the sign. Determine the magnitude and coordinate direction angles of the resultant force.



2. The crate has a mass of 130 kg. Determine the tension developed in each cable for equilibrium.



3. Express the moment of the couple acting on the pipe in Cartesian vector form. What is the magnitude of the couple moment? Take $F = 125$ N.

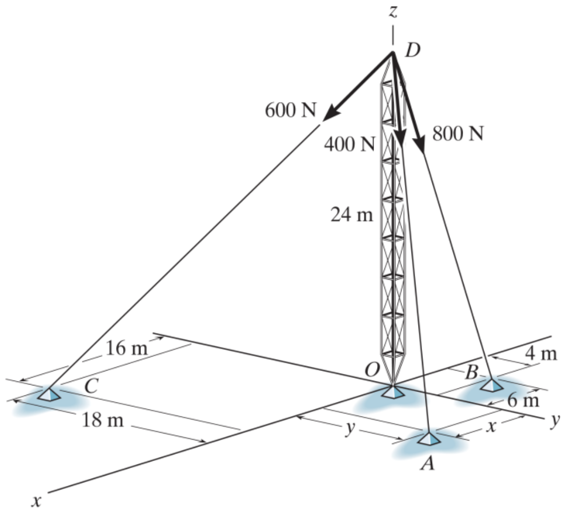


Good Luck

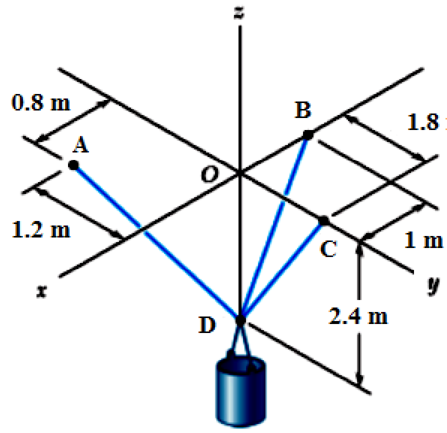
The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Mid Term Exam			
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiner:	**----------------**	**Time:**	**60 min**
Date:	**Oct. 2016**	**Group:**	**---------**

Answer the following questions:

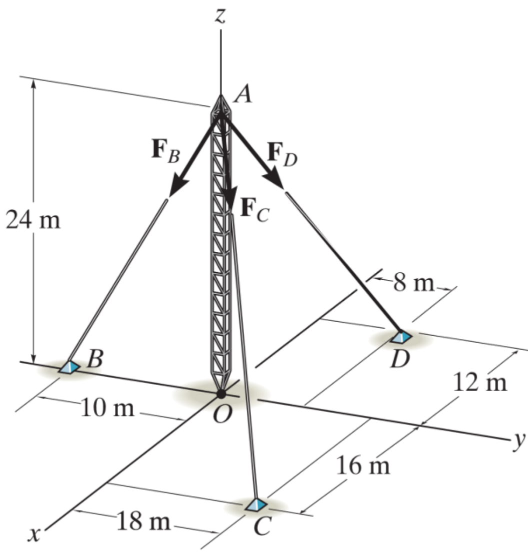
1. The tower is held in place by three cables. If the force of each cable on the tower is shown, determine the magnitude and coordinate direction angles of the resultant force. Take $x = 15\text{ m}$ and $y = 20\text{ m}$.



2. A cylinder of weight $W = 110\text{ N}$ is supported by three cables as shown. Determine the tension in each cable.



3. The tower is supported by three cables. If the forces of these cables acting on the tower are $F_B = 520\text{N}$, $F_C = 680\text{N}$ and $F_D = 560\text{N}$. Determine the resultant moment produced by forces and about point O. Express the result as a Cartesian vector.

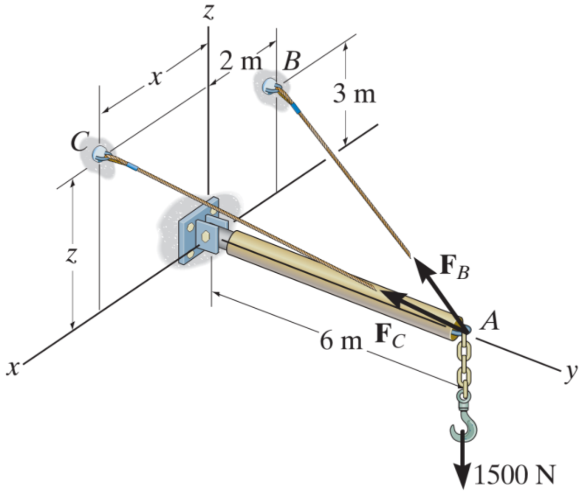


Good Luck

The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Mid Term Exam			
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiner:	**----------------**	**Time:**	**60 min**
Date:	**Oct. 2016**	**Group:**	**---------**

Answer the following questions:

1. Two cables are used to secure the overhang boom in position and support the $1500\ \text{N}$ load. If the resultant force is directed along the boom from point A towards O. determine the magnitudes of the resultant force and forces F_B and F_C. Set $x = 3\ \text{m}$ and $z = 2\ \text{m}$.



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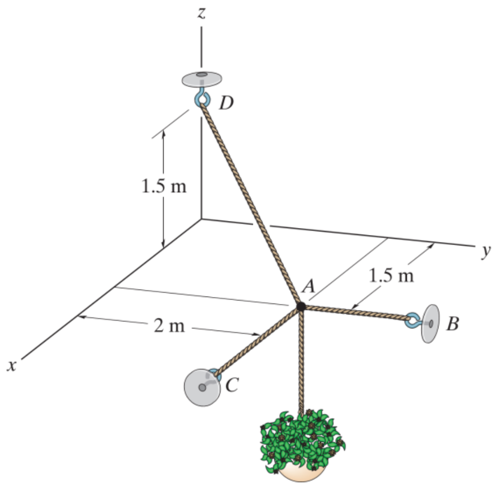
2. The three cables are used to support the $40\ \text{kg}$ flowerpot. Determine the force developed in each cable for equilibrium.



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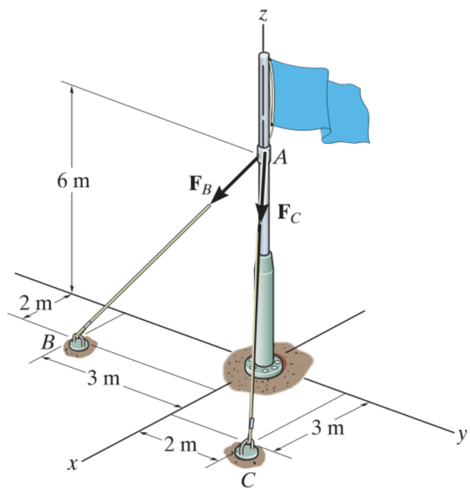
3. If $F_B = 560\ \text{N}$ and $F_C = 700\ \text{N}$, Determine the resultant moment produced by forces and about point O. Express the result as a Cartesian vector.



Good Luck

Appendix B

Final Examinations

The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Academic Year:	2022/2023	Term:	Second Term
Exam:	Final Written Examination	Time:	90 Min.
Subject:	Engineering Mechanics (1)	Code:	ENG 001
Examiners:	Examination Committee	Group:	30, 61, 62

Answer the following questions:

Question (1): (LO1., LO2.) (10 Marks)

The tower is supported by three cables and the forces of these cables are F_B = 300 N, F_C = 250 N and F_D = 400 N . Determine the magnitude and coordinate direction angles of the resultant force.

Question (2): (LO1., LO3.) (10 Marks)

Replace the five forces which acting along the edges of a rectangular block by a resultant force and couple moment at point O. Express the results in Cartesian vector form.

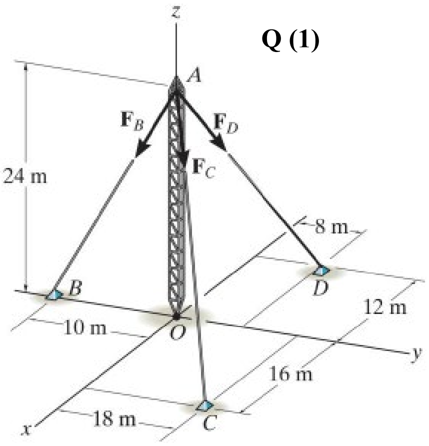
Question (3): (LO4., LO5.) (12 Marks)

The mast is held by a ball and socket at A and two cables CD and BE. The force F=100 kN is applied at H and is parallel to -X axis. Find the reaction at A and the tension in the cables CD and BE.

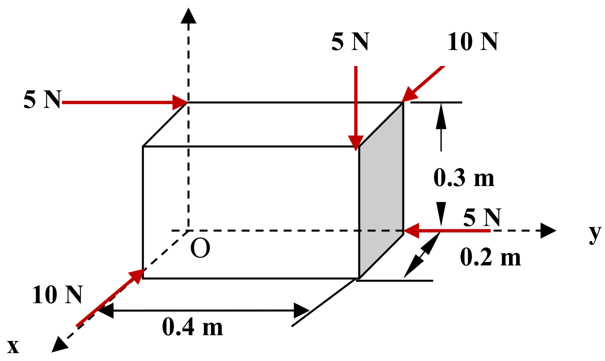
Question (4): (LO2.) (8 Marks)

For supported beam shown, determine the reactions at B and E.

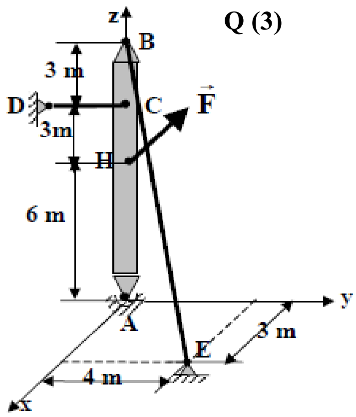
Q (1)



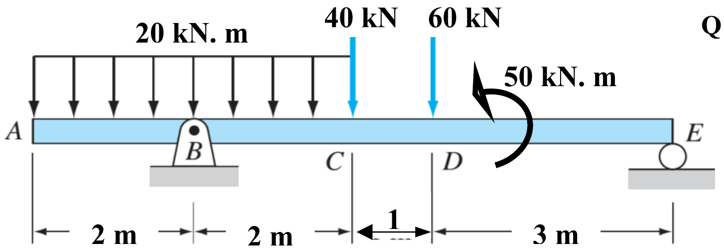
Q (2)



Q (3)



Q (4)



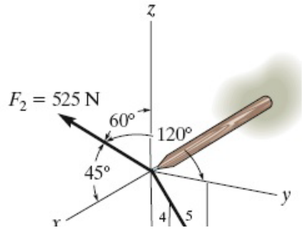
Good luck

Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Academic Year:	**2022/2023**	**Term:**	**Third Term**
Exam:	**Final Written Exam**	**Time:**	**90 min**
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiners:	**Examination Committee**	**Group:**	**1 & 25**

Answer All The Following Questions:

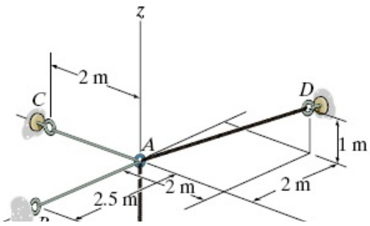
Question (1) (LO1., LO2.) (10 marks)

Determine the magnitude and coordinate direction angles of the resultants of the force.



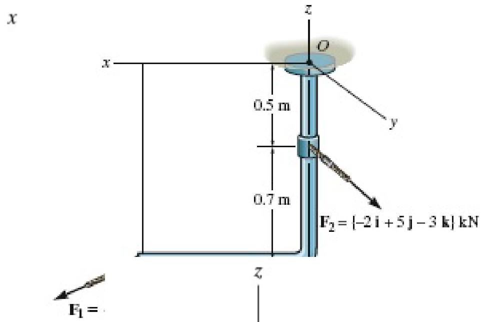
Question(2) (LO1.,LO2.) (10 marks)

Determine the tension in cables AB, AC and AD in order to support 100 kg crate in equilibrium position.



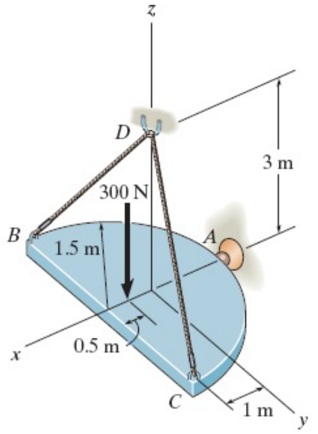
Question (3) (LO.1.,LO3.) (8 marks)

Replace the loadings by an equivalent resultant force and couple moment at point O.



Question (4) (LO4.,LO5.) (12 marks)

Determine the tensions in cables BD and CD and the reactions of the ball and socket support at A.



With my best wishes for success

The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Subject:	Engineering Mechanics (1)	Code:	ENG 001
Examiners:	Examination Committee	Time:	90 min
		Term:	Sep/Jan 2020-2021
Exam Type:	Final exam	Date:	Feb./ 2021

Answer the following questions:

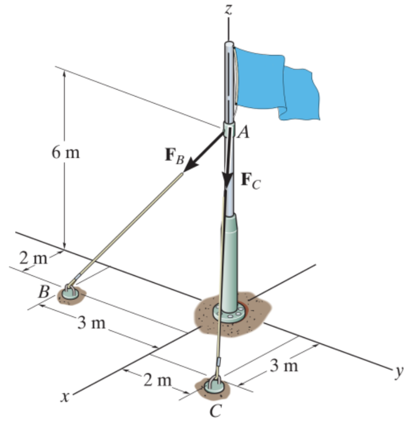
Question (1) [10 marks]

If $F_B = 560\,N$ and $F_C = 700\,N$, determine:

(a) The resultant force in cartesian vector.

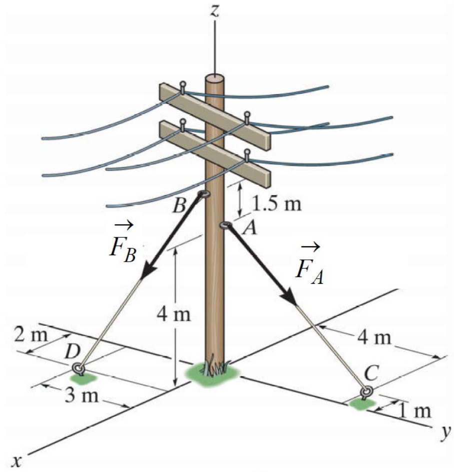
(b) The magnitude of the resultant force.

(c) The coordinate direction angles of the resultant force.



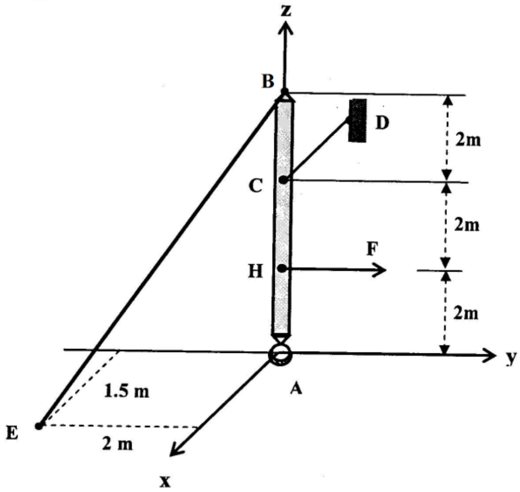
Question (2) [10 marks]

Two forces $\vec{F}_A = -40\,\vec{i} + 170\,\vec{j} - 180\,\vec{k}$ and $\vec{F}_B = 50\,\vec{i} - 80\,\vec{j} - 150\,\vec{k}$ acting on the post. **R**eplace the two forces by a resultant force and moment at point O. Express the results in Cartesian vector form.



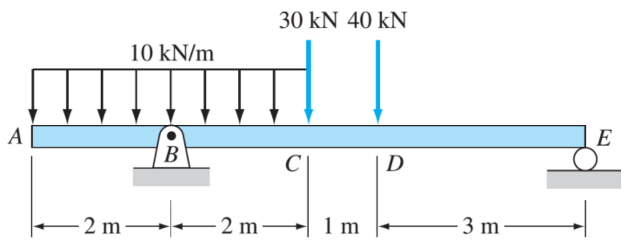
Question (3) [12 marks]

A tower AB is held in vertical position by a ball and socket at A and two cables **CD** and **BE**. An external force of magnitude $F = 600\,N$ is applied parallel to y-axis as shown. Determine the reaction components at A and the tension in each cable. Cable **CD** parallel to x-axis in the negative direction.



Question (4) [8 marks]

For supported beam shown, determine the reactions at B and E.

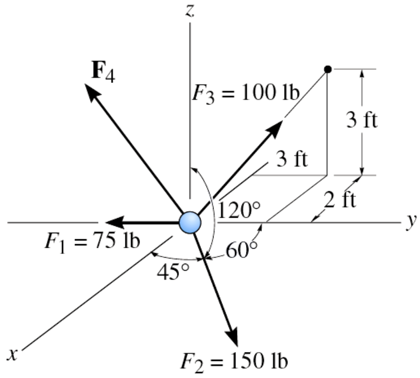


Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Academic Year:	2021/2022	Term:	First Term
Exam:	Final Written Examination	Time:	90 Min.
Subject:	Engineering Mechanics (1)	Code:	ENG 001
Examiners:	Examination Committee	Group:	1 : 40

Answer the following questions:

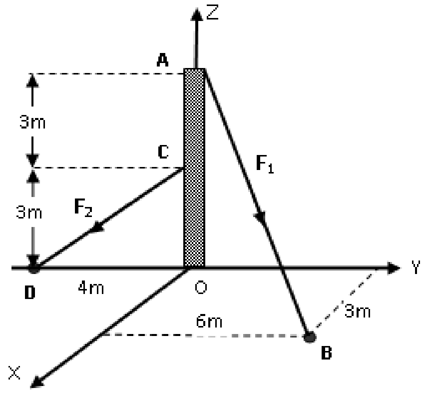
Question (1): **(10 Marks)**

Determine the magnitude and the coordinate direction angles of F_4 for equilibrium of the particle.



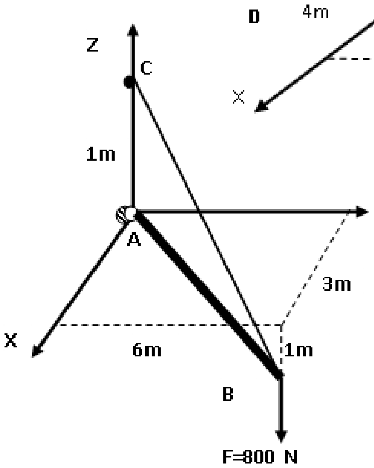
Question (2): **(10 Marks)**

Two forces F_1 and F_2 have magnitudes of 900 N and 500 N, respectively. Replace the two forces acting on the post by a resultant force and couple moment at point O.



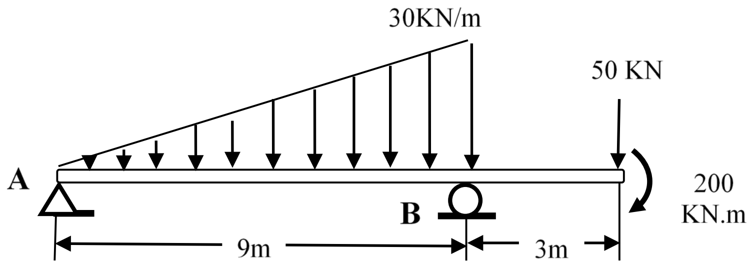
Question (3): **(12 Marks)**

For the shown Figure, determine the force in a cable BC and the reaction of the ball and Socket at A.



Question (4): **(8 Marks)**

For the shown overhanging beam, determine the reaction at A and B



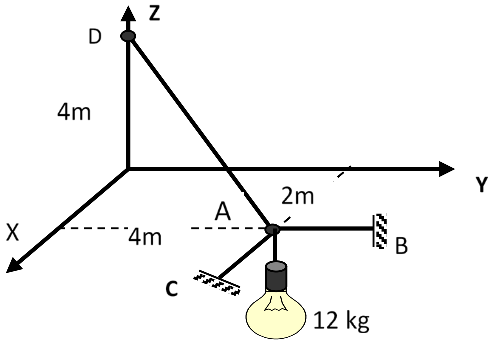
With my best wishes for success

The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Final Exam Evaluation			
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiner:	**Examination Committee**	**Time:**	**90 min**
Date:	**5 / 7 / 2020**	**Group:**	**30 & 51**

Answer the following questions:

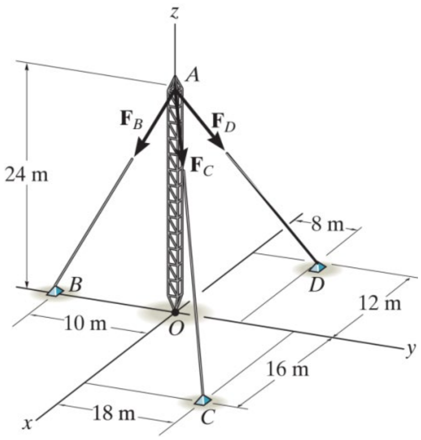
Question (1): [15 Marks]

12 kg lamp is supported by three cables AD, AB and AC, find the tension in each cable for equilibrium of point A.



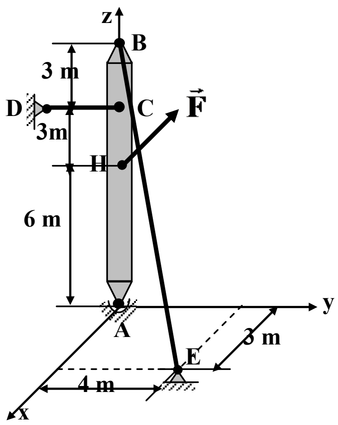
Question (2): [15 Marks]

The tower is supported by three cables. If the forces of these cables acting on the tower are $F_B = 520\text{ N}$, $F_C = 680\text{ N}$ and $F_D = 560\text{ N}$. Determine the resultant moment produced by forces about point *O*. and Express the result as a Cartesian vector.



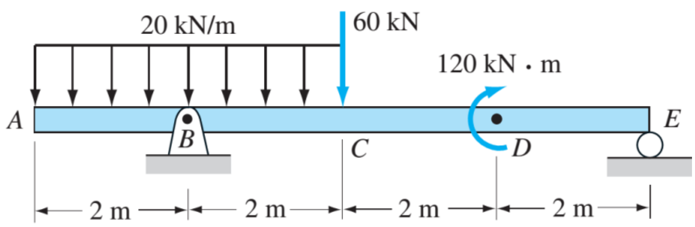
Question (3): [15 Marks]

A mast of height 12 m is held at A by a ball and socket and two cables CD and BE as shown. The weight of the mast can be neglected with respect to the force F = 20 kN applied at H and is parallel to x axis. Find the reaction at A and the tension in the cables.



Question (4): [15 Marks]

For supported beam shown, determine the reactions at B and E.

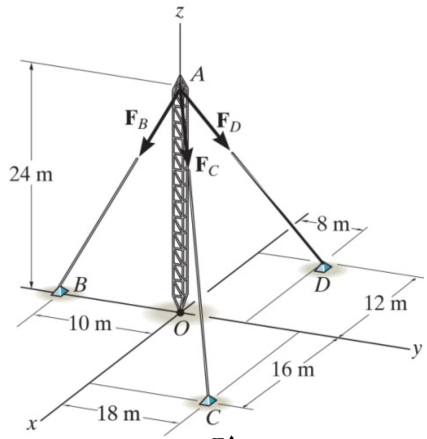


Good Luck

Examination Committee Signature	

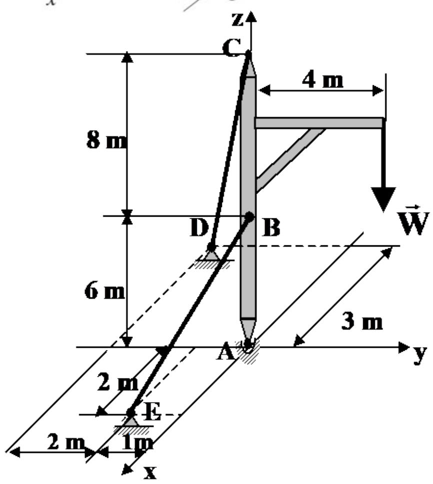
The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Final Exam Evaluation			
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiner:	**Examination Committee**	**Time:**	**90 min**
Date:	**7 / 5 / 2019**	**Group:**	**30,61 : 64**

Answer the following questions:



Question (1): [12 Marks]

The tower is supported by three cables. If the forces of these cables acting on the tower are $F_B = 520\ N$, $F_C = 680\ N$ and $F_D = 560\ N$. Determine the resultant moment produced by forces about point *O*. and Express the result as a Cartesian vector.

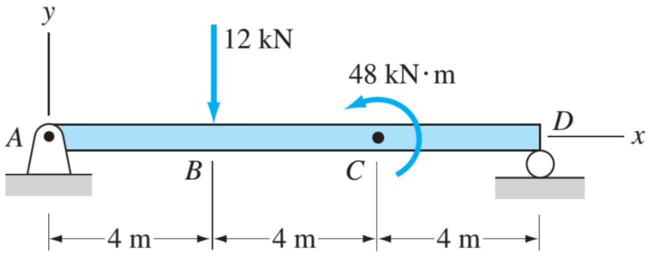


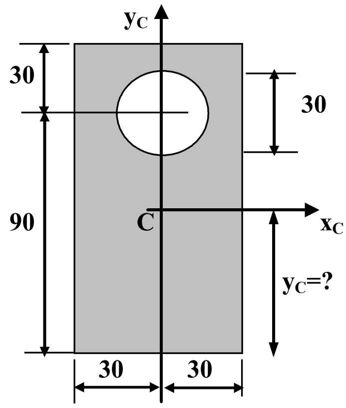
Question (2): [10 Marks]

The shown crane is used to lift a load W = 250 N. Determine the tensions in the cables CD and BE.

Question (3): [8 Marks]

Determine the reaction for the given loaded beam.





Question (4): [10 Marks]

For the given section, determine the location of the centroid, $\mathbf{y_c}$, and the principal centroidal moments of inertia. ($\mathbf{I_{xc}}$ and $\mathbf{I_{yc}}$).
(all dimensions are in mm).

Good Luck

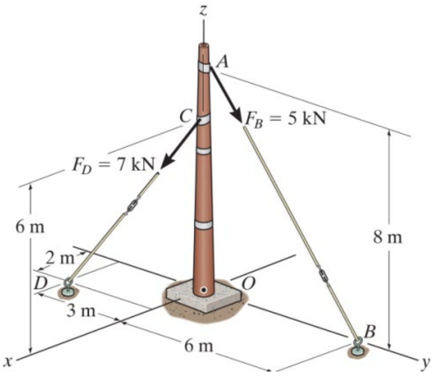
Examiner Committee Signature		

The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Final Exam Evaluation			
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiner:	**Examination Committee**	**Time:**	**90 min**
Date:	**29 / 12 / 2018**	**Group:**	**1 - 20**

Answer the following questions:

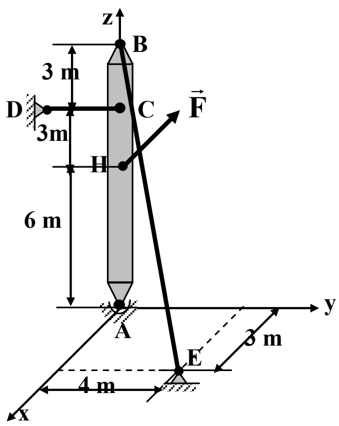
Question (1): [12 Marks]

Replace the two forces acting on the post by equivalent resultant force and couple moment at point **O**. Express the results in Cartesian vector form.



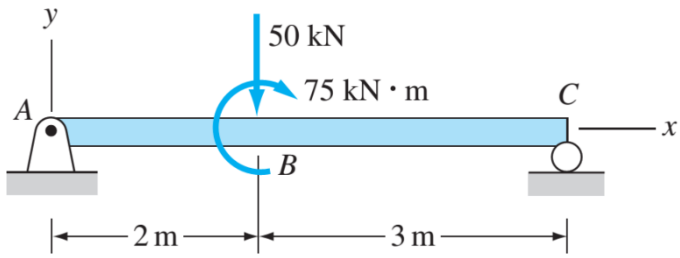
Question (2): [10 Marks]

A mast of height 12 m is held at A by a ball and socket and two cables CD and BE as shown. The weight of the mast can be neglected with respect to the force F = 20 kN applied at H and is parallel to x axis. Find the reaction at A and the tension in the cables.



Question (3): [8 Marks]

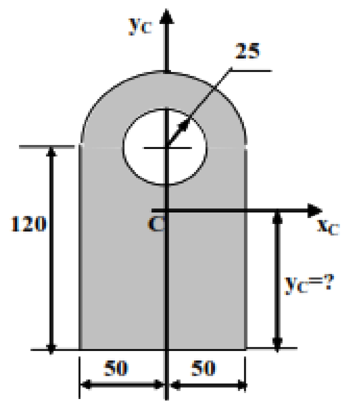
Determine the reaction for the given loaded beam.



Question (4): [10 Marks]

For the given section, determine the location of the centroid, y_c, and the principal centroidal moments of inertia. (I_{xc} and I_{yc}).

(all dimensions are in mm).



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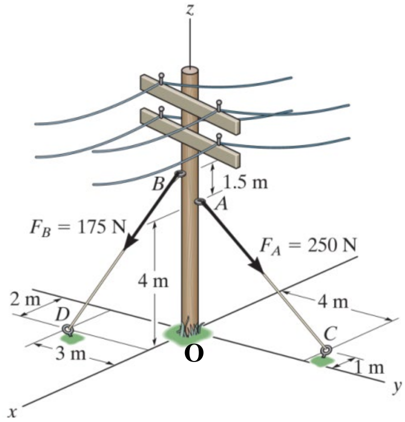
The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Final Exam Evaluation			
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiner:	**Examination Committee**	**Time:**	**90 min**
Date:	**29 / 12 / 2018**	**Group:**	**21 - 35**

Answer the following questions:

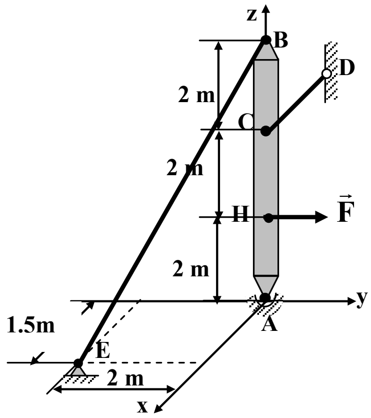
Question (1): **[12 Marks]**

Replace the two forces acting on the post by equivalent resultant force and couple moment at point **O**. Express the results in Cartesian vector form.



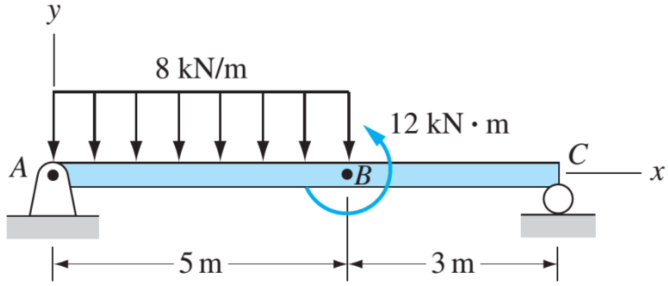
Question (2): **[10 Marks]**

A tower AB is held in vertical position by a ball-and-socket at A and two cables CD and BE. An external force of magnitude F = 600 N is applied as shown. Neglecting the weight of the tower, determine the reaction at A and the tension in each cable.



Question (3): **[8 Marks]**

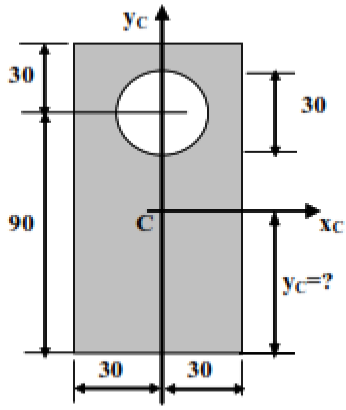
Determine the reaction for the given loaded beam.



Question (4): **[10 Marks]**

For the given section, determine the location of the centroid, $\mathbf{y_c}$, and the principal centroidal moments of inertia. ($\mathbf{I_{xc}}$ and $\mathbf{I_{yc}}$).

(all dimensions are in mm).



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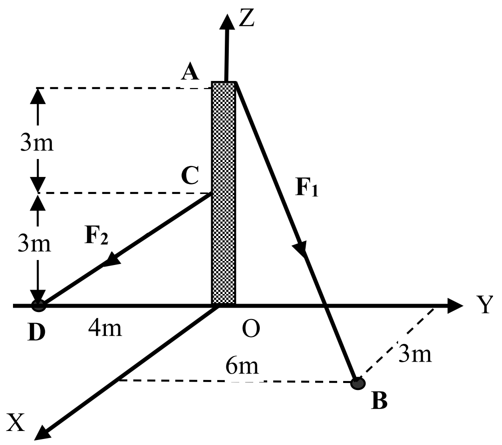
The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Final Exam Evaluation			
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiner:	**Examination Committee**	**Time:**	**90 min**
Date:	**31 / 7 / 2018**	**Group:**	**2**

Answer the following questions:

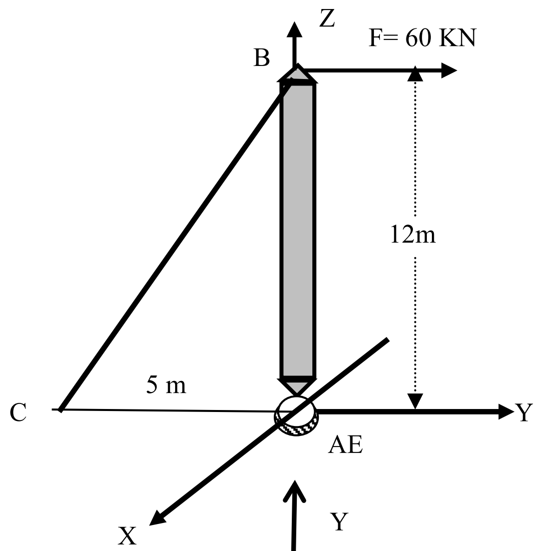
Question (1) **[12 marks]**



Two Forces $\vec{F}_1$ and $\vec{F}_2$ have magnitudes of 90 KN and 50 KN, respectively. Determine,

- i- The resultant of the two forces.
- ii- The magnitude of the resltant.
- iii- The coordinate direction angles of the resultant.
- iv- The equivelent force and moment of the two forces at O.
- v- The magnitude and coordinate direction angles of the above moment.

Question (2) **[10 marks]**

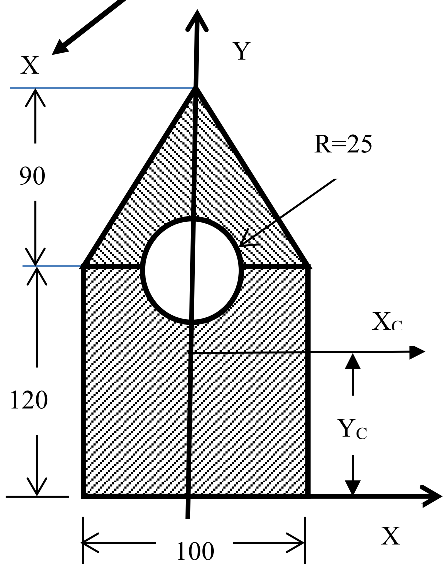


For the shown Figure, determine the force in a cable BC and the reaction of the ball and Socket at A.

Question (3) **[8 marks]**

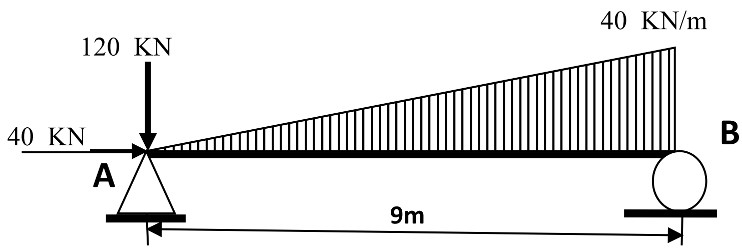
For simply supported beam shown, determine the reactions at A and B

Question (4) **[10 marks]**



a) Determine the centroid (Y_C) for the shown figure.

b) Determine the moment of inertia I_{XC} and I_{YC} for the shown figure.



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The Higher Technological Institute (HTI)			
Department of Mechanical Engineering			
Final Exam Evaluation			
Subject:	**Engineering Mechanics (1)**	**Code:**	**ENG 001**
Examiner:	**Examination Committee**	**Time:**	**90 min**
Date:	**12 / 5 / 2018**	**Group:**	**1 - 6**

Answer the following questions:

Question (1) **[10 marks]**

For the two Forces shown, determine the equivalent force and moment at O.

Question (2) **[10 marks]**

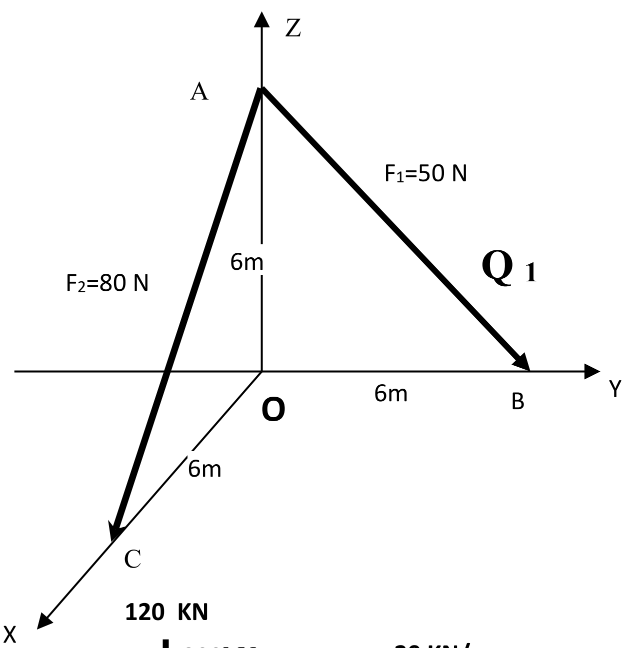
Find the Tension in BC, BD and the reaction for ball and socket at A.

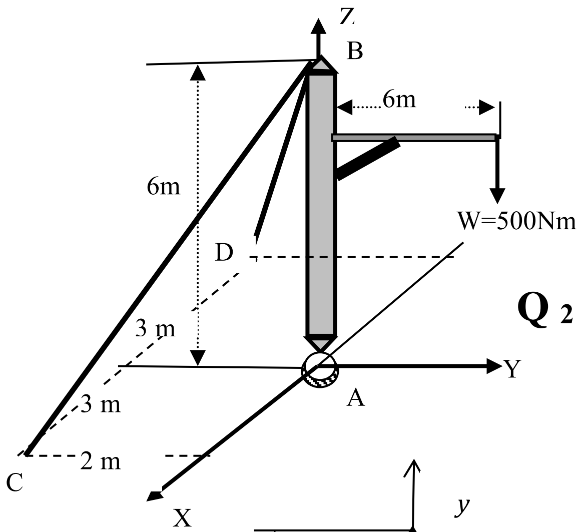
Question (3) **[10 marks]**

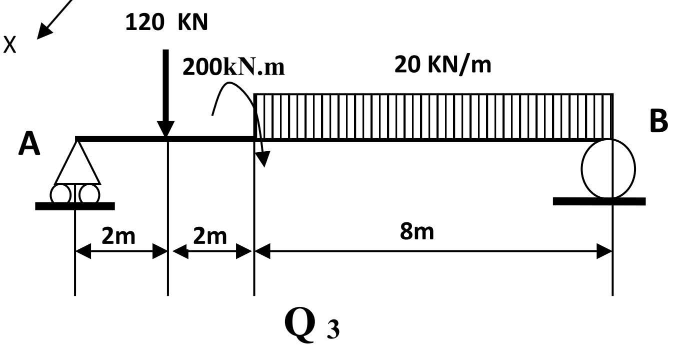
Find the reaction at A and B for the shown simply supported beam.

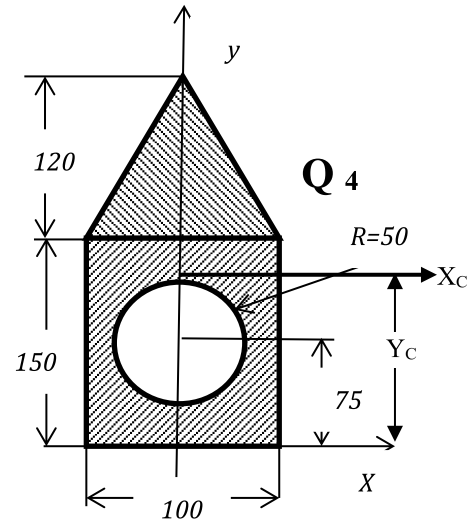
Question (4) **[10 marks]**

Find the Centroid (Y_C) and the moment of inertia (I_x, I_y) of the shown figure (all dimensions in mm).









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